The Eurozone Debt Crisis: 
A New-Keynesian DSGE model with default risk

Daniel Cohen$^{1,2}$, Mathilde Viennot$^1$, Sébastien Villemot$^3$

$^1$Paris School of Economics
$^2$CEPR
$^3$OFCE – Sciences Po

PANORisk workshop – 7 June 2016
Motivation

- Eurozone (EZ) debt crisis does not fit well the literature on sovereign debt models
- Greece:
  - Unexpected shock on 2009 public deficit (final figure: 15.2% GDP)
  - Then, painful and long reduction of deficit (via fiscal austerity)
  - Standard models assume that deficit is a control variable
- Ireland:
  - Debt soared because of contingent liabilities in relation to banking sector
  - Large shock to debt-to-GDP ratio, unrelated to deficit (Ireland was fulfilling all Maastricht criteria before the financial crisis)
  - Standard models assume rather smooth process for GDP

Cohen, Viennot, Villemot (OFCE)
A DSGE with default risk
7 June 2016
Our modelling strategy

- Habit consumption (for making adjustment painful)
- Discontinuous stochastic process for GDP
- Incorporate standard NK features
- Small open economy framework, in 2 flavors:
  - flexible exchange regime
  - monetary union
Outline

1. The model
2. Calibration and baseline results
3. Sensitivity analysis
4. Conclusion
Outline

1. The model
2. Calibration and baseline results
3. Sensitivity analysis
4. Conclusion
Main features

- Small open economy
- Optimizing households who consume, supply labor and invest in physical capital
- Firms produce using labor and capital
- Nominal rigidities: good prices, wages
- Real rigidities: habit consumption, investment cost
- Fiscal authority with debt rule
- Government debt held both domestically and abroad
- Two model flavors:
  - flexible exchange rate (independant monetary policy)
  - monetary union (nominal interest rate determined abroad)
Modelling sovereign default

- The fiscal authority can default on external part of its debt
- In case of default, two costs: GDP loss, financial autarky
- Optimal decision by comparing two value functions
- Technical problem: dimensionality of the problem
- Our (imperfect) solution: satellite model
  ▶ In normal times, agents do not internalize the possibility of a future default (in particular, no endogenous risk premium)
  ▶ But allows us to compute default probabilities on simulated paths
Households

- Program for household $i$:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U_t^i$$

where:

$$u^i(C_t^i, H_t, L_t^i) = \log(C_t^i - H_t) - \frac{\varphi (L_t^i)^{1+\sigma_L}}{1 + \sigma_L}$$

with $H_t = h C_{t-1}$

- Budget constraint:

$$B_t^i + C_t^i = \frac{R_{t-1} + \Delta_{t-1}}{\pi_t} B_{t-1}^i + Y_t^i - I_t^i - \tau_t C_t^i$$

$$Y_t^i = w_t L_t^i + A_t^i + (r_t^k z_t^i - \psi(z_t^i)) K_{t-1}^i + \text{Div}_t^i$$
Euler equation
Symmetric across households

\[ \mathbb{E}_t \left[ \beta \frac{C_t - H_t}{C_{t+1} - H_{t+1}} \frac{1 - \tau_t}{1 - \tau_{t+1}} \frac{R_t + \Delta_t}{\pi_{t+1}} \right] = 1 \]

where \( \Delta_t \) is risk premium.
Labor market

- Differentiated labor varieties
- Standard Calvo pricing
- Indexation of non-reoptimized wages on inflation
- State contingent Arrow-Debreu securities shield against idiosyncratic labor income shock
Capital accumulation

\[ K_t = (1 - \delta) K_{t-1} + \left[ 1 - S \left( \frac{l_t}{l_{t-1}} \right) \right] l_t \]

where \( S \left( \frac{l_t}{l_{t-1}} \right) = \frac{\kappa l_t}{2} \left( \frac{l_t}{l_{t-1}} - 1 \right)^2 \)

