Accelerating the resolution of sovereign debt models using an endogenous grid method

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Summary

- Endogenous default models don't benefit from advanced DSGE resolution techniques (FOC not enough: value function needed, regime switching)
- State-of-the art: slow value function iteration (VFI); equivalent to far away finite horizon
- Idea: use endogenous grid method (EGM) instead
- Features of this method:
  - Use fixed grid for *control* variable
  - Endogenously deduce grid for *state* variable
  - Strength: uses FOC $\Rightarrow$ no maximization (but nonlinear solver)
- Extension to sovereign debt models: grids for both state and control variables need be endogenous $\Rightarrow$ 2EGM
- 2EGM much faster than VFI for similar accuracy

Outline

A canonical sovereign debt model

State of the art

The doubly endogenous grid method (2EGM)

Assessment of VFI and 2EGM

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State of the art

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Assessment of VFI and 2EGM

Model setup

- Tradition of Eaton and Gersovitz (1981), Cohen and Sachs (1986)
- Sovereign country (with representative agent) produces and consumes
- Production is an exogenous stochastic stream
- Difference between production and consumption financed on international markets
  $\Rightarrow$ accumulation of a stock of (short-term) external debt
- The country can make the strategic decision to default
- Default implies financial autarky and cost on output
- Anticipating default, international markets may impose a (model-consistent) risk premium or ration the country

Output process

Output is non stationary, with AR(1) shocks to the stochastic growth trend:

$$\tilde{y}_t = g_t \, \tilde{y}_{t-1}$$

$$\log(g_t) = (1-\rho_g)\left(\log(\mu_g) - \frac{\sigma_g^2}{2(1-\rho_g^2)}\right) + \rho_g \log(g_{t-1}) + \varepsilon_t^g$$

where $\rho_g \in [0,1)$, $\varepsilon_t^g \rightsquigarrow \mathcal{N}(0, \sigma_g^2)$

Agent interactions

- If the sovereign repays:

$$\tilde{c}_t^G = \tilde{y}_t + \tilde{a}_t - \tilde{q}(\tilde{y}_t, \tilde{a}_{t+1})\tilde{a}_{t+1}$$

$$\tilde{V}^G(\tilde{a}_t, \tilde{y}_t) = \max_{\tilde{a}_{t+1}} \left\{ u(\tilde{c}_t^G) + \beta\, \mathbb{E}_t V(\tilde{a}_{t+1}, \tilde{y}_{t+1}) \right\}$$

- If the sovereign defaults:

$$\tilde{c}_t^B = \tilde{y}_t^B = (1-\delta)\tilde{y}_t$$

$$\tilde{V}^B(\tilde{y}_t) = u(\tilde{c}_t^B) + \beta\, \mathbb{E}_t \left[(1-\lambda)\tilde{V}^B(\tilde{y}_{t+1}) + \lambda\, \tilde{V}(0, \tilde{y}_{t+1}) \right]$$

- Optimal choice between repayment and default:

$$\tilde{V}(\tilde{a}_t, \tilde{y}_t) = \max\{\tilde{V}^G(\tilde{a}_t, \tilde{y}_t), \tilde{V}^B(\tilde{y}_t)\}$$

$$\tilde{D}(\tilde{a}_t, \tilde{y}_t) = \mathbb{1}_{\tilde{V}^G(\tilde{a}_t, \tilde{y}_t) < \tilde{V}^B(\tilde{y}_t)}$$

- Investors' zero profit condition (pins down the risk-adjusted interest rate):

$$(1+r)q(\tilde{y}_t, \tilde{a}_{t+1}) = \mathbb{E}_t \left[1 - \tilde{D}(\tilde{a}_{t+1}, \tilde{y}_{t+1}) \right]$$

