The sovereign default puzzle:
A new approach to debt sustainability analysis

Frankfurt joint lunch seminar

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February 20, 2013

Outline

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Goals

- Need for models of debt sustainability analysis (DSA)
- Rich literature on the modeling of sovereign default, with both willingness and ability to repay taken into account
- Delivers rich theoretical insights and good quantitative fit for business cycles of emerging countries
- But fails at delivering realistic debt levels and default incidence, and therefore useless for DSA
- Goal of the present paper: make progress towards DSA-relevant and theoretically-grounded sovereign default models

Canonical model (1/2)

- Tradition of Eaton and Gersovitz (1981), Cohen and Sachs (1986)
- Sovereign country (with representative agent) produces and consumes
- Production is an exogenous stochastic stream
- Difference between production and consumption financed on international markets
  $\Rightarrow$ accumulation of a stock of (short-term) external debt
- The country can make the strategic decision to default
- Default implies financial autarky and cost on output
- Anticipating default, international markets may impose a (model-consistent) risk premium or ration the country

Canonical model (2/2)

- In case of repayment:

$$C_t^r = Q_t - D_t + \tilde{L}(Q_t, D_{t+1})$$

$$J^r(D_t, Q_t) = \max_{D_{t+1}} \left\{ u(Q_t - D_t + \tilde{L}(Q_t, D_{t+1})) + \beta\, \mathbb{E}_t J^*(D_{t+1}, Q_{t+1}) \right\}$$

- In case of default:

$$C_t^d = Q_t^d = (1-\lambda)Q_t$$

$$J^d(Q_t) = u((1-\lambda)Q_t) + \beta\, \mathbb{E}_t \left[(1-x)J^d(Q_{t+1}) + x\, J^*(0, Q_{t+1}) \right]$$

- Optimal choice between repayment and default:

$$J^*(D_t, Q_t) = \max\{J^r(D_t, Q_t), J^d(Q_t)\}$$

$$\tilde{\delta}'(D_t, Q_t) = \mathbb{1}_{J^r(D_t, Q_t) < J^d(Q_t)}$$

- Investors' zero profit condition (pins down the risk-adjusted interest rate):

$$(1+r)\tilde{L}(Q_t, D_{t+1}) = \mathbb{E}_t \left[1 - \tilde{\delta}'(D_{t+1}, Q_{t+1}) \right] D_{t+1}$$

Quantitative sovereign debt models

- Recent trend in the litterature: match quantitative facts with these models (Aguiar and Gopinath, 2006; Arellano, 2008)
- Success for business cycle statistics of emerging countries
  - countercyclical current account
  - countercyclical interest rates
  - consumption more volatile than output
- But failure with respect to debt-to-GDP ratios and default probabilities!
  - either debt ratios too high and probability of default too low...
  - ...or the contrary
  - consequence of the default cost assumed

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The sovereign default puzzle

Paper	Main features	Debt-to-GDP mean ratio (%, annual)	Default probability (%, annual)
Arellano (2008)	Non-linear default cost	1	3.0
Aguiar & Gopinath (2006)	Shocks to GDP trend	5	0.9
Cuadra & Sapriza (2008)	Political uncertainty	2	4.8
Fink & Scholl (2011)	Bailouts and conditionality	1	5.0
Yue (2010)	Endogenous recovery	3	2.7
Mendoza & Yue (2011)	Endogenous default cost	6	2.8
Hatchondo & Martinez (2009)	Long-duration bonds	5	2.9
Benjamin & Wright (2009)	Endogenous recovery	16	4.4
Chatterjee & Eyigungor (2011)	Long-duration bonds	18	6.6

One would want:

- debt-to-GDP ratio of (at least) 40% of yearly GDP
- annual default probability of 3%

