Endogenous Debt Crises[☆,☆☆]

Daniel Cohen[a,*], Sébastien Villemot[b,1]

[a] *Paris School of Economics and CEPR, 48 boulevard Jourdan, 75014 Paris, France*
[b] *OFCE – Sciences Po, 69 quai d'Orsay, 75340 Paris Cedex 07, France*

Abstract

We distinguish two types of debt crises: those that are the outcome of exogenous shocks (to productivity growth for instance) and those that are endogenously created, either by self-fulfilling panic in financial markets or by the reckless behavior of "Panglossian" borrowers. After Krugman, we characterize as "Panglossian" those borrowers who only focus on their best growth prospects, anticipating to default on their debt if hit by an adverse shock, rationally ignoring the risk of default.

We apply these categories empirically to the data. We show that, taken together, endogenous crises are powerful explanations of debt crises, more important for instance than the sheer effect of growth on a country's solvency.

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1. Introduction

International debt crises are (very) costly. Why do we observe that so many countries fall into their trap? Should we not expect more prudent behavior from such countries? The theoretical answer in fact is: it depends. Take the simplest form of financial crisis driven by an exogenous shock. Spreads on sovereign bonds are high because the country is expected to be vulnerable to an earthquake or to a long-lasting commodity shock that is beyond its control. The country should then indeed behave with increased prudence: the greater the debt the country might have to repay, the heavier the cost of the earthquake relative to a favorable state of the nature. Yet, on the other hand, if the expected earthquake is so large that the country knows that it will actually default on its debt, then a "Panglossian attitude" (as Krugman, 1998, has coined it) may become rational: the debt will lose all value

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[*] Corresponding author.

Email addresses: daniel.cohen@ens.fr (Daniel Cohen), sebastien.villemot@sciencespo.fr (Sébastien Villemot)

[1] Sébastien Villemot wrote this article when he was working at CEPREMAP.

after the earthquake, and it would then be absurd not to have borrowed more beforehand. The country then behaves as if the risk of unfavorable shocks can be ignored. Following Dr. Pangloss, the character of Voltaire's book *Candide*, the country acts as if only "the best of all possible worlds" will occur. In this case, debt endogenously leads to debt; we call this the *self-enforcing* case.

Let us now consider the case when crises are driven by the lack of confidence of financial markets towards a given country, making the country financially fragile through self-fulfilling behavior. Self-fulfilling debt crises have been analyzed in different forms. In the model of Cole and Kehoe (1996, 2000), self-fulfilling crises are a variant of liquidity crises, by which a lack of coordination among creditors leads a solvent country to default. As argued by Chamon (2007), however, such crises can readily be avoided when lenders manage to offer contingent loans of the kind organized by venture capitalists: if any individual creditor offers a line of credit, conditionally on other creditors following suit, then liquidity crises can be easily avoided.

Self-fulfilling crises have also been analyzed as the perverse outcome of a snowball effect through which the buildup of debt becomes unmanageable, out of the endogenous fear that it can indeed become unmanageable (Calvo, 1988). Relying on an intuition developed in a simpler model in Cohen and Portes (2006), we show that snowball spirals can only occur in cases where a debt crisis has the potential of damaging the fundamentals of the indebted country. If a crisis reduces the GDP of a country by say 10%, then it is clear that the lack of confidence toward a country can degenerate into a self-fulfilling crisis. If instead the fundamentals are not altered by the crisis, we show that self-fulfilling crises of the Calvo type are (theoretically) impossible.

At the end of this argument, we chose to focus on a simple characterization of a self-fulfilling debt crisis as one that is the outcome of an endogenous weakening of the country's fundamentals. In the self-fulfilling case so defined, it is the crisis that reduces the GDP, originating from the various disruptions that a weakening of the confidence in a country may bring about (capital flight, exchange rate crisis...). In the "earthquake case," the sequence of causation is inverted: the fundamentals are first destroyed, then the crisis occurs.

From the theoretical model that we present, a simple typology of cases is obtained. Below a critical level of debt, a country tends to act prudently, aiming for instance to reduce its debt in response to a permanent adverse shock. Past a critical level of the debt-to-GDP ratio, which can be the outcome of a sequence of repeated unfavorable exogenous shocks, the country will begin to behave in the Panglossian mode, rationally ignoring the bad news, increasing the level of debt to its upper limit in a self-enforcing process. A crisis may then occur either because of the occurrence of another adverse exogenous shock, or because of a self-fulfilling shock, one that endogenously weakens the ability of the country to service its debt.

The data is analyzed with this type of typology in mind. We use a slightly modified version of the database that has been compiled by Kraay and Nehru (2006), which we updated to cover all debt crises that have occurred until 2004. Following and adapting the work of these authors, we first show that the likelihood of a debt crisis is well explained by three factors: the debt-to-GDP ratio, the level of real income per capita, and a measure of overvaluation of the domestic currency.

In order to estimate the risk of a self-fulfilling debt crisis, we then distinguish the law of motion of the debt-to-GDP ratio in normal times from the motion triggered by the onset of the crisis. We define a self-fulfilling crisis as one that would not have happened, had debt-to-GDP simply been driven along the pre-crisis path. We find that self-fulfilling crises, so defined, correspond to a small minority of cases. On average, between 6% and 12% of crises (depending on the methodology) appear to be self-fulfilling. This proportion is clearly not negligible, however, and deserves to be taken seriously.

We also calibrate the strength of the Panglossian effect. We show that countries do appear to have behaved as if the distribution of the risk was truncated, leading the country to ignore risk. The influence of this mechanism on the debt buildup is tested through Monte-Carlo simulation. We show that it is substantial and representing about 12% of the cases (see Arellano, 2008, for similar insights applied to the the case of Argentina).

In this paper, we proceed as follows: in section 2, we set up an horizon model that we solve and analyze, first in the two period case in section 3, then in the general case in section 4. We then turn in section 5 to the presentation of the data supporting our econometric analyses. In section 6, we build and estimate an econometric model which enables us to quantify the importance of both self-fulfilling and self-enforcing crises. Section 7 concludes.

2. A Panglossian theory of debt

In this section we develop a modeling framework which shares many features with the seminal work of Eaton and Gersovitz (1981) and with more recent works such as Aguiar and Gopinath (2006) and Arellano (2008). Compared to these papers, our model mainly differs in two ways. The first difference is how we specify the penalty in case of default: in our model, we decompose this penalty into two components, namely the part which is captured by the investors and the part which is socially lost. The second difference concerns the negotiation process between lenders and investors: in this paper, we assume that the negotiation occurs on the amount to be repaid tomorrow, rather than on the amount to be lent today. These two changes open the possibility for a rich analysis of self-fulfilling crises, as discussed below. For the remaining, our model is fairly standard. In particular, the Panglossian effect that we exhibit is not specific to our setup and is actually present in most models of the literature. Our contribution in this respect is rather to put this mechanism in evidence and then to analyze its empirical relevance.

2.1. The economy

We consider a one-good exchange economy. The country is inhabited by a representative consumer who can tilt consumption away from autarky by borrowing or lending on the international financial markets.

Output produced at time t is a random variable Q_t, driven by a Markovian process. More precisely, the (gross) growth rate of output $g_t = \frac{Q_t}{Q_{t-1}}$ is assumed to be an *i.i.d.* variable, with a cumulative distribution function $\mathcal{F}(g)$. In other words, $\log Q_t$ is a random walk. For the sake of simplicity, the support of g is supposed to lie in a bounded interval of the form $(0, g^{\max}]$.

The world financial markets are characterized by a constant riskless rate of interest r. Lenders are risk-neutral and subject to a zero-profit condition by competition. We further suppose that debt is short-term and needs to be refinanced at every period.

In order to ensure that the wealth of the country is finite, we make the assumption that the average growth rate $\bar{g}$ is less than the gross interest rate $1 + r$.

At any time t, a country that has accumulated a debt D_t may decide to default upon it. When it does so, we assume that the country suffers forever after a negative productivity shock. One can say that the default creates a panic that destroys capital either through an exchange-rate or a banking crisis. We simply assume that post-default output can be written:

$$Q_t^d = \mu\, Q_t$$

in which $\mu \in (0, 1]$. As another cost, we assume that the country is subject to financial autarky, being unable to borrow again later on (a milder form of a sanction would be, more realistically, that the country is barred from the financial market for some time only; analytically, the outcome is formally equivalent).

Once the country has defaulted, creditors will attempt to recover some of their losses. In order to do so, they further reduce the resources of the country in a way which, we assume, is socially efficient: the fraction that they grab is simply subtracted, one for one, from the country's post-default output. Call λ_t the fraction so reduced. We assume that λ_t is itself an *i.i.d.* stochastic variable, in the domain $[0, 1)$ and independent of g_t, which varies with the (legal) strength of the international financial community. We denote by $\mathcal{G}(\lambda_t)$ the cumulative distribution function of λ_t. Creditors therefore capture:

$$P_t = \lambda_t Q_t^d,$$

while the country consequently consumes (given financial autarky):

$$C_t = (1 - \lambda_t) Q_t^d.$$

In the case when μ is equal to one, the outcome may be characterized as an efficient restructuring of the debt, at least from a static point of view (we return to this issue below): creditors are able to capture a fraction of output, which is less than what they are owed, but without imposing a social cost to the economy. When instead, $\mu < 1$, then the implication is that default is socially costly, imposing a loss that is captured by no one.

2.2. Financial markets

The timing of events unfolds as follows. First assume that the country has incurred a debt obligation D_t, falling due at time t, and has always serviced it in full in previous periods. At the beginning of period t, the country learns the value of its output Q_t and the fraction λ_t of post-default output that would be captured were it to default. After observing these variables, the country decides to default or to reimburse its debt.

If the debt is reimbursed in full, the country can contract a new loan, borrowing L_t, which must be repaid at time $t+1$, in the amount of D_{t+1}. In order to avoid coordination problems, we assume, following Chamon (2007), that creditors can commit on L_t and D_{t+1} before the decision to service the debt is known, conditionally on the decision to service the debt being made.

Such financial agreements being concluded, the country eventually consumes, in the event it services its debt in full:

$$C_t^r = Q_t + L_t - D_t$$

Alternatively, in the event of a debt crisis the country's consumption is nailed down to $C_t^d = (1-\lambda_t)Q_t^d$.

We denote by $\mathscr{D}(D_{t+1}, Q_t)$ the default set, *i.e.* the set consisting of all realizations (g_{t+1}, λ_{t+1}) for which the country will decide to default in $t+1$, conditionally on the level of outstanding debt D_{t+1} and past output Q_t. We denote by $\mathscr{R}(D_{t+1}, Q_t)$ the repayment set, *i.e.* the complementary to the default set.

We can then define the risk of a debt crisis in $t+1$, as it is perceived from the perspective of date t:

$$\pi_{t+1|t} = \mathbb{P}(\mathscr{D}(D_{t+1}, Q_t)).$$

The zero-profit condition for creditors may be written as:

$$L_t(1+r) = D_{t+1}(1-\pi_{t+1|t}) + \int\limits_{\mathscr{D}(D_{t+1},Q_t)} V_{t+1}(g\,Q_t, \lambda)\mathrm{d}\mathcal{F}(g)\mathrm{d}\mathcal{G}(\lambda) \tag{1}$$

in which $V_{t+1}(Q_{t+1}, \lambda_{t+1})$ is the discounted present value of all cash-flows that the banks will be able to extract from the country, when they expect to receive forever after $t+1$ an amount $P_{t+1+T} = \lambda_{t+1+T}Q_{t+1+T}^d$ in every period.

Finally, the usual no-Ponzi game condition is supposed to hold so that:

$$\lim_{T\to+\infty} \mathbb{E}_t \frac{D_{t+T}}{(1+r)^{t+T}} = 0.$$

2.3. Preferences

The decision to default or to stay current on the financial markets involves a comparison of two paths that imply expectations over the entire future. In order to address this problem, we assume that the country seeks to solve:

$$J^*(D_t, Q_t, \lambda_t) = \max_{\{C_{t+T}\}_{T\geq 0}} \mathbb{E}_t \left\{ \sum_{T=0}^{\infty} \beta^T u(C_{t+T}) \right\}$$

where β is the discount factor, $C_t > 0$. Note that D_t can be negative if the country builds up foreign assets. We assume that utility is isoelastic, of the form:

$$u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma} \tag{2}$$

where $\frac{1}{\gamma}$ is the inter-temporal elasticity of substitution. In order to incorporate a tendency to accumulate debt, we make the hypothesis that:

$$\beta(1+r)\bar{g}^{-\gamma} < 1 \tag{3}$$

We shall call:

$$J^d(Q_t, \lambda_t) = \mathbb{E}_t \left\{ \sum_{T=0}^{\infty} \beta^T u((1-\lambda_{t+T})Q^d_{t+T}) \right\}$$

the post-default level of utility, which becomes by definition independent of debt, and to which the country is nailed down in case of servicing difficulties.

If instead it were to stay current on its debt obligation, it would obtain:

$$\begin{aligned} J^r(D_t, Q_t) = \max_{L_t, D_{t+1}} \Bigg\{ & u(Q_t - D_t + L_t) + \beta \int\limits_{\mathscr{D}(D_{t+1}, Q_t)} J^d(g_{t+1}\, Q_t, \lambda_{t+1}) \mathrm{d}\mathcal{F}(g_{t+1}) \mathrm{d}\mathcal{G}(\lambda_{t+1}) \\ & + \beta \int\limits_{\mathscr{R}(D_{t+1}, Q_t)} J^r(D_{t+1}, g_{t+1}\, Q_t) \mathrm{d}\mathcal{F}(g_{t+1}) \mathrm{d}\mathcal{G}(\lambda_{t+1}) \Bigg\} \end{aligned}$$

subject to the zero-profit condition (1). Note that $J^r(D_t, Q_t)$ does not depend on the current value of λ_t.

When comparing how much it can get by staying on the markets and the post-default level of welfare, the country picks up its optimum level:

$$J^*(D_t, Q_t, \lambda_t) = \max \left\{ J^r(D_t, Q_t), J^d(Q_t, \lambda_t) \right\}$$

Here $J^*(D_t, Q_t, \lambda_t)$ is a function of the current value of λ_t through the influence of J^d.

Given the Markovian structure of the economy and the isoelastic preferences, the decision to default will depend on whether the debt-to-GDP ratio is above or below a (stochastic) threshold which itself will be a function of the stochastic parameter λ_t. Before showing formally this result and digging deeper into the explicit consequences of the model, let us first analyze a simpler two-period case which will highlight its key properties.

3. A two-period model

3.1. The economy

In order to grasp the logic of the model, let us consider a simple two-period model which will provide the intuition behind our framework. The model follows the intuition of Cohen and Portes (2006).