Detrended model

- Detrending factor:
$$\tilde{\Gamma}_t = \mu_g \tilde{y}_{t-1}$$
- In particular:
$$y_t = \frac{g_t}{\mu_g}$$
- Detrended equations:
$$V(a_t, y_t) = \max\{V^G(a_t, y_t), V^B(y_t)\}$$

$$V^G(a_t, y_t) = \max_{a_{t+1}} \Big\{u(y_t + a_t - q(y_t, a_{t+1})a_{t+1}\, g_t) \\ + \beta\, g_t^{1-\gamma}\, \mathbb{E}_t V(a_{t+1}, y_{t+1})\Big\}$$

$$V^B(y_t) = u((1-\lambda)y_t) + \beta\, g_t^{1-\gamma}\, \mathbb{E}_t \left[(1-\lambda)V^B(y_{t+1}) + \lambda\, V(0, y_{t+1})\right]$$

$$D(a_t, y_t) = \mathbb{1}_{V^G(a_t, y_t) < V^B(y_t)}$$

$$(1 + r)q(y_t, a_{t+1}) = \mathbb{E}_t \left[1 - D(a_{t+1}, y_{t+1})\right]$$

Outline

Value Function Iteration (VFI)

1. Define an interpolation grid $(a_i, y_j)_{(i,j)\in I\times J}$
2. Let $n = 0$, initialize $\hat{V}^{G,(0)}$ and $\hat{V}^{B,(0)}$
3. At each point of the grid, compute $\hat{V}^{G,(n+1)}$ and $\hat{V}^{B,(n+1)}$ by

   $$\hat{V}^{G,(n+1)}(a_i, y_j) = \max_{a'}\{u(y_j + a_i - \hat{q}^{(n+1)}(y_j, a')a'g_j) + \beta g_j^{1-\gamma} \int \hat{V}^{(n)}(a', y')\mathrm{d}F(y'|y_j)\}$$

   $$\hat{V}^{B,(n+1)}(y_j) = u((1-\delta)y_j) + \beta g_j^{1-\gamma} \int \left[(1-\lambda)\hat{V}^{B,(n)}(y') + \lambda\, \hat{V}^{(n)}(0, y')\right] \mathrm{d}F(y'|y_j)$$

   Involves the computation of an integral and a function maximization. Also requires computation of price function $\hat{q}^{(n+1)}$
4. If $\hat{V}^{(n+1)}$ close to $\hat{V}^{(n)}$, stop. Otherwise, let $n = n + 1$ and goto 3

Implementations of VFI

- Discrete State Space (DSS): no interpolation, discretize the state space, the control space and the law of motion of growth
- Alternatively, interpolation with cubic splines
- Hatchondo et al. (2010):
  - DSS is both inefficient and imprecise compared to cubic splines
  - some papers have *qualitatively* wrong results because of DSS
- VFI is slow because it needs an optimization at every point of the grid, at every iteration

Outline

Overview of EGM

- Introduced by Carroll (2006)
- Extended by Fernandez & Villaverde (2007)
- Backward computation of value function, as in VFI
- But uses FOC (Euler equation) instead of objective maximization (Bellman equation)
- As a consequence, much faster
- Euler equation of the canonical model:

$$u'(c_t)\left[q(y_t, a_{t+1}) + a_{t+1}\frac{\partial q}{\partial a_{t+1}}(y_t, a_{t+1})\right] g_t = \beta g_t^{1-\gamma}\mathbb{E}_t \frac{\partial V}{\partial a_{t+1}}(a_{t+1}, y_{t+1})$$

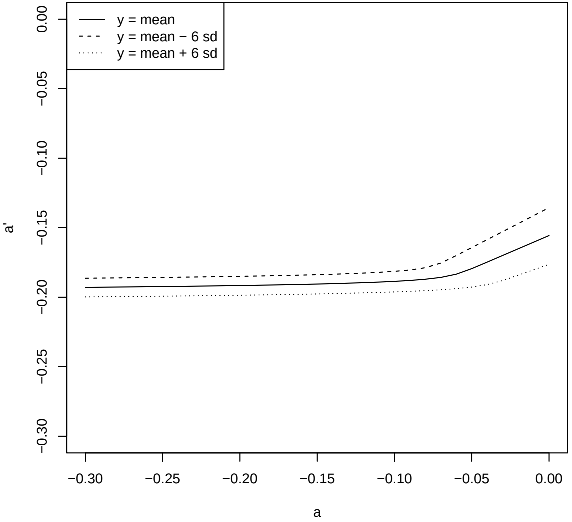
where $c_t = y_t + a_t - q(y_t, a_{t+1})a_{t+1}g_t$