Intuition for solving the puzzle

- In previous models, default frequency and debt levels both determined by a single parameter (cost of default), hence the trade-off
  $\Rightarrow$ need to disconnect the two
- Idea: defaults come *after* a crisis, not the other way round:
  - Default is a decision of the markets, not of the country
  - No such thing as "strategic default" (except Ecuador 2009)
  - Unfoldment of events: crisis $\Rightarrow$ default $\Rightarrow$ extra default costs
  - But extra default costs are lower than in "normal times": the crisis "pre-pays" for the default
  - Makes it possible to have both high default frequencies and high debt levels
- Modeling tool for the eruption of a crisis: Poisson process

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Lévy processes and default

- Brownian process: frequent and infinitesimally small jumps
- Poisson process: infrequent but discrete jumps
- Lévy processes:
  - Lévy process $\simeq$ Brownian process + compound Poisson process
  - generalization in continuous time of random walks
- Theorem:
  - no default if output is a (discretized) Brownian process
  - Brownian motion analog to deterministic case
  - only the Poisson component generates defaults

Discretized Lévy processes

h is the length of a period (continuous time is $h \to 0$)

The Cox-Ross-Rubinstein (CRR) case

$$Q_{t+h} = \begin{cases} e^{\sigma\sqrt{h}} Q_t & \text{with probability } \frac{1}{2} + \frac{\mu}{2\sigma}\sqrt{h} \\ e^{-\sigma\sqrt{h}} Q_t & \text{with probability } \frac{1}{2} - \frac{\mu}{2\sigma}\sqrt{h} \end{cases}$$

As $h \to 0$, converges towards geometric Brownian process of "percentage drift" μ and "percentage volatility" σ

The Poisson case

$$Q_{t+h} = \begin{cases} Q_t & \text{with probability } e^{-p_0 h} \\ k \cdot \tilde{m}_t Q_t & \text{with probability } 1 - e^{-p_0 h} \end{cases}$$

where $\tilde{m}$ has support in $(0, 1)$ and $k = \frac{p_0 h}{1 - e^{-p_0 h}}$.
As $h \to 0$, converges towards geometric compound Poisson process (of rate p_0 and jump size distribution $\tilde{m}_t$)

Default with a Lévy process

- The rest of the model is like the canonical one (except that there is no possibility of redemption)
- Two polar cases for GDP: CRR or Poisson

Theorem (no default in CRR)

In the CRR case, if $h < \frac{1}{\left(\frac{\mu}{\sigma}+4\sigma\right)^2}$, only two cases are possible (for a given initial value of the debt-to-GDP ratio):

- the country immediately defaults;
- the country never defaults (whatever the future path of output).

Theorem (default possible in Poisson)

In the Poisson case, the probability of default between dates t and $t+1$ is inferior to $1 - e^{-p_0}$. The upper bound is reached for some parameter combinations

Simulating the model

Calibration, quarterly

Risk aversion	γ	2
Discount rate	ρ	$\log(0.8)$
Riskless interest rate	r	$\log(1.01)$
Loss of output in autarky (% of GDP)	λ	0.5%
Drift of CRR process	μ	1%
Volatility of CRR process	σ	2.2%
Period size for which CRR and Poisson equivalent	h_0	4

In CRR, no default for $h < h^* \simeq 3.4$ (almost one year)

Simulating the model

Results

Period duration (h, in quarters)	4	2	1	0.33
CRR process				
Default threshold (debt-to-GDP, quarterly, %)	48.4	51.9	68.8	79.3
Default probability in 10 years (%)	35.7	0.0	0.0	0.0
Discretized Poisson process				
Default threshold (debt-to-GDP, quarterly, %)	48.4	47.7	47.6	47.5
Default probability in 10 years (%)	35.1	34.6	34.3	40.0

- Simulation results confirm the theoretical ones
- Note: does not aim at reproducing quantitative facts

Does this generalize to continuous time?