In period 1, output is known, and the country just takes one decision, the amount of debt it borrows.

In period 2, we simplify the stochastic structure of the economy and assume that output can take two values, a good or a bad one, with respective probabilities $1-p$ and p. The only decision left to the country is to repay or default. Let us briefly rephrase, with the notations of the previous section, the logic of the model.

Call L_1 the amount of debt borrowed in period 1 and D_2 the contractual amount to be repaid in period 2. We assume that initial debt is nil: $D_1 = 0$.

If the country does not honor its contractual obligations in period 2, it is subject to a negative output shock of magnitude $1 - \mu$, captured by no-one, and which is the measure of the negative externality associated to the crisis. In the case when $\mu = 1$, default is efficient *ex post*: no loss of resources is imposed on the whole community of creditors and debtors.

The creditors themselves only grasp a portion λ of the post-default output μQ_2. In this simplified example, we take λ to be a deterministic scalar. In the case when $\lambda = 0$, the recovery is nil for the creditors. Note that recovery is stated here as a function of output, as it should be in general: when the debt is too large, creditors attempt to grab a part of the remaining pie.

We can then summarize the model as follows:

In period 1	*In period 2*	
	with prob. $1-p$	with prob. p
Q_1 $C_1 = Q_1 + L_1$	$Q_2 = Q_2^+$ $C_2 = C_2^+ = Q_2^+ - D_2$	$Q_2 = Q_2^-$ (before default) $C_2 = C_2^- = \max\{Q_2^- - D_2, (1-\lambda)\mu Q_2^-\}$

We assume that the country goes on the market to borrow an amount L_1 in response to which the creditors ask for an amount D_2 to be repaid in period 2. Since they are risk neutral, creditors either impose the following safe line of credit D_2^s defined by:

$$L_1(1 + r) = D_2^s \tag{4}$$

when the country is expected to repay its debt in both states of natures, or an unsafe line of credit D_2^u defined by:

$$L_1(1 + r) = (1 - p)D_2^u + p\mu\lambda Q_2^- \tag{5}$$

when a renegotiation is expected to take place in the bad state of nature.

In this simple two period model, the transversality (no-Ponzi) condition is imposed through the fact that the country leaves no debt at the end of period 2 (*i.e.* $D_3 = 0$). In period 2, the country either pays the debt in full or defaults (and pays less than the face value). In our model, D_2 is a function of L_1. The equilibrium can be thought as a characterization of an equilibrium yield function on the loans extended in period 1. The spread ρ overs the riskless rate is a solution to $\rho(L_1) = D_2(L_1)/L_1(1 + r) - 1$.

Last, we assume that the country attempts to solve the following program:

$$\max_{C_1, C_2} u(C_1) + \beta\, \mathbb{E}_1 u(C_2)$$

where the instantaneous utility function u is given by (2).

3.2. The equilibrium

3.2.1. The safe case

Let us first analyze the case when the country repays its debt in both states of nature. This will occur when:

$$D_2 < \kappa\, Q_2^-,$$

in which:

$$\kappa = 1 - (1-\lambda)\mu.$$

In this case, the credit constraint boils down to (4), so that the first order condition can be written:

$$u'(C_1) = \beta(1+r)\{(1-p)u'(C_2^+) + p\, u'(C_2^-)\}$$

or, in explicit form:

$$u'(Q_1 + L_1) = \beta(1+r)\{(1-p)u'(Q_2^+ - L_1(1+r)) + p\, u'(Q_2^- - L_1(1+r))\}$$

In this case, consumption in period 1 is an increasing function of Q_2^-. When the bad state of nature is awful, with a low Q_2^-, the country is prudent, and consumes less in the first period. It will have to repay for its debt in a worst case scenario and that reduces its propensity to borrow.

3.2.2. The unsafe case

Consider now the case when the equilibrium value of D_2 is such that $D_2 > \kappa Q_2^-$. In that case the country will not repay its debt in the second period when the bad state materializes.

Creditors, *ex ante*, tie debt and repayment according to (5), so that the first order condition boils down to:

$$u'(C_1) = \beta(1+r)u'(C_2^+) \tag{6}$$

This is the same Euler equation as in the riskless rate, with only one (good) state of nature! The occurrence of the bad state of nature is not taken into account, at least directly. The structure is equivalent to the Blanchard-Yaari (Blanchard, 1985; Yaari, 1965) model, where the probability of death of the individuals does not appear—at least directly—in the Euler equation.

The critical difference with Blanchard-Yaari (BY henceforth) is that the probability of death, *i.e.* of default here, is endogenous in our model when it is biological in BY. The other difference is that the government is infinitely lived in BY when it is the sovereign itself which is subject to the risk of death in our framework. The core analogy between the two models remains identical however as far as the decisions to save or borrow are concerned. Rational agents truncate the perception of the future, when they take debt or investment decisions, to the subset of events when they are still alive (or solvent). This truncation creates a Panglossian factor upon which we shall rely, below, to offer an empirical test of our model.

When $\lambda > 0$, Q_2^- does play a role, indirectly: the larger it is, the higher the supply curve, allowing the country to borrow in more favorable terms. In the extreme case when $\lambda = 0$, the case with no recovery, Q_2^- entirely disappears from the picture. The country behaves as if the best outcome only where to occur, ignoring the bad outcome, to the extent that it has no bearing on its decision to borrow.

This is the *Panglossian* effect that we alluded to. The country is totally indifferent to the bad states of nature, while in the safe equilibrium it was instead very concerned about it.

In the model above we assume that a renegotiation occurs after a default, so that some recovery is granted to the creditors. The recovery comes in proportion to the ability of the creditors to extract a flow of payments from the debtors before the courts. In our model, the recovery is a function of the state of the economy, and does not depend at all on the debt due. The idea is that the debt being too large anyway, it has to be cut to be in line with the fundamentals.

One can think of situations however where the recovery is a combination of the nominal value due and of the fundamentals. The bargaining power of the creditors for instance may be proportionate to how many they are or to what is at stake for them. In that case, one may write that the recovery V is the sum of two terms:

$$V = (1-\gamma)\,\lambda\mu Q_2^- + \gamma D_2$$

in which γ is some positive number smaller than one. In that case, the first order condition is driven by

$$(1+r)\frac{\partial L_1}{\partial D_2} = (1-p) + p\,\gamma$$

which is translated into a new Euler equation:

$$u'(C_1) = \beta(1+r)\frac{(1-p)u'(C_2^+) + p\,\gamma\,u'(C_2^-)}{1-p(1-\gamma)}$$

As we move from $\gamma = 1$ to $\gamma = 0$, we switch from the standard Euler first order condition to the Panglossian case.

3.2.3. The risk of multiple equilibria

In a standard setup, the interest rate charged by investors is entirely determined the probability of default, via the risk premium: the higher the risk, the higher the interest rate.

But the reverse causality can very well be also at work. One may have situations where two equilibria are possible: a "good equilibrium" where the investors ask for a low interest rate, leading to a low debt-to-GDP tomorrow—and therefore a low risk of default, consistently with the low interest rate—, and a "bad equilibrium" where investors ask for a high interest rate—consistently leading to a high level of risk.

Multiple equilibria are possible in this model whenever $\mu < 1$. In the context of our model, a country can be safe, if priced as such, and unsafe, if regarded as risky, whenever default leads to an inefficient loss of output. This would be the case if we could simultaneously have:

$$D_2^s < \kappa\,Q_2^-,$$

which means that the country will not renegotiate its debt in the second period if offered a safe line of credit, and:

$$D_2^u > \kappa\, Q_2^-$$

which means that it would renegotiate the debt, if priced at the risk adjusted rate. Plugging the value of D_2^s and D_2^u into these two inequalities, one finds that multiple equilibria are feasible when the amount borrowed in the first period stands in the following range:

$$\kappa\, Q_2^- - p(1-\mu)Q_2^- < L_1(1+r) < \kappa\, Q_2^-$$

The first inequality means that the repayment is too large to be honored at the risk adjusted interest, when the second means that the riskless rate is indeed self-enforcing. We demonstrate the generality of this insight in the next section. The range of multiple equilibria becomes larger as μ gets smaller. In the limiting case when $\mu = 1$, no multiple equilibria are feasible. The intuition is fairly straightforward: multiple equilibria are possible when the fear of a crisis has the potential to destroy the fundamentals upon which the debt is repaid, self-fulfillingly reducing the ability of the country to pay. In the case when the fundamentals are immune to the judgment of the market, only one equilibria stands out.

3.2.4. Summary

In the end, this simplified model shows that debt crises can be the endogenous effects of two forces: a *self-enforcing* and a *self-fulfilling* mechanism.

Figure 1 shows what is at stake. In the efficient case, the supply of credit is continuous and monotonic, the "fundamentals" upon which the country repays its debt are not affected by the decision to default, and just one equilibrium is possible. In the inefficient case, a bad credit rating has the potential to destroy output endogenously with probability p. Two equilibria, corresponding to the two values of these fundamentals, can arise simultaneously.

4. Recursive equilibrium

4.1. Definition and basic properties

We now define a recursive equilibrium in the infinite horizon model. Such an equilibrium consists of a set of policy functions for the country and the investors. We assume that agents act sequentially and that the government does not have commitment.

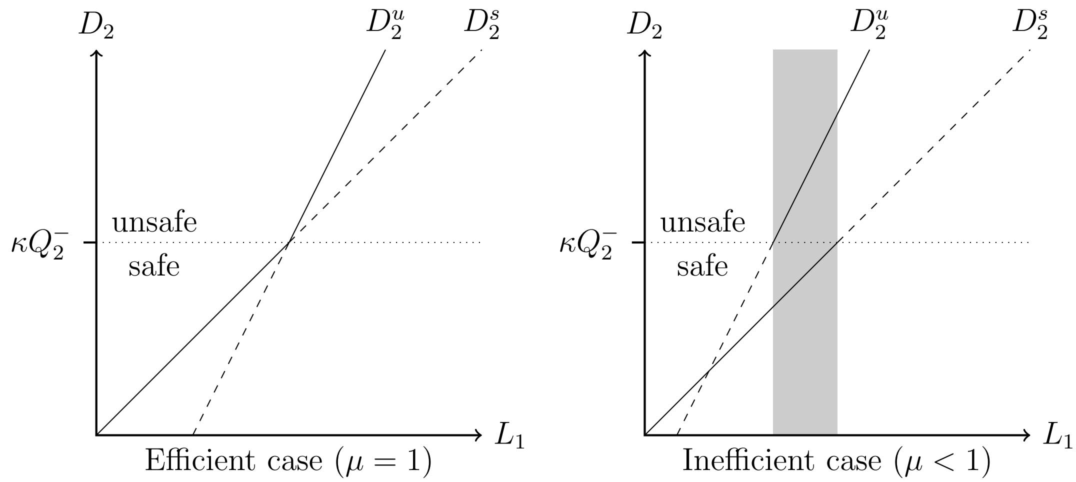
As in the two-period model, we also make the assumption that in the process of negotiating debt contracts, the country first announces the amount L that it wants to borrow today, and the investors reply with the amount D' that they ask tomorrow for that loan.

Definition 1 (Recursive equilibrium)**.** *A recursive equilibrium is defined by default and repayment sets $\mathscr{D}$ and $\mathscr{R}$, value functions J^r, J^d, J^* for the country, a policy function $\tilde{D}'$ and a default value function V for the investors, such as:*

- *The value function J^d in case of default satisfies*

$$J^d(Q,\lambda) = u((1-\lambda)\mu Q) + \beta \int J^d(g'\, Q, \lambda')\mathrm{d}\mathcal{F}(g')\mathrm{d}\mathcal{G}(\lambda') \tag{7}$$

Figure 1: The risk of multiple equilibria



The dashed lines correspond to pairs (L_1, D_2) that cannot be realized in equilibrium. Conversely the solid lines correspond to possible equilibria. The gray area in the inefficient case shows the region where multiple equilibria are possible.

- *Given default and repayment sets $\mathscr{D}$ and $\mathscr{R}$ and investors' policy function $\tilde{D}'$, the value function J^r in case of repayment satisfies:*

$$J^r(D,Q) = \max_{L \in \mathscr{L}(Q),\, L \geq D-Q} \left\{ u(Q - D + L) + \beta \int_{\mathscr{D}(\tilde{D}'(L,Q),Q)} J^d(g'\,Q, \lambda') \mathrm{d}\mathcal{F}(g') \mathrm{d}\mathcal{G}(\lambda') \right.$$
$$\left. + \beta \int_{\mathscr{R}(\tilde{D}'(L,Q),Q)} J^r(\tilde{D}'(L,Q), g'\,Q) \mathrm{d}\mathcal{F}(g') \mathrm{d}\mathcal{G}(\lambda') \right\} \quad (8)$$

  where $\mathscr{L}(Q)$ characterizes the domain of definition of $\tilde{D}'$.

- *J^* is the maximum of J^r and J^d, and the default and repayment sets verify:*

$$(g', \lambda') \in \mathscr{D}(D', Q) \;\Leftrightarrow\; (g', \lambda') \notin \mathscr{R}(D', Q) \;\Leftrightarrow\; J^d(g'\,Q, \lambda') > J^r(D', g'\,Q)$$

- *The value that investors can extract in case of default satisfies:*

$$V(Q,\lambda) = \lambda \mu Q + \frac{1}{1+r} \int V(g'\,Q, \lambda') \mathrm{d}\mathcal{F}(g') \mathrm{d}\mathcal{G}(\lambda') \quad (9)$$

- *Given the default and repayment sets $\mathscr{D}$ and $\mathscr{R}$, the policy function of investors satisfies the zero-profit condition for all $L \in \mathscr{L}(Q)$:*

$$L(1+r) = \tilde{D}'(L,Q)\, \mathbb{P}[\mathscr{R}(\tilde{D}'(L,Q),Q)] + \int_{\mathscr{D}(\tilde{D}'(L,Q),Q)} V(g'\,Q, \lambda') \mathrm{d}\mathcal{F}(g') \mathrm{d}\mathcal{G}(\lambda') \quad (10)$$

Just as in the two period model, the policy function of investors is equivalent to defining an equilibrium interest spread over the riskless rate $\rho(L, Q)$ as a solution to: $\rho(L, Q) = D'(L, Q)/L(1 + r) - 1$.

At this point, it is important to note that our model is constructed in such a way that homogeneous equilibria, as defined below, are possible.