\[ \mathbb{E}_t \left[ \frac{1}{\beta} \left( \frac{C_{t+1} - H_{t+1}}{C_t - H_t} \frac{1 - \tau_{t+1}}{1 - \tau_t} \right) \right] q_t = q_{t+1} (1 - \delta) + z_{t+1} r^k_{t+1} - \psi(z_{t+1}) \]

\[ q_t \left[ 1 - S \left( \frac{l_t}{l_{t-1}} \right) \right] - 1 + \beta \mathbb{E}_t q_{t+1} \left( \frac{C_t - H_t}{C_{t+1} - H_{t+1}} \frac{1 - \tau_t}{1 - \tau_{t+1}} \right) \]

\[ = q_t S' \left( \frac{l_t}{l_{t-1}} \right) \frac{l_t}{l_{t-1}} \]

\[ r^k_t = \psi'(z_t) \]
Production

- Final good firms:
  \[ Y_t = \left( \int_0^1 y_{j,t} \frac{\epsilon-1}{\epsilon} \, dj \right)^{\frac{\epsilon}{\epsilon-1}} \]

- Intermediate good firms:
  \[ y_{j,t} = A_t (z_t K_{j,t-1})^{\alpha_K} M_t^{\alpha_M} L_{jt}^{1-\alpha_K-\alpha_M} \]

with standard Calvo pricing
Fiscal policy

- Budget constraint:
  \[ B_t + D_t + \tau_t C_t = \frac{R_{t-1} + \Delta_{t-1}}{\pi_t} B_{t-1} + \frac{R^*_{t-1} + \Delta_{t-1}}{\pi_t} \frac{E_t}{E_{t-1}} D_{t-1} + G_t \]

- Fiscal rule:
  \[ \tau_t C_t - G_t - \text{Int}_t = \alpha_B \left( B_{t-1} + \frac{E_t}{E_{t-1}} D_{t-1} - \overline{BD}_t \right) \]
  where
  \[ \text{Int}_t = \left( \frac{R_{t-1} + \Delta_{t-1}}{\pi_t} - 1 \right) B_{t-1} + \left( \frac{R^*_{t-1} + \Delta_{t-1}}{\pi_t} - 1 \right) \frac{E_t}{E_{t-1}} D_{t-1} \]
External sector

- Exports:
  \[ X_t = \varepsilon_t^\psi Y_t^* \]

- Balance of payments equilibrium:
  \[ D_t = \frac{R_{t-1}^* + \Delta_{t-1}}{\pi_t} \frac{E_t}{E_{t-1}} D_{t-1} + \varepsilon_t M_t - X_t \]
Monetary policy and exchange rate

Flexible exchange rate (FLEX)

- Taylor rule:

\[
\frac{R_t}{\bar{R}} = \left( \frac{R_{t-1}}{\bar{R}} \right)^{\rho_{\pi}} \left( \frac{\pi_t}{\bar{\pi}} \right)^{r_{\pi}(1-\rho_{\pi})}
\]

- UIP:

\[
R_t + \Delta_t = \mathbb{E}_t \left( R_t^* \frac{E_{t+1}}{E_t} \right) + \vartheta \left( e^{(D_t - \bar{D})} - 1 \right)
\]

- Risk premium:

\[
\Delta_t = 0
\]
Monetary policy and exchange rate

Monetary union (EMU)

- No autonomous monetary policy:
  \[ R_t = R_t^* \]

- Real exchange rate:
  \[ \frac{E_t}{E_{t-1}} = \frac{\pi_t^*}{\pi_t} \]

- Risk premium (computed on external part of debt):
  \[ \Delta_t = \psi_{RP} \left( e^{D_t - \bar{D}} - 1 \right) \]
Satellite default model

- After a default, proportional cost on GDP:
  \[ Y_t^d = (1 - \lambda Q) Y_t \]

- Government budget constraint becomes:
  \[ B_t + T_t = R_{t-1} + \Delta_{t-1} \frac{B_{t-1}}{\pi_t} + G_t \]