- Undergoing work with Sylvain Carré
- Preliminary answer: no
- But this is because of pathological reasons: a (geometric) Brownian process can go to 0 almost instantly
- Highly improbable events, so the default probability must still be very small
- Quantification work to come

Typology of debt crises

1. Failure to adjust in real time to a smooth shock
   $\Rightarrow$ the solution is to have a more efficient monitoring of intra-annual deficit (when $\mu/\sigma \simeq 1$, the time window is one month)
2. A discontinuous shock
   $\Rightarrow$ this is the real challenge

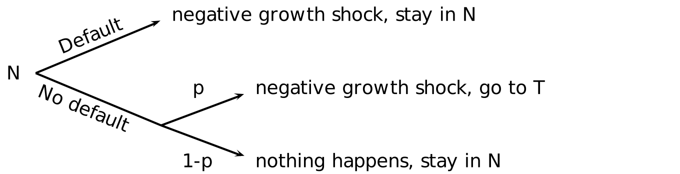
Previous models did not take this distinction into account.

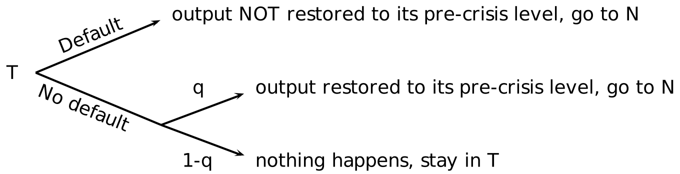
Outline

Model outline

- Growth has a Brownian and a Poisson component
- Brownian component = usual business cycle AR(1) process
- Poisson component = exogenous risk of being hit by a confidence shock which has real and lasting negative consequences
- Confidence can be restored if no default during crisis
  $\Rightarrow$ markets act like a "trembling hand"
- Regime switching model in the spirit of Hamilton (1989)
- Recovery value for investors in case of default
  $\Rightarrow$ raises sustainable debt-levels

Law of motion of the economy





N is "normal times", T is "trembling times"
p is the probability of a confidence shock, q that of a confidence restoration

The growth rate

Growth equal to:

$$g_t = e^{y_t} + z_t$$

Brownian component:

$$y_t = \mu_y + \rho_y(y_t - \mu_y) + \varepsilon_t \qquad \varepsilon_t \rightsquigarrow \mathcal{N}(0, \sigma_y^2)$$

Poisson component: (μ_z is the size of the shock on impact)

State in $t-1$	If repayment in $t-1$	If default in $t-1$
Normal (N)	$\begin{cases} z_t = \rho_z z_{t-1} & \text{prob. } 1-p \\ z_t = \rho_z z_{t-1} - \mu_z & \text{prob. } p \end{cases}$	$z_t = \rho_z z_{t-1} - \mu_z$
Trembling (T)	$\begin{cases} z_t = \rho_z z_{t-1} & \text{prob. } 1-q \\ z_t = \rho_z z_{t-1} + \mu_z & \text{prob. } q \end{cases}$	$z_t = \rho_z z_{t-1}$

Calibration

Risk aversion	γ	2
Discount factor	β	0.95
World riskless interest rate	r	1%
Probability of settlement after default	x	10%
Loss of output in autarky (% of GDP)	λ	2%
Probability of entering "trembling times"	p	1.5%
Probability of exiting "trembling times"	q	5%
Recovery value (% of yearly GDP)	V	25%
Size of "Poisson" shock to growth	μ_z	1%
Auto-correlation of "Poisson" component of growth	ρ_z	0.8
Mean of "Brownian" component of growth	μ_g	1.006
Standard deviation of "Brownian" component of growth	σ_y	3%
Auto-correlation of "Brownian" component of growth	ρ_y	0.17