Definition 2 (Homogeneous recursive equilibrium)**.** *A recursive equilibrium is said* homogeneous *if it satisfies the following relationships for $a > 0$:*

$$
\begin{aligned}
J^r(a\,D, a\,Q) &= a^{1-\gamma} J^r(D, Q) \\
J^d(a\,Q, \lambda) &= a^{1-\gamma} J^d(Q, \lambda) \\
J^*(a\,D, a\,Q, \lambda) &= a^{1-\gamma} J^*(D, Q, \lambda) \\
\tilde{D}'(a\,L, a\,Q) &= a\,\tilde{D}'(L, Q) \\
V(a\,Q, \lambda) &= a\,V(Q, \lambda) \\
\mathscr{L}(a\,Q) &= a\,\mathscr{L}(Q) \text{ (with obvious notation)}
\end{aligned}
$$

The possibility of homogeneous recursive equilibria stems from three specific features of our model: the isoelasticity of the utility function, the specific form of the output process (*i.i.d.* in growth rates), and the proportionality of default costs.

It is theoretically possible that our model has recursive equilibria that are not homogeneous, but such equilibria are more of the nature of mathematical curiosities rather than economically relevant objects. In the following, we will therefore assume that there exists at least a homogeneous equilibria, which satisfies standard regularity conditions, and we establish several results that apply to these homogeneous recursive equilibria.

Proposition 1. *Default occurs if and only if debt-to-GDP ratio is higher than a given threshold $d^*(\lambda)$.*

Proof. Immediate consequence of the homogeneity of value functions. □

When λ is revealed at any time t, being a common knowledge to both the debtor and the lenders, the flow of new loans is either sufficient to allow the country to keep servicing its outstanding stock of debt, or instead it is instantaneously dried out as creditors understand that the country will not service its debt.

Definition 3 (Market shock)**.** *We refer to a change of the default threshold $d^*(\lambda)$ as a* market shock*, to the extent that it measures the shift in the perceived enforcement technology in the hands of creditors. One can think of the perceived international legal environment as the underlying fundamental of this shock (see for example the Argentina versus Elliott case).*

Finally, we introduce a definition, which will be useful for characterizing the case of multiple equilibria:

Definition 4 (Smooth default)**.** *We call the* smooth default *case the situation where the default threshold is equal to what the investors can extract in case of default,* i.e. *when $d^*(\lambda) = V(1, \lambda)$.*

Clearly, a smooth default is only possible when $\mu = 1$, *i.e.* when a default leads to an efficient restructuring of the debt from a static point of view (there is still an inefficiency related to the loss of access to financial markets).[2] The reciprocal is not true: it is possible to have statically efficient defaults which are not smooth. Think of a country with a low rate of time preference (lower than the riskless interest rate), and with a linear utility function. It is easy to show that, in that case, the country will be willing to repay a debt higher than what investors would extract in case of default (simply because the country has a lower discount rate than investors).

4.2. The risk of multiple equilibria

Formally, a multiple equilibrium in the interest rate is a situation where, for a given L_t, there are two values $D^1_{t+1} < D^2_{t+1}$ verifying the zero-profit condition (10).

Proposition 2. *Multiple equilibria in the interest rate are impossible in the smooth default case.*

Proof. See Appendix B.2. □

This result is the generalization of the result obtained in the two-period model of section 3. What drives the multiple equilibrium case is the fact that the crisis endogenously destroys part of the fundamentals upon which the debt is repaid. This may be the key reason why corporate self-fulfilling debt crises are a curiosity. To the extent that an appropriate bankruptcy procedure exists, the risk that a financial crisis can, out of its own making, endanger the value of a firm is much reduced. This kind of multiple equilibria in the interest rate, also called the "snowball effect," have been studied by Calvo (1988).

Note that multiple equilibria in the interest rate are possible in our model because the country announces L and the investors reply with some D' which satisfies the zero-profit condition; as noted by Chamon (2007, footnote 7), such multiple equilibria are impossible in the reverse setup, where the country announces D' and the investors reply with the corresponding L.

4.3. Dynamics

We now derive the Euler equation of a country which is current on its debt at the time of the decision. For the sake of simplicity, we switch back to the notation using time subscripts.

The first order condition of the maximization in (8) leads to:

$$u'(C_t) = \beta \frac{\partial D_{t+1}}{\partial L_t} \mathbb{E}_t \left[u'(C_{t+1}) \,|\mathscr{R}(D_{t+1}, Q_t) \right] \tag{11}$$

(using the fact that J^r and J^d are equal at the default threshold). Here the term $\mathbb{E}_t \left[u'(C_{t+1}) \,|\mathscr{R}(D_{t+1}, Q_t) \right]$ stands for the expectation of $u'(C_{t+1})$, from the perspective of date t, conditionally on the decision to repay at date $t+1$. This equation is the analog of equation (6) in the two-period model.

[2] See proposition 3 in Appendix B.1 for a proof.

Assume in the following that λ is a constant (the general case is treated in Appendix B.2). This corresponds to the case when there is a given deterministic threshold for debt to be repaid in full. With obvious notations, the zero-profit condition (10) for creditors may be written as:

$$L_t(1+r) = D_{t+1}(1-\pi_{t+1|t}) + \int_0^{g^*_{t+1}} V(g_{t+1}\, Q_t)\mathrm{d}\mathcal{F}(g_{t+1})$$

where $g^*_{t+1} = \frac{D_{t+1}}{d^*\, Q_t}$ is the growth threshold under which the country defaults.

Taking the derivative with respect to D_{t+1}, one gets:

$$(1+r)\frac{\partial L_t}{\partial D_{t+1}} = (1-\pi_{t+1|t}) - \frac{\partial \pi_{t+1|t}}{\partial D_{t+1}}[D_{t+1} - V(g^*_{t+1}\, Q_t)]$$

or:

$$\frac{\partial L_t}{\partial D_{t+1}} = \frac{1}{1+r}\left(1-\pi_{t+1|t} - \xi_{t+1|t}\right)$$

in which $\xi_{t+1|t}$ is a scalar which is nil in the smooth default case (*i.e.* when $D_{t+1} = V(g^*_{t+1}Q_t)$) and positive otherwise.[3] The term $\xi_{t+1|t}$ can be coined as the "default wedge." Note that this term did not appear in the simplified two period model, for which $\frac{\partial \pi_{t+1|t}}{\partial D_{t+1}} = 0$ given the discrete structure of probability of default.

The term $1-\pi_{t+1|t}-\xi_{t+1|t}$ is the marginal price of debt, in the sense of Bulow and Rogoff (1988): it is the total value to creditors of having the face value of the country's debt raised by one dollar. In this case, it is equal to the probability of repayment, minus an extra loss incurred by the investors when the default is not smooth.

Substituting $\frac{\partial L_t}{\partial D_{t+1}}$ for its value and using the approximation $\frac{1-\pi_{t+1|t}}{1-\pi_{t+1|t}-\xi_{t+1|t}} \simeq 1+\xi_{t+1|t}$, we can then rewrite the Euler equation (11) as:

$$u'(C_t) = \beta(1+r)(1+\xi_{t+1|t})\mathbb{E}_t\left[u'(C_{t+1})\,|\mathscr{R}(D_{t+1}, Q_t)\right] \tag{12}$$

The term $\xi_{t+1|t}$ tends to raise the marginal utility of consumption at time t and consequently reduces the propensity to borrow. To the extent that default entails a social loss, the benefit of borrowing against future risk is reduced, decreasing the desirability of debt in consequence.

4.4. A linear approximation

We now write the first-order linear approximation of the model presented so far. We linearize around the balanced growth path, *i.e.* the steady state of the detrended version of the model. The detrending factor of a variable at date t is Q_t, so that $\hat{X}_t = \frac{X_t}{Q_t}$. Using these notation, equation (12) can be rewritten as:

$$u'(\hat{C}_t) = \beta(1+r)(1+\xi_{t+1|t})\mathbb{E}_t\left[g_{t+1}^{-\gamma}u'(\hat{C}_{t+1})\,\middle|\mathscr{R}(\hat{D}_{t+1}, 1)\right] \tag{13}$$

[3] See Appendix B.2 for a proof.

Because of assumption (3), this equation cannot be satisfied at the deterministic steady state. This means that in the absence of stochastic shocks, the solution is a corner one: the country borrows up to the default limit, before it becomes rationed by the investors. Using the law of motion of debt, assuming a constant debt-to-GDP ratio where the country is indifferent between default and repayment, one gets the following deterministic steady state:

$$\bar{\hat{D}} = \frac{(1+r)(1-(1-\lambda)\mu)}{1+r-\bar{g}}$$

We now proceed to a linearization around that point. Let us note: $u'(\hat{C}_t) = -\frac{\partial J^r}{\partial D}(\hat{D}_t, 1) = a_0 - a_1\,(\hat{D}_t - \bar{\hat{D}})$. Using the approximation $g_{t+1}^{-\gamma} \simeq 1 - \gamma(g_{t+1} - 1)$, we may then rewrite equation (13) as:

$$a_0 - a_1(\hat{D}_t - \bar{\hat{D}}) = a_2\left[a_0 + a_0\gamma\,(g^+_{t+1|t} - 1) - a_1(\hat{D}^*_{t+1} - \bar{\hat{D}})\right] \tag{14}$$

in which $g^+_{t+1|t} = \mathbb{E}_t\left[g_{t+1}\,\middle|\,\mathscr{R}(\hat{D}^*_{t+1}, 1)\right]$, $a_2 = \beta(1+r)(1+\xi)$ (neglecting here the variability of the factor ξ), and $\hat{D}^*_{t+1}$ is the corresponding first best decision regarding debt. Let us denote:

$$\Xi_{t+1|t} = g^+_{t+1|t} - \bar{g} = \pi_{t+1|t}(g^+_{t+1|t} - g^-_{t+1|t}), \tag{15}$$

in which $g^-_{t+1|t} = \mathbb{E}_t\left[g_{t+1}\,\middle|\,\mathscr{D}(\hat{D}^*_{t+1}, 1)\right]$. The Euler equation can then be written as:

$$\hat{D}^*_{t+1} = a_3 + a_4\,\hat{D}_t + a_5\,\Xi_{t+1|t} \tag{16}$$

where a_3, a_4 and a_5 are obvious to deduce from (14) and (15).

The term $a_5\Xi_{t+1|t}$ is the Panglossian term, which measures the way creditors truncate their forecasting set.

Equation (16) determines the optimal debt target, but in practice this level cannot be achieved for two reasons: first, there is some inertia in the adjustment process, for institutional or political reasons; second, as shown by Campos et al. (2006), there is a lot of extrinsic noise in the level of debt, due to either unforeseen contingencies debt or unpredicted valuation effects. We therefore assume that the debt target D^*_{t+1} differs from the actual debt D_{t+1}, and that the two are related by the following equation:

$$D_{t+1} = (1-\rho)D^*_{t+1} + \rho\,D_t + \varepsilon^d_{t+1}Q_{t+1}$$

where ε^d_{t+1} is an *i.i.d.* shock and $\rho \in [0,1]$ measures the inertia of the adjustment. Detrending this equation, and using the linearization $1/g_{t+1} \simeq 2 - g_{t+1}$, one gets:

$$\hat{D}_{t+1} = (1-\rho)\hat{D}^*_{t+1} + \rho\,(2 - g_{t+1})\hat{D}_t + \varepsilon^d_{t+1} \tag{17}$$

The deviation of the debt adjustment from the "desired" pattern has also been modeled by Alesina and Tabellini (1990) and more recently by Beetsma and Mavromatis (2014).

These papers have modeled the distortion brought by the political decision process in the debt build up. In this paper we just assume that unbiased deviations may occur, due to valuation effects (exchange rates, change in the price of assets abroad...). In our model, the key source of distortion is the Panglossian effect which tilts the incentives of policymakers towards more debt.

Combining equations (16) and (17), we therefore get:

$$\hat{D}_{t+1} = a_6 + a_7\,\hat{D}_t - a_8\,g_{t+1}\hat{D}_t + a_9\,\pi_{t+1|t}\,(g^+_{t+1|t} - g^-_{t+1|t}) + \varepsilon^d_{t+1} \tag{18}$$

where a_6, a_7, a_8 and a_9 are obvious to deduce from (16) and (17). Equation (18) will be used in the empirical section 6 as the basis for the debt dynamics.

They novelty introduced by our model is the factor $a_9\,\pi_{t+1|t}\,(g^+_{t+1|t} - g^-_{t+1|t})$ (the remaining terms are consistent with a variety of models). Let us emphasize its role and where it comes from. The core idea of the model presented above is the Panglossian factor which states that a borrower tends to truncate its vision of the future to the subset of events in which it will remain solvent. Therefore, a positive driving force of the decision to borrow is the discrepancy between the average growth of the economy and the prevailing growth rate in the better states of nature where the country remains solvent. Writing the average growth rate as $\bar{g} = (1 - \pi_{t+1|t})g^+_{t+1|t} + \pi_{t+1|t}\,g^-_{t+1|t}$, the Panglossian term that we factor in is simply: $g^+_{t+1|t} - \bar{g} = \pi_{t+1|t}(g^+_{t+1|t} - g^-_{t+1|t})$. The larger it is, the more debt the country is willing to take, *ceteris paribus*. This is the key factor that we now bring to the data.

5. Dataset

Our empirical strategy relies on a dataset of "debt distress" and "normal times" episodes, following the methodology of Kraay and Nehru (2006).

More precisely, for a given year, a country is considered to be in debt crisis if at least one of the following three conditions holds:

1. The country receives debt relief from the Paris Club in the form of a rescheduling and/or a debt reduction.
2. The sum of its principal and interest in arrears is large relative to the outstanding debt stock.
3. The country receives substantial balance of payments support from the IMF through a non-concessional Standby Arrangement (SBA) or an Extended Fund Facility (EFF).

For the last two conditions, we choose the same thresholds as do Kraay and Nehru (2006): a country is considered to be in crisis if its arrears are above 5% of the total stock of its outstanding debt, or if the total amount agreed to under SBA/EFF arrangements is above 50% of the country's IMF quota. Moreover, a country receiving Paris Club relief for a given year is also considered to be in crisis for the following two years since the relief decision is typically based on three-years balance of payments projections by the IMF.[4]

[4]The following data sources are used for creating the dataset:

The set of countries over which are made the computations consists of the 135 developing countries defined by the World Bank, from which we removed the 38 countries that have absolutely no access to private financial markets.[5] We choose to remove them since their situation of indebtedness is somewhat different from that of the rest of the developing world (in particular, they have a much higher proportion of concessional lending). From the standpoint of the model, they probably fall into the category of countries that have no access to risky markets, and their debt dynamics must consequently be different.

We are therefore left with a sample of 97 countries. From the time angle, our data cover the period 1970–2004.