EGM on canonical model

1. Define a fixed grid for *tomorrow*'s assets $(a'_i)_{i \in I}$, and one for today's output $(y_j)_{j \in J}$
2. Let $n = 0$. Choose initial values for $\hat{V}^{G,(0)}$ and $\hat{V}^{B,(0)}$. Choose initial grid, $\left(a_{ij}^{(0)}, y_j\right)_{(i,j) \in I \times J}$ for $\hat{V}^{G,(0)}$. The grid $a_{ij}^{(n)}$ will vary, hence the name of EGM
3. Compute $\hat{V}^{B,(n+1)}$ as in VFI (backward iteration)
4. Compute $\hat{V}^{G,(n+1)}$: for every (a'_i, y_j), use Euler equation to find c consistent with a'_i (involves a nonlinear solver and two numerical differentiations but *no maximization*)
5. Deduce today's assets $a_{ij}^{(n+1)}$ with resource constraint, and deduce $\hat{V}^{G,(n+1)}$ at $(a_{ij}^{(n+1)}, y_j)$
6. If $\hat{V}^{(n+1)}$ close to $\hat{V}^{(n)}$, stop. Otherwise, set $n = n + 1$ and goto 3

Why EGM fails on canonical sovereign debt model

Choice function for tomorrow's level of debt, given today's level



2EGM

- Idea: make the grid for tomorrow's assets also endogenous
- Iteratively adapt that grid so that it converges towards ergodic set
- More precisely, find upper and lower limits for tomorrow's assets, so that today's assets fall in the endogenous grid of previous period
- Implementation: dichotomy-based algorithm
- Grid for both today's and tomorrow's assets is endogenous, hence the 2EGM name
- Extra bonus: approximation of the solution only computed on ergodic set

Outline

Comparison devices

- Comparison dimensions: speed, ease of implementation, accuracy (moments, average Euler errors)
- On the canonical model
- And on the "trembling times" model of Cohen and Villemot (2012)
  - Growth has a Brownian and a Poisson component
  - Poisson component = exogenous risk of being hit by a confidence shock which has real negative consequences
  - Confidence can be restored if no default during crisis $\Rightarrow$ markets act like a "trembling hand"
  - Recovery value for investors in case of default $\Rightarrow$ raises sustainable debt-levels
  - State space of dim. 3, shocks of dim. 2

Comparison results

Model	*Canonical*		*Trembling*	
Solution characteristics				
Method	VFI	2EGM	VFI	2EGM
Grid points	15×30	15×30	10^3	10^3
Convergence criterion	10^{-6}	10^{-6}	$10^{-1.7}$	10^{-3}
Lines of C++ code	1,000	1,080	1,423	1,525
Solution time				
Single thread	54.4s	5.8s	3,588s	413s
8 threads	15.9s	3.1s	1,396s	195s
Moments				
Rate of default (%, per year)	0.86	0.86	1.24	2.50
Mean D/Q (%, annualized)	4.68	4.68	38.58	38.17
Euler errors (in $\log_{10}$ *units)*				
Mean	-4.38	-4.20	-1.99	-2.08
Max	-3.47	-3.39	-0.98	-0.46

Conclusion

- 2EGM faster than VFI by a factor between 5 and 10, for same accuracy level
- Same complexity of implementation
- Future work:
  - Merging of DSGE elements (following Mendoza and Yue, 2012)
  - Computationally: sparse grid methods

Thanks for your attention!

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