Resolution method

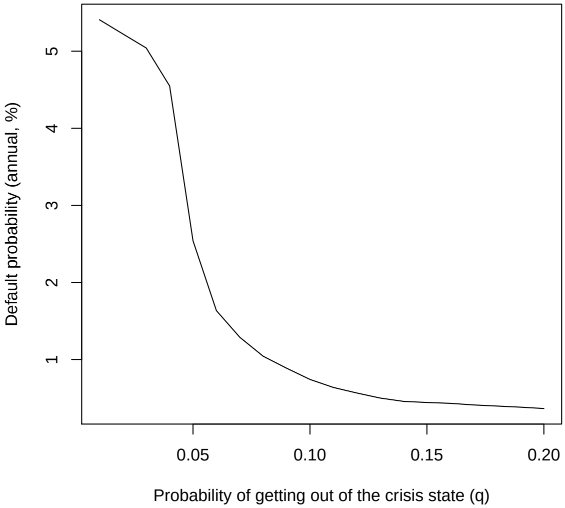
- State space of dimension 3: (D, y, z)
- 4 value functions: default versus repayment, normal versus trembling times
- Special care has been given to the numerical solution, given the problems raised by Hatchondo et al. (RED, 2010)
- Value function iteration too slow (curse of dimensionality) and imprecise
- Use of an extension of the endogenous grid method
- For more details, see Villemot (2012)

Simulated moments

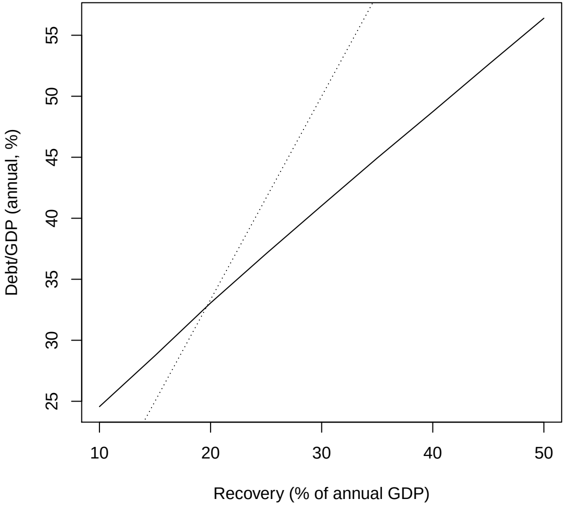
	Benchmark	With no Poisson
Rate of default (%, per year)	2.50	0.26
Mean debt output ratio (%, annualized)	38.17	46.82
$\sigma(Q)$ (%)	4.45	4.42
$\sigma(C)$ (%)	6.04	6.89
$\sigma(TB/Q)$ (%)	2.63	3.47
$\sigma(\Delta)$ (%)	0.57	0.18
$\rho(C, Q)$	0.92	0.89
$\rho(TB/Q, Q)$	-0.41	-0.49
$\rho(\Delta, Q)$	-0.60	-0.41
$\rho(\Delta, TB/Q)$	0.64	0.90

TB = trade balance, Δ = risk premium

Default probability, as a function of q



Mean debt-to-GDP as a function of recovery V



Self-fulfilling reinterpretation

- When q is low, Poisson shocks always trigger a default
- A self-fulfilling reinterpretation becomes possible, à la Cole and Kehoe (1996, 2000)
- Suppose two equilibria are possible:
  - a "bad" equilibrium where investors think the country will default and whose panic destroy the country's fundamental, self-fulfillingly making the country default
  - a "good" equilibrium, where investors think that the country will repay and where the country therefore repays
- For low values of q, the Poisson shock can therefore be reinterpreted as a sunspot, triggering the coordination on the "bad" equilibrium

Outline

Analysis at business cycle frequencies

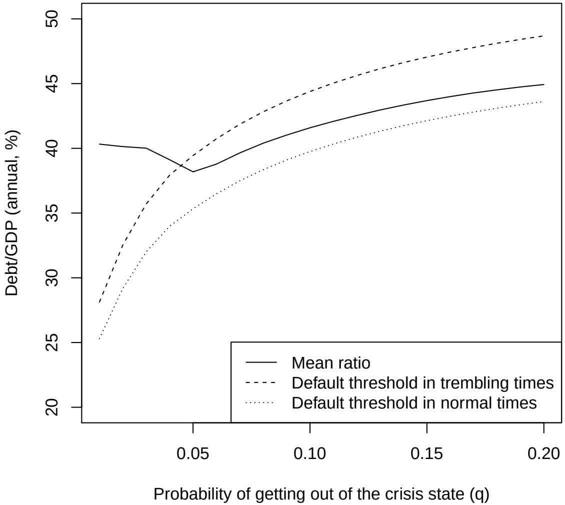
- Assume here that the switch between normal and trembling times corresponds to the business cycle
- Hamilton (1989) on US data for 1952–1984:
  $p = 9.5\%$ and $q = 24.5\%$
- Goodwin (1993) on 8 advanced economies for 1960–2000:
  $p \in [1\%, 9\%]$, $q \in [21\%, 49\%]$
- Model simulations:

p (quarterly)	1%	1%	10%	10%
q (quarterly)	20%	50%	20%	50%
Rate of default (yearly)	0.38%	0.27%	0.32%	0.29%
Mean D/Q (annualized)	45%	47%	43%	46%

$\Rightarrow$ trembling times for debt crises are less frequent and more severe downturns than are business cycles downturns

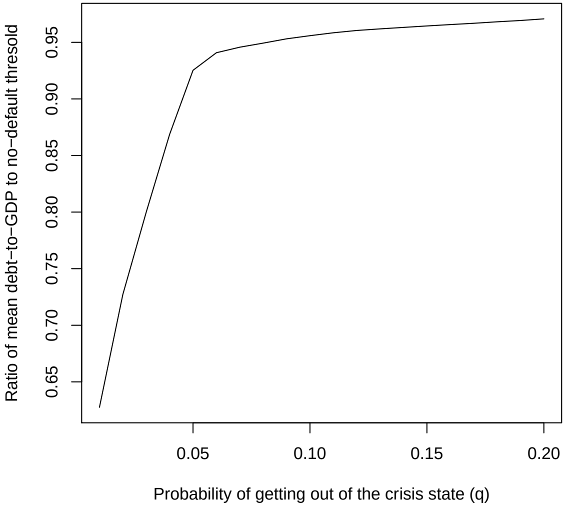
Mean debt-to-GDP and credit ceilings

As function of q



Credit ceilings

As a fraction of equilibrium levels in normal times



Welfare costs of imposing credit ceilings

Calculation à la Lucas (1987)

q (quarterly)	1%	5%	10%	20%
Unconstrained welfare	−18.273	−18.510	−18.524	−18.570
Constrained welfare	−18.573	−18.581	−18.578	−18.573
Cost of ceiling (as a permanent GDP loss)	1.64%	0.39%	0.30%	0.02%

- Lucas (2003): cost of fluctuations $\simeq$ 0.1% of GDP
- Cost insignificant for large q
- But large for low q

$\Rightarrow$ ceilings may be worth a try for intermediate q if default has systemic importance

Other remarks

- Size of the Poisson shock (μ_z)
  - benchmark (with emerging countries in mind): GDP level permanently lowered by 3.8%
  - This is big, but not so compared to the Greek case
  - For eurozone, the cost may be higher (due to monetary union)
  - The model can then deliver higher sustainable debt levels
- Sovereign debt held by foreigners:
  - 70% for Greece, Portugal, Ireland
  - But very low for Japan
  - Policy lesson: have debt held by domestic entities
  - Not captured by our model, but would be an interesting extension

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Conclusion

- A critical parameter: the speed at which the country exits from "trembling times"
- Rapid reaction from policymakers is needed
- Credit ceilings should be contingent and can be costly in terms of welfare
- The mess created by the management of the eurozone crisis probably changed the perception that markets have of this ability to react $\Rightarrow$ raised default risk

Future work

- Improve understanding (and possibly modelling) of recovery parameter q
- Develop a support tool for debt sustainability analysis
  - Based on the trembling times model
  - Requires empirical work on cross-country data as input
  - Would permit to create calibrations for various country profiles
- Incorporate endogenous and theoretically-grounded sovereign risk premium into standard NK models
  - Standard NK ingredients (nominal side to be as second step)
  - Distinction between domestic and foreign sovereign debt
  - Welfare-maximizing social planner vs fiscal rule
  - Necessity to improve on solution algorithms

Thanks for your attention!

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