Prior to the elimination of certain observations in our econometric estimations (due to missing data), our sample of episodes consists of 70 distress episodes, and 223 normal times episodes.[6] The average default episode lasts 13.3 years and exhibits a GDP loss from peak to through of 1.9%.[7]

In one regression (column (5) of Table 1, p. 21) we extend the data to include the most recent financial crises (up to 2011), although the nature of the post-Lehman financial crisis is not quite that of the proposed model.

To summarize, the differences between our dataset and that of Kraay and Nehru are twofold: first, we update their data to 2004, which is relatively minor but allows us to

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- the World Bank's *Global Development Finance* for data on debt levels and payment arrears (World Bank, 2006a);
- the Paris Club website for information on debt reliefs (`http://www.clubdeparis.org`);
- the IMF's *International Financial Statistics* for data on SBA/EFF commitments (International Monetary Fund, 2006);
- the World Bank's *World Development Indicators* for general macroeconomic variables (World Bank, 2006b);
- the *Penn World Table* for data on Purchasing Power Parity (PPP) variables (Heston et al., 2006).

[5] We define market access as in Gelos et al. (2004, 2011). The countries we removed are those that never accessed international credit markets between 1980 and 2000, in accordance with the authors' definition. The complete country list can be found on page 29 of Gelos et al. (2004).

[6] As visible in Table A.4 of Appendix A, the Argentina default of 2001 is not present in our sample. This is because, as Kraay and Nehru (2006), we require that a crisis episode be preceded by three years without a crisis (in order to ensure exogeneity of regressors). In this specific case, Argentina is in crisis according to our criteria between 1983 and 1995, then between 1998 and 2009. The two year delay (1996 and 1997) between the two crisis episodes does not fulfill our criteria, and the second episode is therefore excluded from the database.

[7] One could argue that our third criterion for detecting episodes of distress (the balance of payments support by the IMF) does not characterize an outright default that has the potential of destroying a country's fundamentals. We therefore computed statistics on default episodes where only the first two criteria (arrears of payment and Paris club) are used. The number of default episodes is then 68, against 314 normal times episodes. The average default episode duration is 14.3 years, and the loss of GDP from peak to through is 1.6%. The fact that these statistics are similar to those of our main dataset shows that the IMF support criterion is not the main determinant behind the construction of our dataset of default episodes.

include the Ecuadorian debt crisis of 2000 for instance. Second, we restrict our analysis to the emerging countries that have access to private credit markets.

The interested reader can refer to Cohen and Valadier (2011) for additional insights on a very close dataset and for various descriptive statistics and econometric results extracted from it.

Another dataset of interest is that of Standard & Poor's (Beers and Chambers, 2006). It contains fewer events (59 ones), which are more severe than ours (when analyzed through the magnitude of GDP fall). Our database can then be thought of as debt distress rather than strict default as in the more demanding S&P database.

6. The econometric model

6.1. The estimated equations

Our empirical framework is given by the following system of three simultaneous equations. Since these three equations exhibit a circular dependency, there is an identification issue, which is dealt with in the following section.

$$d_{it} = X^d_{i,t-1}\,\eta^d + g_{it}\,X^{d,g}_{i,t-1}\,\eta^{d,g} + \varepsilon^d_{it} \tag{19}$$

$$g_{it} = X^g_{i,t-1}\,\eta^g + \delta_{it}\,X^{g,\delta}_{i,t-1}\,\eta^{g,\delta} + \varepsilon^g_{it} \tag{20}$$

$$\delta_{it} = 1_{\{X^{\delta}_{i,t-1}\,\eta^{\delta} + d_{it}\,X^{\delta,d}_{i,t-1}\,\eta^{\delta,d} + \varepsilon^{\delta}_{it} > 0\}} \tag{21}$$

where i indexes countries, t indexes time, d_{it} is the debt-to-GDP ratio, g_{it} is the percentage year-on-year growth rate of nominal US$ GDP, δ_{it} is a dummy indicating a debt crisis; the various components of $X_{i,t-1} = (X^d_{i,t-1}, X^{d,g}_{i,t-1}, X^g_{i,t-1}, X^{g,\delta}_{i,t-1}, X^{\delta}_{i,t-1}, X^{\delta,d}_{i,t-1})$ are row vectors of exogenous variables and the various components of $\eta = (\eta^d, \eta^{d,g}, \eta^g, \eta^{g,\delta}, \eta^{\delta}, \eta^{\delta,d})$ are column vectors of parameters of corresponding sizes; ε^d_{it}, ε^g_{it}, and $\varepsilon^{\delta}_{it}$ are stochastic exogenous shocks.

Equation (19) reflects the theoretical debt dynamics equation (18), and the shock ε^d_{it} is therefore interpreted as a unforeseen deviation from the Euler equation, for the reasons explained in section 4.4.

In the growth equation (20), the shock ε^g_{it} is the driver of the country's growth exogenous uncertainty. Depending on the occurrence of a debt crisis, growth can be endogenously reduced, as captured by the incidence of the δ_{it} variable on growth.

Finally, in the debt crisis equation (21), the shock $\varepsilon^{\delta}_{it}$ corresponds to the variability of the threshold level of debt default; it is the empirical counterpart of the shock on λ_t in the theoretical model, since this latter variable has an impact on the default threshold.[8] We have already referred to this shock as the *market shock* (see definition 3): it represents the ability of market investors to divert country resources in case of default, and more generally

[8] As can be seen from proposition 3 in Appendix B.1.

it captures market nervousness on property rights, contagion effects, and global financial crises.[9]

The following normal distribution is assumed for the three shocks (which in addition are supposed to be independent and identically distributed over periods and countries):

$$\begin{pmatrix} \varepsilon_{it}^{d} \\ \varepsilon_{it}^{g} \\ \varepsilon_{it}^{\delta} \end{pmatrix} \rightsquigarrow \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_d^2 & 0 & 0 \\ 0 & \sigma_g^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right)$$

Since equation (21) which defines the crisis dummy is essentially a probit, its identifiability is guaranteed by setting the variance of $\varepsilon_{it}^{\delta}$ to unity.

6.2. Identification and multiple equilibria

Since there is a circular dependency between the three endogenous variables, the econometric model cannot be identified at this stage. Indeed, for a given set of exogenous $X_{i,t-1}$ and for a given draw of the random variables ε_{it}^{d}, ε_{it}^{g} and $\varepsilon_{it}^{\delta}$, the model does not rule out the possibility of having two vectors $(d_{it}, g_{it}, \delta_{it})$ satisfying equations (19), (20) and (21): of these two vectors, one would be a no-crisis scenario ($\delta_{it} = 0$), and the other a crisis scenario ($\delta_{it} = 1$). This feature is precisely the possibility of multiple equilibria that we are trying to modelize.

Let g_{it}^{0} and d_{it}^{0} be the growth and the debt-to-GDP ratio conditional to no crisis occurring ($\delta_{it} = 0$). Conversely, let g_{it}^{1} and d_{it}^{1} be the growth and the debt-to-GDP ratio conditional to a crisis occurring ($\delta_{it} = 1$). We shall refer to d_{it}^{0} as to "pre-crisis" debt level and to d_{it}^{1} as to "post-crisis" level.

With these notations, a solution to equations (19), (20) and (21) is either $(d_{it}^{0}, g_{it}^{0}, 0)$ or $(d_{it}^{1}, g_{it}^{1}, 1)$.

In order to address the identification issue, a stochastic variable (with only two possible values) is introduced, which determines which equilibrium to choose: it is a *sunspot* variable, as it is sometimes called in the literature, *i.e.* a variable with no relation to economic fundamentals but which makes agents coordinate on one equilibrium when several are possible. We therefore introduce a fourth random variable ζ_{it} following a Bernoulli distribution of parameter p (that is: $\mathbb{P}(\zeta_{it} = 1) = p$ and $\mathbb{P}(\zeta_{it} = 0) = 1 - p$). The variable ζ_{it} is the sunspot whose role is to discriminate between the two equilibria when both are possible.

Given these extensions, it is now possible to describe how the model behaves. We assume that the parameter restrictions described in Appendix C are satisfied. Then, for a given set of exogenous $X_{i,t-1}$, and for a given draw of random variables ε_{it}^{d}, ε_{it}^{g}, $\varepsilon_{it}^{\delta}$ and ζ_{it}, only three cases are possible:

- The *crisis equilibrium*, inexorably driven by economic fundamentals, occurs when the crisis indicator is lighted at the pre-crisis debt levels d_{it}^{0}, *i.e.* when : $X_{i,t-1}^{\delta}\eta^{\delta} +$

[9] Of course, this shock captures all omitted variables in the debt crisis equation. But since we already control for the effect of debt and growth, we argue that a large fraction of the variance of that residual is explained by factors outside the control of the country, and therefore mostly market-related.

$d^0_{it}\, X^{\delta,d}_{i,t-1}\eta^{\delta,d} + \varepsilon^{\delta}_{it} > 0$. In that case, a no-crisis equilibrium is impossible, and because of equations (C.3), (C.4) and (C.5), one has $X^{\delta}_{i,t-1}\eta^{\delta} + d^1_{it}\, X^{\delta,d}_{i,t-1}\eta^{\delta,d} + \varepsilon^{\delta}_{it} > 0$, *i.e.* a crisis is triggered.

- The *no-crisis equilibrium* occurs when the crisis indicator is off even at the (worst) post-crisis debt levels d^1_{it}, *i.e.* when: $X^{\delta}_{i,t-1}\eta^{\delta} + d^1_{it}\, X^{\delta,d}_{i,t-1}\eta^{\delta,d} + \varepsilon^{\delta}_{it} < 0$. A crisis equilibrium is impossible, and because of equations (C.3), (C.4) and (C.5), one has $X^{\delta}_{i,t-1}\eta^{\delta} + d^0_{it}\, X^{\delta,d}_{i,t-1}\eta^{\delta,d} + \varepsilon^{\delta}_{it} < 0$, *i.e.* no crisis occurs.

- The *multiple equilibria case*, when $X^{\delta}_{i,t-1}\eta^{\delta} + d^1_{it}\, X^{\delta,d}_{i,t-1}\eta^{\delta,d} + \varepsilon^{\delta}_{it} > 0 > X^{\delta}_{i,t-1}\eta^{\delta} + d^0_{it}\, X^{\delta,d}_{i,t-1}\eta^{\delta,d} + \varepsilon^{\delta}_{it}$. Both equilibria are possible: at pre-crisis level it is avoided, at post-crisis level it is triggered. The outcome is given by the sunspot: $\delta_{it} = \zeta_{it}$ (and g_{it} and d_{it} are set accordingly, *i.e.* $g_{it} = g^{\delta_{it}}_{it}$ and $d_{it} = d^{\delta_{it}}_{it}$). A *self-fulfilling crisis* occurs if $\zeta_{it} = 1$: it could have been avoided had the sunspot had been different, since the fundamentals are compatible with a no-crisis equilibrium.

We model the risk of multiple equilibria as the risk that the good growth regime is turned into the bad growth one. The good growth regime can only be sustained when debt is low, the other one when the debt is high. But being on a "high" or "low" debt path is itself dependent on the growth regime, hence the risk of multiple equilibria. Given the fact that debt is paid out of GDP at large, we focus on growth of income rather than on income per head.

The derivation of the likelihood function of the model can be found in Appendix D.

6.3. Estimating the self-fulfilling effect

The econometric model presented in sections 6.1 and 6.2 is estimated with full information maximum likelihood (FIML) on the dataset of debt crisis episodes presented in section 5. Details about the estimation procedure can be found in Appendix E.

Table 1 reports the results for various specifications. The present section discusses only columns (1) and (2); the remaining ones will be discussed in the following section.

All exogenous variables are taken in $t-2$ (*i.e.* two years before the beginning of the episode). The parameter p governing the self-fulfilling risk is calibrated: its estimation has not been possible with a reasonable accuracy. Two different values have been used for its calibration: $p = 1$, which reflects the assumption that, when two equilibria are possible, the market always chooses the worst of the two; and $p = 0.5$, which means that, when there is a possibility of a self-fulfilling crisis, a coin is flipped and the crisis takes place half of the time.

The first panel of the table reports the debt-to-GDP ratio dynamics (equation (19)), the second panel reports the growth dynamics (equation (20)), and the third panel reports the crisis probability (equation (21)).

The current debt-to-GDP ratio is explained by past debt-to-GDP ratio and by the interaction of *current* growth with past debt-to-GDP ratio (under the category $\eta^{d,g}$). This

Table 1: Estimation results

	(1)	(2)	(3)	(4)	(5)
Debt/GDP ratio dynamics					
η^d: Debt/GDP $(t-2)$	1.204***	1.205***	1.104***	1.104***	1.021***
	(0.023)	(0.023)	(0.075)	(0.073)	(0.143)
η^d: Crisis prob $\times$ Growth gap $\hat{g}$ $(t/t-2)$			0.821**	0.825**	1.544*
			(0.262)	(0.266)	(0.746)
η^d: Growth $(t-2)$ $-$ Mean Growth $(t-2/t-4)$				-0.017	
				(0.212)	
$\eta^{d,g}$: Debt/GDP $(t-2)$ $\times$ Growth (t)	-1.722***	-1.719***	-1.651***	-1.669***	-1.492***
	(0.214)	(0.210)	(0.320)	(0.317)	(0.453)
σ_d	0.124***	0.125***	0.120***	0.121***	0.120***
	(0.006)	(0.006)	(0.008)	(0.008)	(0.009)
Growth dynamics					
η^g: Log per capita PPP real GDP $(t-2)$	-0.023**	-0.025**	-0.023**	-0.023**	-0.026***
	(0.008)	(0.008)	(0.007)	(0.007)	(0.007)
η^g: Growth $(t-2)$	0.281**	0.277**	0.281**	0.278**	0.298***
	(0.101)	(0.101)	(0.086)	(0.087)	(0.083)
η^g: Constant	0.268***	0.290***	0.271***	0.270***	0.294***
	(0.064)	(0.064)	(0.059)	(0.060)	(0.054)
$\eta^{g,\delta}$: Debt crisis dummy (t)	-0.059***	-0.077***	-0.062***	-0.061***	-0.061***
	(0.015)	(0.014)	(0.015)	(0.015)	(0.014)
σ_g	0.094***	0.093***	0.094***	0.094***	0.092***
	(0.004)	(0.004)	(0.004)	(0.004)	(0.006)
Debt crisis determinants					
η^δ: Log per capita PPP real GDP (t-2)	-0.365**	-0.426**	-0.356**	-0.363**	-0.340**
	(0.132)	(0.133)	(0.135)	(0.135)	(0.124)
η^δ: US$ GDP / PPP GDP (t-2)	1.477**	1.582**	1.454**	1.475**	1.128*
	(0.535)	(0.530)	(0.525)	(0.525)	(0.537)
η^δ: Constant	0.237	0.705	0.202	0.261	0.468
	(1.071)	(1.070)	(1.085)	(1.084)	(0.974)
$\eta^{\delta,d}$: Debt/GDP (t)	2.883***	2.971***	2.815***	2.801***	2.113***
	(0.456)	(0.465)	(0.429)	(0.430)	(0.362)
Calibrated parameter					
p: Sunspot Bernoulli parameter	1.000	0.500	1.000	1.000	1.000
Self-fulfilling probability	0.111	0.077	0.111	0.110	0.086
Self-enforcing probability			0.124	0.124	0.157
Number of observations	253	253	251	248	288
Log-likelihood	301.683	300.872	306.744	300.338	353.093
AIC	-579.366	-577.744	-587.489	-572.675	-680.186

second term is meant to capture the accounting effect of growth in the denominator of the debt-to-GDP ratio.