- Other equations remain essentially the same

- This defines a default value function \( J^d \)

- Default threshold: \( D \) such that \( J^d = J^r \) (given other state variables)

- Default occurs when \( J^d > J^r \) (given the state variables)

- Simulation of 10,000 points for computing default probability

- Simplification: possibility of default not anticipated by agents
Outline

1. The model
2. Calibration and baseline results
3. Sensitivity analysis
4. Conclusion
Calibration (selected parameters)

For a small country within the Euro area. Standard values for most parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption habit</td>
<td>$h$</td>
<td>0.85</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.995</td>
</tr>
<tr>
<td>Total debt target</td>
<td>$\bar{BD}_t$</td>
<td>2.4$Y_t$</td>
</tr>
<tr>
<td>Back to equilibrium debt targets (fiscal rule)</td>
<td>$\alpha_B$</td>
<td>1/80</td>
</tr>
<tr>
<td>Risk premium in UIP (only FLEX)</td>
<td>$\vartheta$</td>
<td>0.001</td>
</tr>
<tr>
<td>Risk premium on debt (only EMU) $\Delta_t$</td>
<td>$\psi_{RP}$</td>
<td>0.008</td>
</tr>
<tr>
<td>External debt target</td>
<td>$\bar{D}$</td>
<td>0.3$\bar{Y}$</td>
</tr>
<tr>
<td>Loss of output in autarky in the FLEX model (% of GDP)</td>
<td>$\lambda_Q$</td>
<td>0.03</td>
</tr>
<tr>
<td>Loss of output in autarky in the EMU model (% of GDP)</td>
<td>$\lambda_Q$</td>
<td>0.04</td>
</tr>
</tbody>
</table>
## Default probabilities and debt thresholds

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Default probability</th>
<th>Mean external debt</th>
<th>Default threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline ((\bar{D} = 0.3\bar{Y}))</td>
<td>FLEX 0%</td>
<td>7.5%</td>
<td>60%</td>
</tr>
<tr>
<td></td>
<td>EMU 0.5%</td>
<td>7.5%</td>
<td>128%</td>
</tr>
<tr>
<td>(D = 0.8\bar{Y})</td>
<td>FLEX 0.5%</td>
<td>20%</td>
<td>60%</td>
</tr>
<tr>
<td></td>
<td>EMU 2.2%</td>
<td>20%</td>
<td>117%</td>
</tr>
</tbody>
</table>
Outline

1. The model
2. Calibration and baseline results
3. Sensitivity analysis
4. Conclusion
Sensitivity to habit consumption ($h$)
Baseline calibration ($\bar{D} = 0.3\bar{Y}$)

**FLEX model**

**EMU model**
Sensitivity to habit consumption ($h$)
Medium external debt ($\bar{D} = 0.8\bar{Y}$)

**FLEX model**

**EMU model**
Sensitivity to external debt target ($\bar{D}$)

- **FLEX model**
- **EMU model**

![Graphs showing default probability vs. Duip for FLEX and EMU models](image-url)
Sensitivity to total debt target ($BD$)

**FLEX model**

**EMU model**
Sensitivity to speed of convergence ($\alpha_B$)

FLEX model

EMU model
Outline

1. The model
2. Calibration and baseline results
3. Sensitivity analysis
4. Conclusion
Main preliminary results

- Critical differences between flexible regime and monetary union
- Default thresholds larger in flexible economy...
- ...but thresholds more likely to be reached in monetary union
- Fast speed of convergence increases defaults in flexible regime, diminishes them in monetary union
- In EMU, external debt plays a critical role for stabilization...
- ...as a consequence, debt more volatile and default risks are more important
Future work

- Incorporate possibility of redemption after default
- Analyze impact of a discrete shock on debt-to-GDP ratio
- Allow default on total debt (and not just external debt)
- Handle (some) nonlinearities