Current growth is explained by three factors: past growth, the occurrence of a crisis—which lowers the level of growth by a constant amount—, and the level of real GDP per capita—in order to capture the international convergence effect.

The occurrence or not of a debt crisis is explained by the *current* debt-to-GDP ratio, the level of real GDP per capita, and the overvaluation of the exchange rate (measured as the ratio of GDP expressed in current US$ to GDP expressed in international PPP US$). The level of real GDP per capita is included because richer countries seem to exhibit less crises in the data; the overvaluation of the exchange rate is meant to capture the fact that currency misalignment increases the risk of a currency crisis which in turn increases the risk of a debt crisis, since debt is generally denominated in foreign currency.

In Table 1, one can see in columns (1) and (2) that most of the parameters of interest are estimated with the expected sign and with a good accuracy.

As expected, the debt dynamics exhibits high inertia (the coefficient on past debt-to-GDP is close to unity), and the interaction of *current growth* with past debt-to-GDP ratio has a strong effect (with a coefficient close to -2, which is logical given the fact that lagged variables are taken two periods in the past).

The growth dynamics has some serial auto-correlation, though not very high. The convergence effect of poor countries clearly appears. And, as expected, a debt crisis lowers the level of growth by more than 5% on average.

The estimators for the determinants of debt crises are also consistent: debt crises are made more likely by a high *current* debt-to-GDP ratio, low real income level and overvaluation of the domestic currency.

In addition to the results of parameter estimations, the tables also report information about the percentage of crises that were of a self-fulfilling nature. Indeed, with this econometric model, it is possible for a given crisis to compute the *a posteriori* probability that it was of a self-fulfilling nature, by opposition to being solely driven by fundamentals and exogenous shocks (see below section 6.5). The line entitled "Self-fulfilling probability" in the tables reports the mean of that probability over all the crises in the dataset. In column (1) where p is calibrated to 1—that is, when the markets are considered as "panic prone"—about 11% of debt crises are reported as being self-fulfilling. In column (2), where p is calibrated to 0.5, the proportion of self-fulfilling crises is consistently almost halved, being around 7%.

6.4. Estimating the Panglossian effect

Since our theoretical model predicts that some countries will adopt a prudent behavior while others will accumulate debt, ignoring the risk of a crisis in the Panglossian mode (see section 4.3), this hypothesis is tested in the data. More precisely, using equation (15), we construct a proxy variable for the Panglossian effect which we define as $\pi_{it}(g_{it}^{+} - g_{it}^{-})$; this expression is then introduced as an explanatory variable in the debt dynamics equation (18).

The first step for constructing this variable consists in estimating π_{it}, the probability of a debt crisis for country i at date t, given variables in $t-2$. For that purpose, a simple probit is estimated on the dataset of episodes, where the probability of a debt crisis is a function of several exogenous variables.[10] This makes it possible to compute at every date the probability of a debt crisis two periods ahead, as predicted by the probit model, independently of the actual realization or not of a crisis.[11]

The second step consists in estimating the *growth gap* $\hat{g}_{it} = g_{it}^{+} - g_{it}^{-}$, *i.e.* the expected growth conditionally on no crisis occurring minus the expected growth conditionally on a crisis occurring. For a given π_{it}, the corresponding growth gap $g_{it}^{+} - g_{it}^{-}$ is approximated by taking the mean growth rate (across the whole data sample) above and below the quantile π_{it}.[12]

The Panglossian variable thus constructed is used in the estimations of columns (3) and (4) of Table 1. Note that since the Panglossian effect is a generated regressor, the standard errors of the parameter estimates—as generated by the FIML estimator—need to be corrected to take into account the sampling error of the first step probit. For that purpose, we implemented the generic method proposed by Murphy and Topel (1985) for two stage maximum likelihood estimation.

The estimation reported in column (3) shows that the Panglossian effect enters in the debt dynamics equation, consistently with the theoretical model. Its coefficient has the expected sign and is significant at the 0.2% level.

The table also reports information about the percentage of crises that are of a self-enforcing nature, *i.e.* that are the direct consequence of the Panglossian effect. More precisely, after having canceled the self-fulfilling effect, one can compute the probability that a crisis would not have occurred if the Panglossian effect had not been operative between $t-2$ and t. The self-fulfilling and the self-enforcing probabilities thus computed are additive by construction. This leads, on average, to a self-enforcing probability of about 12%.

Note that the self-enforcing probabilities reported here only take into account the impact of the Panglossian effect between dates $t-2$ and t. In section 6.6 below we present another quantitative measure of the importance of the Panglossian effect, taking into account its cumulative effect over a longer period.

Robustness checks are also performed in order to show that what is measured with the

[10] Those variables are: the debt-to-GDP ratio, the log of per capita real PPP GDP, the total debt service-to-exports ratio and the overvaluation of the exchange rate (measured by US$ GDP to PPP GDP ratio). All exogenous are taken two years before the beginning of the episode. The methodology is exactly that of Kraay and Nehru (2006), using a slightly different set of exogenous variables.

[11] One possible criticism against this methodology is that the crisis probability as defined by a probit is not consistent with the crisis probability as defined by the larger system of simultaneous equations. The main reason for adopting this methodology is that estimating a model-consistent probability is a very difficult problem from a computational point of view: it involves the computation of a fixed point in the maximum likelihood estimation, and there is no well-known methodology for computing the standard errors of the coefficients thus estimated. From an economic point of view, the methodology that we adopt is equivalent to the hypothesis that agents in the economy only know the probit model, but not the simultaneous equations model, and use the probit model to form their expectations about the future. This hypothesis is not fully satisfactory, but can nevertheless be justified by the fact that crisis forecasting is generally done with very simple models, as the probit one, both in policy institutions and in credit rating agencies.

[12] Note that this method is rigorously exact when a common factor drives (up to uncorrelated disturbances) the determinant of growth and that of the probability of default.

Panglossian variable is indeed the effect exhibited in the theoretical model and not a proxy for another economic mechanism.

One may argue that the Panglossian variable is simply a proxy for "bad news," and that the increase in debt that is measured when such a bad news occurs would also be predicted by standard inter-temporal consumption smoothing. To test that hypothesis, we construct a measure of the business cycle, equal to growth in $t-2$ minus mean growth over $t-4$ to $t-2$. If the inter-temporal consumption smoothing hypothesis was true, this variable should enter in the debt dynamics, since it captures temporary shocks. On the contrary, the results in column (4) show that this variable is not significant and does not diminish the explanatory power of the Panglossian variable.

We also performed other robustness checks not reported on Table 1, but available upon request. First, one may argue that what is captured in the Panglossian effect is simply the mechanical effect of the risk premium asked by investors when the level of risk is higher. We therefore tested the Panglossian variable against the variable $\pi_{it}\frac{D_{i,t-2}}{Q_{i,t-2}}$, which is a proxy for the risk premium effect (since the risk premium is supposed to be highly correlated with the crisis probability). The results show that the Panglossian variable remains significant—though at a lower level—while the risk premium variable is not significant and has the wrong sign. Secondly, following Moral-Benito (2012) who shows that the most robust determinants to growth are the price of investment goods, distance to major world cities, and political rights, we tested the explanatory power of those variables in the growth equation of specification (3). It turns out that none of these three variables is significant in our estimation, whether taken collectively or individually, and that the quantitative and qualitative results of our baseline estimation remain essentially unchanged.

Finally, in column (5) we present estimates of the same model, with data including the latest financial crisis (up to 2011). The point estimate of the coefficient of the Panglossian effect is larger, although less accurately measured. Although the results do not reject our (pre-Lehman) model of sovereign risk, more work is clearly needed to account for the specific way by which the Lehman/Greek crisis has differentially affected the sovereign bond market.

We also tried additional robustness check, by incorporating other measures of debt crisis such as the S&P data base. Our results (available on request) show that a default, in the S&P definition, entails a larger (negative) economic impact, with identical debt dynamics almost unchanged with respect to the results presented here.

6.5. A posteriori self-fulfilling probabilities

For each crisis in the sample, it is possible to compute the *a posteriori* probability that it was self-fulfilling.[13] The results of this computation are given in Table 2. The crises are ordered by their likelihood of being self-fulfilling episodes. The probabilities are computed on the basis of specification (3) in Table 1. Note that in this specification, the self-fulfilling parameter p is calibrated to 1; a lower value would give correspondingly lower self-fulfilling probabilities. The values reported can therefore be considered as upper bounds of the real

[13]See Appendix D for the methodological details.

probabilities. Also note that, for the crises that start after 2002, the reported probabilities have been computed off-sample (because specification (3) in Table 1 incorporates only crises up to 2002).

Table 2: Individual crises self-fulfilling probabilities

Country	Year	Crisis length	Self-fulfill prob. (in %)
Jordan	1989	16	0.2
Somalia	1981	24	1.4
Rwanda	1994	11	1.4
Congo, Rep.	1985	20	1.6
Nigeria	1986	19	1.9
Cote d'Ivoire	1981	16	3.1
Guinea-Bissau	1981	23	3.7
Benin	2009	3	3.8
Madagascar	1980	25	4.5
Congo, Dem. Rep.	1976	29	4.6
Turkey	1978	7	4.6
Uruguay	1983	4	5.0
Ethiopia	1991	14	5.1
Benin	1983	16	5.4
Benin	1970	9	5.9
Chile	1983	7	6.5
India	1981	3	6.6
Dominican Republic	2003	5	7.2
Egypt, Arab Rep.	1977	4	7.8
Uruguay	2002	3	7.9
Mexico	1983	10	8.0
Sudan	1977	28	9.0
Gabon	1986	19	9.1
Peru	1977	4	9.9
Ghana	1970	7	10.0
Solomon Islands	2002	3	10.3
Brazil	1998	7	10.3
Kenya	1975	3	10.6
Pakistan	1972	5	10.8
Senegal	1980	23	10.8
Philippines	1976	3	10.9
Paraguay	1986	9	11.5

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Country	Year	Crisis length	Self-fulfill prob. (in %)
Brazil	1983	3	11.6
Niger	1983	22	11.9
Ecuador	2000	5	12.4
Dominica	2005	5	12.5
Kenya	1992	5	12.6
Bangladesh	1979	3	12.9
Honduras	1979	23	13.0
Egypt, Arab Rep.	1984	12	13.0
Colombia	1999	3	13.2
Ukraine	2008	3	13.4
Dominican Republic	1983	17	14.2
Sri Lanka	2005	3	14.5
Turkey	1999	6	14.5
Kenya	2000	3	14.7
Indonesia	1970	3	14.7
Ecuador	1983	14	14.8
Jamaica	1977	24	14.9
Comoros	1987	18	15.2
Tunisia	1986	6	15.3
Ghana	1996	3	15.5
Algeria	1994	4	15.7
Chile	1972	5	15.8
Morocco	1980	15	15.9
Trinidad and Tobago	1988	5	16.0
Thailand	1997	3	16.3
Costa Rica	1980	16	16.3
Cameroon	1987	18	16.6
Pakistan	1980	4	17.0
Kyrgyz Republic	2002	3	18.1
Pakistan	1994	10	18.7
Venezuela, RB	1989	4	19.3
Indonesia	1997	8	19.6
El Salvador	1990	3	19.9
Argentina	1983	13	20.3

In words, the Jordan crisis of 1989 or the Rwandan crisis of 1994 were almost surely not created by a self-fulfilling process. They could not have been avoided by simply restoring confidence.

In contrast, the crises of Argentina in 1983, El Salvador in 1990 or Indonesia in 1997 may have been self-fulfilling. There is about one chance in five that they could have been avoided if confidence had been maintained and panic avoided.

6.6. Simulating the model

We now turn to the simulation of the estimated model. The strategy is to simulate the dynamic model described by equations (19), (20) and (21) over several periods, for a given trajectory of the random draws $(\varepsilon_{it}^d, \varepsilon_{it}^g, \varepsilon_{it}^\delta)$ and of the exogenous values X_{it}, given the estimated parameters η.

More precisely, we simulate the specification reported in column (3) of Table 1, for given values of both the set of exogenous and of parameters (as obtained by maximum-likelihood estimation). The log of per capital PPP real GDP and of the US$ GDP to PPP GDP ratio are set constant across time and equal to the sample mean. The starting point of the simulations is a debt-to-GDP ratio of 60%. The probability π_{it} used for the Panglossian effect is recomputed at each period, using the simple probit described in section 6.4. We simulate 2500 series of 5 periods (*i.e.* of 10 years, since lagged variables are taken 2 years earlier).

The dynamics of the model are affected by four shocks that may be switched off for comparison purposes: shocks to the law of motion of debt (ε_{it}^d), to growth (ε_{it}^g), to the crisis equation (ε_{it}^δ), plus the sunspot (ζ_{it}). We also consider simulations where the Panglossian effect is switched off (just by removing the corresponding term in the debt equation). Thus, there is a total of $2^5 = 32$ possible combinations according to whether some of these five effects are activated or not.

When the five effects are activated, 89.4% of the simulations exhibit a crisis episode in at least one of the 5 simulation periods. This high occurrence rate of crisis is the consequence of the relatively high level of the debt-to-GDP ratio that has been chosen as the starting point for simulations.

In order to compute the contribution of each of these five effects to these crises, each of them is shut off one by one, and we observe by how much the number of crises diminishes, which gives the contribution of each one.[14]

Table 3 reports the contribution of each effect so computed: it shows the percentage of crisis episodes that can be considered as a direct consequence of each effect.

One can see that the largest contributor is the market shock ε_{it}^δ which explains more than 55% of crises. This shows that if there was a known and deterministic debt-to-GDP threshold above which default occurs, then the risk of a debt crisis would be more than halved. The remaining 45% of crises are directly due to the law of motion of the country's debt ratio. Within that category, the Panglossian effect (12%) is of the same order of magnitude as the debt shock (15%), which means that the "strategic" accumulation of excessive debt is about as important as the uncontrolled part of this accumulation. The lack of predictability of growth (11%) is also of the same order of magnitude as the Panglossian effect. In our simulations, the self-fulfilling part of the debt crises comes last (6%); although this is the lesser part of our decomposition, it nevertheless represents more than half of the risk that is due to the vagaries of the growth rate.

[14] An issue is that the results depend on the order in which the effects are shut down: this problem is solved by making these computations for the $5! = 120$ possible orders, and by computing the average contributions.

Table 3: Simulated contributions of shocks and Panglossian effect to crises

Effect	Contribution
Market shock ($\varepsilon_{it}^{\delta}$)	55.8%
Debt shock (ε_{it}^{d})	15.2%
Panglossian effect	12.0%
Growth shock (ε_{it}^{g})	11.0%
Self-fulfilling effect (ζ_{it})	6.1%
Total	100.0%

Monte Carlo simulations of the estimated model (column (3) of Table 1). Results computed over 2,500 simulations of a 10-year duration and starting from a debt-to-GDP ratio of 60%.

7. Conclusion

We have tried to distinguish two attitudes towards debt: the attitude of prudent borrowers, who attempt to stabilize their debt at low levels, even in the event of an adverse shock, and Panglossian borrowers, who only take into account the best scenarios possible, rationally anticipating to default on their debt if hit by an unfavorable shock (or by a sequence of them). We have shown empirically that this distinction is consistent with the data.

We also have distinguished two types of debt crises: those that are the effect of an exogenous shock, and those that are created in a self-fulfilling manner by the financial markets themselves. We have shown that the large majority of crises are of the first kind, although the probability of self-fulfilling cases is not negligible.

These results have a few policy implications that we leave to future work. For one thing, if the earthquake model is correct, then there is room for improving the stability of financial markets by the use of more conditional sovereign lending, contingent on other lenders following suit. It indeed remains a question to understand why sovereign debt arrangements contain so few contingency clauses.

Regarding the self-fulfilling case, if our results can be trusted, while the now old debate on sovereign debt restructuring remains important, it may be relatively less so than finding more innovative sources of financing.

Aguiar, M., Gopinath, G., June 2006. Defaultable debt, interest rates and the current account. Journal of International Economics 69 (1), 64–83.
URL http://ideas.repec.org/a/eee/inecon/v69y2006i1p64-83.html

Alesina, A., Tabellini, G., July 1990. A Positive Theory of Fiscal Deficits and Government Debt. Review of Economic Studies 57 (3), 403–14.
URL http://ideas.repec.org/a/bla/restud/v57y1990i3p403-14.html

Arellano, C., June 2008. Default risk and income fluctuations in emerging economies. The

American Economic Review 98 (3), 690–712.
URL http://ideas.repec.org/a/aea/aecrev/v98y2008i3p690-712.html

Beers, D. T., Chambers, J., Sep. 2006. Sovereign defaults at 26-year low, to show little change in 2007. Tech. rep., Standard & Poor's.

Beetsma, R., Mavromatis, K., 2014. An analysis of eurobonds. Journal of International Money and Finance 45 (C), 91–111.
URL http://ideas.repec.org/a/eee/jimfin/v45y2014icp91-111.html

Blanchard, O. J., April 1985. Debt, deficits, and finite horizons. Journal of Political Economy 93 (2), 223–47.
URL http://ideas.repec.org/a/ucp/jpolec/v93y1985i2p223-47.html

Bulow, J. I., Rogoff, K., Jun. 1988. Sovereign debt restructurings: Panacea or pangloss? NBER Working Papers 2637, National Bureau of Economic Research, Inc.
URL http://ideas.repec.org/p/nbr/nberwo/2637.html

Byrd, R. H., Lu, P., Nocedal, J., Zhu, C., Zhu, C., 1994. A limited memory algorithm for bound constrained optimization. SIAM Journal on Scientific Computing 16, 1190–1208.

Calvo, G. A., September 1988. Servicing the public debt: The role of expectations. American Economic Review 78 (4), 647–61.
URL http://ideas.repec.org/a/aea/aecrev/v78y1988i4p647-61.html

Campos, C. F., Jaimovich, D., Panizza, U., September 2006. The unexplained part of public debt. Emerging Markets Review 7 (3), 228–243.
URL http://ideas.repec.org/a/eee/ememar/v7y2006i3p228-243.html

Chamon, M., January 2007. Can debt crises be self-fulfilling? Journal of Development Economics 82 (1), 234–244.
URL http://ideas.repec.org/a/eee/deveco/v82y2007i1p234-244.html

Cohen, D., Portes, R., Mar. 2006. Toward a lender of first resort. IMF Working Papers 06/66, International Monetary Fund.
URL http://ideas.repec.org/p/imf/imfwpa/06-66.html

Cohen, D., Valadier, C., Feb. 2011. 40 years of sovereign debt crises. CEPR Discussion Papers 8269, Centre for Economic Policy and Research.
URL http://ideas.repec.org/p/cpr/ceprdp/8269.html

Cole, H. L., Kehoe, T. J., November 1996. A self-fulfilling model of mexico's 1994–1995 debt crisis. Journal of International Economics 41 (3-4), 309–330.
URL http://ideas.repec.org/a/eee/inecon/v41y1996i3-4p309-330.html

Cole, H. L., Kehoe, T. J., January 2000. Self-fulfilling debt crises. Review of Economic Studies 67 (1), 91–116.
URL http://ideas.repec.org/a/bla/restud/v67y2000i1p91-116.html

Eaton, J., Gersovitz, M., April 1981. Debt with potential repudiation: Theoretical and empirical analysis. Review of Economic Studies 48 (2), 289–309.
URL http://ideas.repec.org/a/bla/restud/v48y1981i2p289-309.html

Gelos, R. G., Sahay, R., Sandleris, G., Dec. 2004. Sovereign borrowing by developing countries: What determines market access? IMF Working Papers 04/221, International Monetary Fund.
URL http://ideas.repec.org/p/imf/imfwpa/04-221.html

Gelos, R. G., Sahay, R., Sandleris, G., March 2011. Sovereign borrowing by developing countries: What determines market access? Journal of International Economics 83 (2), 243–254.
URL http://ideas.repec.org/a/eee/inecon/v83y2011i2p243-254.html

Heston, A., Summers, R., Aten, B., September 2006. Penn world table version 6.2. Tech. rep., Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania.

International Monetary Fund, October 2006. International Financial Statistics Yearbook, 2006. IMF.

Kraay, A., Nehru, V., 2006. When is external debt sustainable? World Bank Economic Review 20 (3), 341–365.
URL http://ideas.repec.org/a/oup/wbecrv/v20y2006i3p341-365.html

Krugman, P. R., 1998. What happened to Asia? http://web.mit.edu/krugman/www/DISINTER.html.

Moral-Benito, E., May 2012. Determinants of Economic Growth: A Bayesian Panel Data Approach. The Review of Economics and Statistics 94 (2), 566–579.
URL http://ideas.repec.org/a/tpr/restat/v94y2012i2p566-579.html

Murphy, K. M., Topel, R. H., October 1985. Estimation and inference in two-step econometric models. Journal of Business & Economic Statistics 3 (4), 370–79.
URL http://ideas.repec.org/a/bes/jnlbes/v3y1985i4p370-79.html

R Development Core Team, 2011. R: A Language and Environment for Statistical Computing. R Foundation for Statistical Computing, Vienna, Austria, ISBN 3-900051-07-0.
URL http://www.R-project.org

World Bank, June 2006a. Global Development Finance 2006: The Development Potential of Surging Capital Flows. World Bank.

World Bank, 2006b. World Development Indicators 2006. World Bank.

Yaari, M., 1965. Uncertain lifetime, life insurance, and the theory of the consumer. Review of Economic Studies 32 (2), 137–150.

Appendix A. Debt crises

Tables A.4 and A.5 present the complete list of the crisis episodes identified according to the methodology of section 5.

In Table A.4, for each crisis episode, the first three columns give the country, the year of the crisis outbreak, and the number of years it lasted. The columns labeled "Type of crisis" tell whether the crisis was characterized by a Paris Club relief, accumulated arrears or IMF intervention (or several of these options). The last column is the debt-to-GDP ratio at three points in time (3 years before the outbreak, in the year of the outbreak and three years later)

Table A.5 gives other macroeconomic indicators about the country: the debt-to-PPP-GDP ratio (at the same three points in time), the debt service-to-exports ratio, the mean annual growth before the crisis and the mean effective interest rate charged on the debt before the crisis.

Table A.4: List of crisis episodes (with debt indicators)

Country	Year	Length	Type of crisis			D/GDP		
			Paris Club	*Arrears*	*SBA/EFF*	$t-3$	t	$t+3$
Indonesia	1970	3	Y	N	N	46.9	46.9	42.3
Benin	1970	9	N	Y	N	12.5	12.5	11.7
Ghana	1970	7	N	Y	N	25.8	25.8	30.6
Guinea	1970	35	Y	Y	Y	NA	NA	NA
Chile	1972	5	Y	Y	Y	33.1	30.7	76.4
Pakistan	1972	5	Y	N	N	34.0	43.7	50.7
Tanzania	1972	33	Y	Y	Y	NA	NA	NA
Kenya	1975	3	N	N	Y	27.6	39.6	41.0
Congo, Dem. Rep.	1976	29	Y	Y	Y	13.2	30.2	30.0
Philippines	1976	3	N	N	Y	27.4	35.3	48.3
Jamaica	1977	24	Y	Y	Y	60.4	51.7	71.4
Egypt, Arab Rep.	1977	4	N	N	Y	24.5	80.2	83.5
Peru	1977	4	Y	N	Y	38.8	64.4	45.4
Sudan	1977	28	Y	Y	Y	28.9	35.1	68.0
Panama	1978	3	N	N	Y	50.4	93.8	77.7
Turkey	1978	7	Y	N	Y	10.9	22.3	28.9
Honduras	1979	23	Y	Y	Y	27.1	52.6	63.5

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Country	Year	Length	Type of crisis			D/GDP		
			Paris Club	*Arrears*	*SBA/EFF*	$t-3$	t	$t+3$
Bangladesh	1979	3	N	N	Y	19.8	19.5	28.0
Mauritius	1979	3	N	N	Y	NA	NA	53.3
Costa Rica	1980	16	Y	Y	Y	42.9	56.8	133.1
Madagascar	1980	25	Y	Y	Y	33.1	30.6	57.9
Senegal	1980	23	Y	Y	Y	31.7	49.3	83.8
Morocco	1980	15	Y	Y	Y	50.8	51.7	93.5
Pakistan	1980	4	Y	N	Y	50.0	41.9	41.9
India	1981	3	N	N	Y	12.4	12.1	16.5
Romania	1981	5	Y	N	Y	NA	NA	NA
Cote d'Ivoire	1981	16	Y	Y	Y	48.6	96.5	124.9
Guinea-Bissau	1981	23	Y	Y	N	43.9	97.8	183.1
Somalia	1981	24	Y	Y	Y	91.8	151.0	190.0
Argentina	1983	13	Y	Y	Y	35.3	44.2	47.3
Niger	1983	22	Y	Y	Y	34.4	52.7	74.3
Benin	1983	16	Y	Y	N	30.2	68.1	74.0
Brazil	1983	3	Y	N	Y	30.4	48.5	40.7
Chile	1983	7	Y	N	Y	43.8	90.7	119.3
Dominican Republic	1983	17	Y	Y	Y	30.2	34.0	60.2
Ecuador	1983	14	Y	Y	Y	50.4	67.9	90.5
Mexico	1983	10	Y	N	Y	29.5	62.5	77.9
Uruguay	1983	4	N	N	Y	16.4	64.8	66.7
Egypt, Arab Rep.	1984	12	Y	Y	Y	94.3	105.1	109.0
Congo, Rep.	1985	20	Y	Y	Y	91.8	141.2	184.9
Lebanon	1986	6	N	Y	N	NA	NA	37.7
Sao Tome and Principe	1986	19	Y	Y	N	83.9	122.9	291.9
Gabon	1986	19	Y	Y	Y	27.0	57.1	80.0
Nigeria	1986	19	Y	Y	Y	50.2	109.9	126.3
Paraguay	1986	9	N	Y	N	25.2	58.9	54.6
Tunisia	1986	6	N	N	Y	48.6	65.9	69.0
Cameroon	1987	18	Y	Y	Y	37.2	37.9	59.7
Comoros	1987	18	N	Y	N	97.5	103.5	71.8
Trinidad and Tobago	1988	5	Y	Y	Y	19.6	46.7	46.7
Vietnam	1988	17	Y	Y	Y	0.4	2.4	243.4
Jordan	1989	16	Y	Y	Y	78.2	177.2	150.0
Venezuela, RB	1989	4	N	N	Y	58.3	76.8	64.7
El Salvador	1990	3	Y	N	N	50.2	44.8	29.3
Seychelles	1990	15	N	Y	N	69.4	49.7	38.7

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Country	Year	Length	Type of crisis			D/GDP		
			Paris Club	*Arrears*	*SBA/EFF*	$t-3$	t	$t+3$
Ethiopia	1991	14	Y	Y	N	99.9	95.8	181.7
Kenya	1992	5	Y	Y	N	71.2	83.9	80.8
Algeria	1994	4	Y	N	Y	62.3	71.1	64.5
Rwanda	1994	11	Y	Y	N	42.4	126.6	60.1
Pakistan	1994	10	Y	N	Y	51.4	52.8	48.2
Ghana	1996	3	Y	N	N	76.7	83.6	83.3
Indonesia	1997	8	Y	Y	Y	61.0	63.1	87.5
Thailand	1997	3	N	N	Y	45.3	72.7	64.9
Brazil	1998	7	N	N	Y	22.8	30.7	45.5
Colombia	1999	3	N	N	Y	29.7	39.9	40.7
Turkey	1999	6	N	N	Y	44.1	55.6	71.3
Kenya	2000	3	Y	N	N	49.3	48.4	45.6
Ecuador	2000	5	Y	N	Y	65.2	86.0	62.0
Kyrgyz Republic	2002	3	Y	N	N	139.0	115.3	NA
Solomon Islands	2002	3	N	Y	N	49.7	79.5	NA
Uruguay	2002	3	N	N	Y	35.3	86.4	NA
Dominican Republic	2003	5	Y	N	Y	19.4	34.2	25.6
Dominica	2005	5	N	Y	N	76.1	76.7	67.2
Sri Lanka	2005	3	Y	N	N	57.2	47.0	38.6
Ukraine	2008	3	N	Y	Y	39.1	54.6	82.8
Benin	2009	3	N	Y	NA	13.8	20.0	27.2

Table A.5: List of crisis episodes (with other macro indicators)

Country	Year	D/PPP-GDP			TDS/X	Growth	Interest rate
		$t-3$	t	$t+3$		$t-3\ldots t-1$	$t-3\ldots t-1$
Indonesia	1970	16.0	16.0	16.0	13.0	6.9	1.0
Benin	1970	5.7	5.7	6.0	3.4	2.6	1.0
Ghana	1970	12.6	12.6	13.4	10.8	3.2	2.1
Guinea	1970	14.6	14.6	22.8	NA	NA	1.3
Chile	1972	17.2	17.5	27.6	27.3	4.9	3.5
Pakistan	1972	14.5	14.8	14.3	33.2	5.8	2.0
Tanzania	1972	8.6	49.4	63.7	NA	NA	0.9
Kenya	1975	12.0	21.3	23.6	8.6	9.0	3.5
Congo, Dem. Rep.	1976	9.8	28.9	26.0	9.2	2.1	3.4
Philippines	1976	7.3	11.2	16.9	22.5	6.0	3.3

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Country	Year	D/PPP-GDP			TDS/X	Growth	Interest rate
		$t-3$	t	$t+3$		$t-3\ldots t-1$	$t-3\ldots t-1$
Jamaica	1977	43.8	41.8	43.2	37.2	-3.7	6.9
Egypt, Arab Rep.	1977	7.3	26.0	25.9	16.3	8.7	1.7
Peru	1977	19.7	27.0	18.6	42.5	4.9	5.0
Sudan	1977	18.7	29.2	46.1	12.7	14.6	1.8
Panama	1978	30.2	53.5	47.8	NA	1.5	4.4
Turkey	1978	9.2	19.3	18.2	19.1	7.0	5.6
Honduras	1979	13.2	28.1	33.4	29.1	10.3	5.1
Bangladesh	1979	5.0	5.4	6.7	28.8	5.1	1.9
Mauritius	1979	2.7	12.0	14.9	NA	NA	3.3
Costa Rica	1980	20.6	31.5	43.8	22.0	6.7	5.0
Madagascar	1980	18.4	24.3	32.2	37.7	3.2	2.5
Senegal	1980	15.8	29.6	31.2	7.1	0.1	3.8
Morocco	1980	24.6	29.6	30.6	19.1	4.4	4.8
Pakistan	1980	14.8	12.7	10.6	31.9	5.3	2.5
India	1981	4.2	4.0	4.5	16.1	2.4	2.7
Romania	1981	1.9	13.5	7.8	NA	NA	4.3
Cote d'Ivoire	1981	37.6	62.3	47.5	16.4	0.8	6.5
Guinea-Bissau	1981	22.9	44.9	67.5	10.5	-0.3	1.0
Somalia	1981	16.1	27.3	36.2	3.0	-1.0	0.3
Argentina	1983	15.9	23.7	24.6	107.4	-2.2	8.8
Niger	1983	23.4	19.0	26.9	22.9	-0.0	9.2
Benin	1983	21.5	30.8	29.3	9.1	6.3	3.0
Brazil	1983	15.2	18.9	15.1	69.4	1.8	12.1
Chile	1983	29.1	42.1	41.4	43.0	0.9	11.7
Dominican Republic	1983	15.6	15.9	17.6	29.8	3.9	9.1
Ecuador	1983	22.0	24.5	28.0	34.0	2.4	9.4
Mexico	1983	20.2	25.8	26.8	52.7	5.8	12.0
Uruguay	1983	12.3	24.4	23.7	19.6	-0.8	9.2
Egypt, Arab Rep.	1984	26.1	29.5	37.6	19.9	7.0	4.3
Congo, Rep.	1985	56.3	75.8	123.5	20.2	12.1	5.7
Lebanon	1986	NA	NA	NA	NA	NA	8.0
Sao Tome and Principe	1986	46.5	76.5	117.5	24.2	NA	1.7
Gabon	1986	19.1	35.7	51.0	11.7	3.6	7.9
Nigeria	1986	37.5	42.2	36.9	53.8	-0.1	9.4
Paraguay	1986	12.8	16.3	14.1	13.7	1.3	4.0
Tunisia	1986	17.1	22.0	21.5	22.2	5.4	5.9
Cameroon	1987	15.4	21.0	28.1	15.9	7.4	6.2

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Country	Year	D/PPP-GDP			TDS/X	Growth	Interest rate
		$t-3$	t	$t+3$		$t-3\dots t-1$	$t-3\dots t-1$
Comoros	1987	20.4	34.0	26.9	28.9	2.8	1.2
Trinidad and Tobago	1988	13.8	23.9	24.0	11.0	-4.0	7.6
Vietnam	1988	NA	NA	28.2	NA	3.4	0.5
Jordan	1989	44.9	70.3	63.2	32.8	2.7	6.0
Venezuela, RB	1989	38.9	29.7	27.7	43.7	5.3	8.6
El Salvador	1990	14.7	14.2	10.5	34.6	1.8	3.9
Seychelles	1990	43.8	31.8	24.0	9.1	7.0	5.2
Ethiopia	1991	46.7	47.2	37.2	48.6	0.9	1.0
Kenya	1992	25.8	25.6	22.3	37.2	3.4	4.6
Algeria	1994	24.7	26.8	22.4	68.9	-0.5	7.1
Rwanda	1994	10.3	25.0	16.9	16.5	-1.6	1.3
Pakistan	1994	10.3	10.1	9.5	25.4	4.8	3.6
Ghana	1996	23.6	25.7	23.9	24.4	4.1	2.1
Indonesia	1997	16.9	16.5	17.1	30.4	7.9	5.0
Thailand	1997	18.4	26.3	19.7	14.0	8.0	4.3
Brazil	1998	15.1	20.7	18.0	39.7	3.4	6.2
Colombia	1999	13.5	15.2	13.0	36.6	2.0	6.4
Turkey	1999	25.0	28.1	33.6	28.0	5.9	5.7
Kenya	2000	17.7	16.0	16.7	22.1	2.0	3.0
Ecuador	2000	29.3	24.6	26.7	31.2	-0.0	6.0
Kyrgyz Republic	2002	12.3	10.7	NA	20.9	4.8	3.3
Solomon Islands	2002	15.5	20.5	NA	5.0	-7.9	1.9
Uruguay	2002	20.7	34.3	NA	27.6	-2.6	6.8
Dominican Republic	2003	8.5	12.0	11.1	6.0	4.4	4.8
Dominica	2005	70.3	64.9	54.4	10.5	2.8	3.0
Sri Lanka	2005	19.7	18.0	19.0	11.9	5.1	2.0
Ukraine	2008	12.2	23.7	NA	12.9	6.0	4.1
Benin	2009	6.8	11.6	NA	4.8	4.5	2.8

Appendix B. Extra theoretical results and proofs

This section establishes extra results for the theoretical model.

Appendix B.1. Statics

Lemma 1. *Lenders will not lend today more than the present value of the wealth expected tomorrow,* i.e. *one has:*

$$\forall L \in \mathscr{L}(Q),\ (1+r)L \leq \frac{\bar{g}Q}{1-\frac{\bar{g}}{1+r}}$$

Proof. This is the intuitive consequence of the fact that debt is repaid out of the country's GDP, that consumption must be positive and that Ponzi games are excluded. □

Corollary 1. *The country always defaults if its debt is higher than its wealth,* i.e. *one has:*

$$d^*(\Lambda) \leq \frac{1}{1 - \frac{\bar{g}}{1+r}}$$

Proof. Suppose the economy is in a state (D, Q, λ) such that:

$$D > \frac{Q}{1 - \frac{\bar{g}}{1+r}}$$

If the country decides to repay, lemma 1 shows that it can borrow at most $L = \frac{\bar{g}Q}{(1+r)\left(1 - \frac{\bar{g}}{1+r}\right)}$.

It is easy to see that country consumption $Q - D + L$ cannot be positive in that case. So default is the only option. □

Lemma 2. *The country does not default if it has access to a contract which gives him a higher* current *consumption level than in case of default. Formally, given D, Q and λ, if there exists $L \in \mathscr{L}(Q)$ such that $L - D \geq -\lambda\mu Q - (1-\mu)Q$, then default is not optimal,* i.e. *$J^r(D, Q) \geq J^d(Q, \lambda)$.*

Proof. Let $L \in \mathscr{L}(Q)$ such that $L - D \geq -\lambda\mu Q - (1-\mu)Q$. By definition, one has:

$$\begin{aligned} J^r(D, Q) \geq u(Q - D + L) + \beta & \int\limits_{\mathscr{D}(\tilde{D}'(L,Q),Q)} J^d(g' Q, \lambda') \mathrm{d}\mathcal{F}(g') \mathrm{d}\mathcal{G}(\Lambda') \\ & + \beta \int\limits_{\mathscr{R}(\tilde{D}'(L,Q),Q)} J^r(\tilde{D}'(L, Q), g' Q) \mathrm{d}\mathcal{F}(g') \mathrm{d}\mathcal{G}(\Lambda') \end{aligned}$$

Then:

$$\begin{aligned} J^r(D, Q) - J^d(Q, \lambda) \geq\ & u(Q - D + L) - u((1-\lambda)\mu Q) \\ & + \beta \int\limits_{\mathscr{R}(\tilde{D}'(L,Q),Q)} (J^r(\tilde{D}'(L, Q), g' Q) - J^d(g' Q, \lambda)) \mathrm{d}\mathcal{F}(g') \mathrm{d}\mathcal{G}(\lambda') \end{aligned}$$

Since, by definition, J^r is greater than J^d over the repayment set $\mathscr{R}$, one has:

$$J^r(D, Q) - J^d(Q, \lambda) \geq u(Q - D + L) - u((1-\lambda)\mu Q)$$

So $J^r(D, Q) - J^d(Q, \lambda) \geq 0$ by definition of L, and since u is increasing. □

This leads to a lower bound on the default threshold:

Proposition 3. *The country never defaults if debt is lower than what the investors can extract in case of default plus the one-period loss of output due to the negative productivity shock,* i.e. *one has:*

$$d^*(\lambda) \geq V(1, \lambda) + 1 - \mu$$

Proof. Suppose the economy is in a state (D, Q, λ) such that $D \leq V(Q, \lambda) + (1-\mu)Q$. Then let:

$$L = \frac{1}{1+r} \int V(g' Q, \lambda') \mathrm{d}\mathcal{F}(g') \mathrm{d}\mathcal{G}(\lambda')$$

It is easy to see that $L \in \mathscr{L}(Q)$. Indeed, if the country asks for that level today, the investors can ask for $\tilde{D}'(L, Q) = \frac{g^{\max} Q}{1 - \frac{g}{1+r}}$, which verifies the zero-profit condition (because at such a level of indebtment, the country will default tomorrow with probability one because of corollary 1).

Given that value for L, one has $L - D \geq -\lambda\mu Q - (1-\mu)Q$ (because $D \leq V(Q, \lambda) + (1-\mu)Q$, and using (9)).

By lemma 2, default is therefore not optimal. The country decides to repay in that situation.

The result follows using the homogeneity properties.

□

Appendix B.2. Dynamics

This subsection performs the same derivations as section 4.3, but using more rigorous notations, and in the general case where λ is stochastic.

The envelope theorem in equation (8) leads to:

$$\frac{\partial J^r}{\partial D}(D, Q) = -u'(\tilde{C}(D, Q))$$

where $\tilde{C}(D, Q)$ is the optimal consumption level in case of repayment.

Combining this with the first order condition of the maximization in (8), and using the fact that J^r and J^d are equal at the default threshold, one gets:

$$u'(\tilde{C}(D, Q)) = \beta \frac{\partial \tilde{D}'}{\partial L}(\tilde{L}(D, Q), Q) \int\limits_{\mathscr{R}(\tilde{D}'(\tilde{L}(D,Q),Q),Q)} u'(\tilde{C}(\tilde{D}'(\tilde{L}(D, Q), Q), g' Q)), g' Q) \mathrm{d}\mathcal{F}(g') \mathrm{d}\mathcal{G}(\lambda') \tag{B.1}$$

where $\tilde{L}(D, Q) = \tilde{C}(D, Q) + D - Q$ is the optimal level of borrowing in case of repayment. This is the analog of equation (6) in the two period model.

We now turn to the computation of $\frac{\partial \tilde{D}'}{\partial L}$, and by the way we will prove proposition 2.

The function $\tilde{D}'(L, Q)$ is determined by the implicit equation (10). This equation can be rewritten as:

$$f(L, Q, \tilde{D}'(L, Q)) = 0$$

where:

$$f(L, Q, D') = D' \, \mathbb{P}[\mathscr{R}(D', Q)] + \int\limits_{\mathscr{D}(D', Q)} V(g' Q, \lambda') \mathrm{d}\mathcal{F}(g') \mathrm{d}\mathcal{G}(\lambda') - L(1+r)$$

The implicit function theorem states that there is a unique solution to this implicit equation if the derivative of f with respect to D' is non negative, and that in that case we have $\frac{\partial \tilde{D}'}{\partial L} = \frac{\frac{\partial f}{\partial L}}{\frac{\partial f}{\partial D'}}$.

Using the specific structure of $\mathscr{D}$ and $\mathscr{R}$, one can rewrite f as:

$$f(L, Q, D') = \int \left(\int_{\frac{D'}{d^*(\lambda')Q}}^{g^{\max}} D' \mathrm{d}\mathcal{F}(g') + \int_0^{\frac{D'}{d^*(\lambda')Q}} V(g'Q, \lambda') \mathrm{d}\mathcal{F}(g') \right) \mathrm{d}\mathcal{G}(\lambda') - L(1+r)$$

Taking the derivative with respect to D', one gets:

$$\frac{\partial f}{\partial D'}(L, Q, D') = \mathbb{P}[\mathscr{R}(D', Q)] - \frac{D'}{Q} \int \frac{d^*(\lambda') - V(1, \lambda')}{d^*(\lambda')^2} \mathrm{d}\mathcal{G}(\lambda')$$

In the general case the sign of this derivative is not constant, since both terms in the expression are positive (the second term is positive because of proposition 3). But if we assume that $d^*(\lambda') = V(1, \lambda')$ (smooth default), we have:

$$\frac{\partial f}{\partial D'}(L, Q, D') = \mathbb{P}[\mathscr{R}(D', Q)] \geq 0$$

So the derivative is non null, except in the case where $\mathbb{P}[\mathscr{R}(D', Q)] = 0$ (but in this latter case, the zero profit condition implies that $L = \frac{1}{1+r} \int V(g'Q, \lambda') \mathrm{d}\mathcal{F}(g') \mathrm{d}\mathcal{G}(\lambda')$). Hence in the smooth default case, the derivative is non null everywhere except on a set of points of empty interior. Using the implicit function theorem, this implies that there is a unique continuous function verifying the zero-profit condition. This proves proposition 2.

Moreover, the derivative of the investors' decision rule $\tilde{D}'$ is:

$$\frac{\partial \tilde{D}'(L, Q)}{\partial L} = \frac{\frac{\partial f}{\partial L}}{\frac{\partial f}{\partial D'}} = \frac{1+r}{\mathbb{P}[\mathscr{R}(\tilde{D}'(L, Q), Q)] - \xi(\tilde{D}'(L, Q), Q)}$$

where:

$$\xi(D', Q) = \frac{D'}{Q} \int \frac{d^*(\lambda') - V(1, \lambda')}{d^*(\lambda')^2} \mathrm{d}\mathcal{G}(\lambda')$$

Recall that $\xi(D', Q) \geq 0$ because of proposition 3, and that $\xi(D', Q) = 0$ in the smooth default case. The Euler equation (B.1) can then be rewritten as:

$$u'(\tilde{C}(D, Q)) = \frac{\beta(1+r)}{\mathbb{P}[\mathscr{R}(\tilde{D}'(L, Q), Q)] - \xi(\tilde{D}'(L, Q), Q)} \times \int_{\mathscr{R}(\tilde{D}'(\tilde{L}(D,Q),Q),Q)} u'(\tilde{C}(\tilde{D}'(\tilde{L}(D, Q), Q), g'Q)) \mathrm{d}\mathcal{F}(g') \mathrm{d}\mathcal{G}(\lambda')$$

Along an equilibrium path, this means that we have (switching to the notation using time subscripts):

$$u'(C_t) = \beta(1+r)\left(\frac{1-\pi_{t+1|t}}{1-\pi_{t+1|t}-\xi_{t+1|t}}\right)\mathbb{E}_t\left[u'(C_{t+1})\,|\mathscr{R}(D_{t+1},Q_t)\right]$$

where $\xi_{t+1|t} = \xi(D_{t+1}, Q_t)$, $\pi_{t+1|t}$ is the probability of default in $t+1$ from the perspective of date t, and the term $\mathbb{E}_t\left[u'(C_{t+1})\,|\mathscr{R}(D_{t+1},Q_t)\right]$ stands for the expectation of $u'(C_{t+1})$, from the perspective of date t, *conditionally on the decision to repay at date* $t+1$.

This equation reveals the core of the Panglossian theory. First consider the smooth default case where $\xi_{t+1|t} = 0$. In that case, the equation boils down to:

$$u'(C_t) = \beta(1+r)\mathbb{E}_t\left[u'(C_{t+1})\,|\mathscr{R}(D_{t+1},Q_t)\right]$$

When it decides its level of indebtment, the country only takes into account the consequences of its decision for the subset of events where growth is high and makes default non-optimal. It then rationally ignores risk: this is the Panglossian effect.

Appendix C. Parameter restrictions

Let's begin by some easy to establish relationships for the growth and debt levels in the default and repayment cases, derived from equations (19)–(21):

$$\begin{aligned} g^0_{it} &= X^g_{i,t-1}\eta^g + \varepsilon^g_{it} \\ g^1_{it} &= X^g_{i,t-1}\eta^g + X^{g,\delta}_{i,t-1}\eta^{g,\delta} + \varepsilon^g_{it} = g^0_{it} + X^{g,\delta}_{i,t-1}\eta^{g,\delta} \end{aligned} \tag{C.1}$$

$$\begin{aligned} d^0_{it} &= X^d_{i,t-1}\eta^d + g^0_{it}X^{d,g}_{i,t-1}\eta^{d,g} + \varepsilon^d_{it} \\ d^1_{it} &= X^d_{i,t-1}\eta^d + g^1_{it}X^{d,g}_{i,t-1}\eta^{d,g} + \varepsilon^d_{it} = d^0_{it} + X^{g,\delta}_{i,t-1}\eta^{g,\delta}X^{d,g}_{i,t-1}\eta^{d,g} \end{aligned} \tag{C.2}$$

In the light of these relationships, we choose to impose the following parameter restrictions stemming from economic theory:

$$\forall i,t:\ X^{d,g}_{i,t-1}\eta^{d,g} < 0 \tag{C.3}$$

$$\forall i,t:\ X^{g,\delta}_{i,t-1}\eta^{g,\delta} < 0 \tag{C.4}$$

$$\forall i,t:\ X^{\delta,d}_{i,t-1}\eta^{\delta,d} > 0 \tag{C.5}$$

Constraint (C.4), combined with (C.1), implies that $g^1_{it} < g^0_{it}$: growth is always lower in a crisis scenario than in a no-crisis scenario, *ceteris paribus*.

Constraint (C.3) means that the debt-to-GDP ratio is a decreasing function of growth. Combined with (C.4) and (C.2), it implies that $d^0_{it} < d^1_{it}$: the debt-to-GDP ratio is always worse in a crisis scenario than in a no-crisis scenario, *ceteris paribus*.

Constraint (C.5) simply states that the probability of a debt crisis—as given by equation (21)—is an increasing function of the debt-to-GDP ratio.

Appendix D. Likelihood derivation

In this section we derive the likelihood function of the econometric model described in section 6. The likelihood of a single observation $(d_{it}, g_{it}, \delta_{it})$ is $\mathcal{L}_\Theta(d_{it}, g_{it}, \delta_{it}|X_{i,t-1})$ given the exogenous values $X_{i,t-1}$ and the vector of parameters $\Theta = (\eta^d, \eta^{d,g}, \eta^g, \eta^{g,\delta}, \eta^\delta, \eta^{\delta,d}, \sigma_d, \sigma_g, p)$.

For the remaining of this subsection, the i and t subscripts are dropped for the sake of simplicity.

Let's note φ the probability density function of the standard normal distribution (zero mean and unit variance) and Φ its cumulative density function.

Given (d, g, δ), ε^d and ε^g can be immediately inferred. The likelihood function is therefore, by independence of the four shocks $(\varepsilon^d, \varepsilon^g, \varepsilon^\delta, \zeta)$:

$$\mathcal{L}_\Theta(d, g, \delta|X) = \mathbb{P}_\Theta(\varepsilon^d = d - X^d\eta^d - gX^{d,g}\eta^{d,g})\, \mathbb{P}_\Theta(\varepsilon^g = g - X^g\eta^g - \delta X^{g,\delta}\eta^{g,\delta})\, \mathbb{P}_\Theta(\delta|d, X)$$

The first two factors are:

$$\mathbb{P}_\Theta(\varepsilon^d = d - X^d\eta^d - gX^{d,g}\eta^{d,g}) = \frac{1}{\sigma_d}\varphi\left(\frac{d - X^d\eta^d - gX^{d,g}\eta^{d,g}}{\sigma_d}\right)$$

$$\mathbb{P}_\Theta(\varepsilon^g = g - X^g\eta^g - \delta X^{g,\delta}\eta^{g,\delta}) = \frac{1}{\sigma_g}\varphi\left(\frac{g - X^g\eta^g - \delta X^{g,\delta}\eta^{g,\delta}}{\sigma_g}\right)$$

The third factor is discussed below.

Appendix D.1. Crisis case

If $\delta = 1$, one knows that $d = d^1$ and $g = g^1$. Then:

$$\begin{aligned}\mathbb{P}(\delta = 1|d^1, X) \;&=\; \mathbb{P}(X^\delta\eta^\delta + d^0X^{\delta,d}\eta^{\delta,d} + \varepsilon^\delta > 0) + \\ &\quad\; p\,\mathbb{P}(X^\delta\eta^\delta + d^1X^{\delta,d}\eta^{\delta,d} + \varepsilon^\delta > 0 > X^\delta\eta^\delta + d^0X^{\delta,d}\eta^{\delta,d} + \varepsilon^\delta)\end{aligned}$$

In this equation, the first term corresponds to a crisis driven solely by fundamentals and exogenous shocks, and the second term to the self-fulfilling case.

Using (C.2), it can be rewritten as:

$$\begin{aligned}\mathbb{P}(\delta = 1|d^1, X) \;&=\; \Phi[X^\delta\eta^\delta + (d^1 - X^{g,\delta}\eta^{g,\delta}X^{d,g}\eta^{d,g})X^{\delta,d}\eta^{\delta,d}] + \\ &\quad\; p\left\{\Phi(X^\delta\eta^\delta + d^1X^{\delta,d}\eta^{\delta,d}) - \Phi[X^\delta\eta^\delta + (d^1 - X^{g,\delta}\eta^{g,\delta}X^{d,g}\eta^{d,g})X^{\delta,d}\eta^{\delta,d}]\right\}\end{aligned}$$

For a given crisis observation, it is therefore possible to compute the *a posteriori* probability that the crisis is of a self-fulfilling nature (by opposition to a crisis solely driven by fundamentals and exogenous shocks). This probability is computed as the measure of the set of events where the (unobservable) trigger of default ε^δ_{it} is such that $X^\delta_{i,t-1}\eta^\delta + d^1_{it}\,X^{\delta,d}_{i,t-1}\eta^{\delta,d} + \varepsilon^\delta_{it} > 0 > X^\delta_{i,t-1}\eta^\delta + d^0_{it}\,X^{\delta,d}_{i,t-1}\eta^{\delta,d} + \varepsilon^\delta_{it}$. Since only crisis episodes are considered, the value of d^1_{it} is directly observable, and that of d^0_{it} can be easily found using equation (C.2). It is then straightforward to compute the probability using the assumption that ε^δ_{it} is be normally distributed:

$$\mathcal{S}_\Theta(d^1, X) = \frac{p\left\{\Phi(X^\delta\eta^\delta + d^1X^{\delta,d}\eta^{\delta,d}) - \Phi[X^\delta\eta^\delta + (d^1 - X^{g,\delta}\eta^{g,\delta}X^{d,g}\eta^{d,g})X^{\delta,d}\eta^{\delta,d}]\right\}}{\mathbb{P}_\Theta(\delta = 1|d^1, X)}$$

Appendix D.2. No-crisis case

If $\delta = 0$, one knows that $d = d^0$. Then:

$$\begin{aligned}\mathbb{P}(\delta = 0|d^0, X) &= \mathbb{P}(X^\delta\eta^\delta + d^1 X^{\delta,d}\eta^{\delta,d} + \varepsilon^\delta < 0) + \\ &\quad (1-p)\,\mathbb{P}(X^\delta\eta^\delta + d^1 X^{\delta,d}\eta^{\delta,d} + \varepsilon^\delta > 0 > X^\delta\eta^\delta + d^0 X^{\delta,d}\eta^{\delta,d} + \varepsilon^\delta)\end{aligned}$$

In this equation, the first term corresponds to the no-crisis equilibrium driven by strong fundamentals, and the second term to the self-fulfilling case in which the country escapes the crisis.

Using (C.2), it can be rewritten as:

$$\begin{aligned}\mathbb{P}(\delta = 0|d^0, X) &= 1 - \Phi[X^\delta\eta^\delta + (d^0 + X^{g,\delta}\eta^{g,\delta}X^{d,g}\eta^{d,g})X^{\delta,d}\eta^{\delta,d}] + \\ &\quad (1-p)\left\{\Phi[X^\delta\eta^\delta + (d^0 + X^{g,\delta}\eta^{g,\delta}X^{d,g}\eta^{d,g})X^{\delta,d}\eta^{\delta,d}] - \Phi(X^\delta\eta^\delta + d^0 X^{\delta,d}\eta^{\delta,d})\right\}\end{aligned}$$

Appendix E. Estimation methodology

The model is estimated with full information maximum (log-)likelihood, *i.e.* by computing the following:

$$\underset{\Theta\in\mathcal{B}}{\operatorname{argmax}} \sum_{(i,t)} \log \mathcal{L}_\Theta(d_{it}, g_{it}, \delta_{it}|X_{i,t-1})$$

where $\mathcal{B}$ is a set of constraints over parameters to ensure that constraints (C.3), (C.4) and (C.5) are satisfied and that $\sigma_d > 0$, $\sigma_g > 0$. The parameter p is calibrated.

The programs performing the estimations are written using the R environment for statistical computing.[15]

Appendix E.1. Dealing with constraints

The constrained-optimization algorithm that we use is the L-BFGS-B method (Byrd et al., 1994), which allows box constraints (*i.e.* each variable can be given a lower and/or upper bound). The constraints over σ_d and σ_g already fit into this category.

Constraints (C.3), (C.4), (C.5) (respectively over $\eta^{d,g}$, $\eta^{g,\delta}$, $\eta^{\delta,d}$) are enforced by replacing them with tighter constraints, in the following way:

- First, we only choose constant sign regressors in $X^{d,g}$, $X^{g,\delta}$, $X^{\delta,d}$ (that is, all elements of a given column in these matrices have a constant sign).
- Second, every component of $\eta^{d,g}$, $\eta^{g,\delta}$, $\eta^{\delta,d}$ is constrained to have the sign that will enforce the constraint.

Therefore, constraints (C.3), (C.4), (C.5) are clearly satisfied, and the constraints over $\eta^{d,g}$, $\eta^{g,\delta}$, $\eta^{\delta,d}$ can be dealt with by the L-BFGS-B algorithm.

[15] See `http://www.r-project.org` and R Development Core Team (2011).

Appendix E.2. Non-concavity

The second issue is the fact that the log-likelihood function is not globally concave, which implies that different initial values in the optimization algorithm can lead to different local maxima.

This problem is dealt with using a simple randomization algorithm. The following procedure is repeated 50,000 times:

- Generate a random initial value for the maximization algorithm. We alternate between two algorithms for generating this point (each algorithm is used half of the time):
  - Draw a totally random point. For unconstrained parameters $(\eta^d, \eta^g, \eta^\delta)$, a standard normal distribution is used. For sign-constrained parameters $(\eta^{d,g}, \eta^{g,\delta}, \eta^{\delta,d}, \sigma_d, \sigma_g)$, a χ^2_1 distribution is used (multiplied by -1 for the relevant components of $\eta^{d,g}$, $\eta^{g,\delta}$, $\eta^{\delta,d}$).
  - Draw a point in the neighborhood of the point which has the highest likelihood so far. For all parameters, a normal distribution centered around that point is used, using the same standard error than the maximum likelihood estimator.
- Run the L-BFGS-B algorithm using the initial value thus generated.
- If the result has a greater log-likelihood than the previous best point, keep it, otherwise discard it.

The results obtained in this way exhibit good numerical stability.