École des Hautes Études en Sciences Sociales

Thèse

pour l'obtention du grade
de docteur en sciences économiques

Présentée et soutenue publiquement par

Sébastien Villemot

le 19 octobre 2012

Essays on Modelling the Sovereign Default Risk

Directeur

Monsieur Daniel Cohen
Professeur à l'École Normale Supérieure

Jury

Monsieur Michel Juillard
Conseiller scientifique à la Banque de France, rapporteur

Monsieur Pierre-Olivier Gourinchas
Professeur à l'Université de Californie à Berkeley, rapporteur

Monsieur Xavier Ragot
Chargé de recherche au Centre National de la Recherche Scientifique

Monsieur Thierry Verdier
Directeur d'études à l'École des Hautes Études en Sciences Sociales

Monsieur Jeromin Zettelmeyer
Directeur de la recherche à la Banque Européenne pour la Reconstruction et le Développement

Abstract

This thesis contributes to the literature on sovereign debt and default risk, building on theoretical models of strategic default and on more recent developments of the quantitative sovereign debt literature.

The first contribution is to suggest a solution to the "sovereign default puzzle:" most quantitative sovereign debt models predict a default at very low debt-to-GDP thresholds, in clear contradiction with what is observed in the data. Starting from the observation that countries generally do not want to default but are rather forced into it by the markets, I present a model which can replicate the key stylized facts regarding sovereign risk.

As another contribution, I establish a typology of debt crises in three categories: those crises that are the consequence of exogenous shocks, those that are *self-fulfilling* prophecies, and those *self-enforcing* crises that are the consequence of a rational tendency to over-borrow when the risk of a negative shock is high. The estimated proportion of self-fulfilling and self-enforcing crises in the data is about 10% in each case.

I also study how sovereign default can be understood in the context of small open economy real business cycle models. The conclusion is that these models oscillate between two polar cases: default is either inexistent or too frequent, depending on the chosen parameter values. These models are therefore not well suited for studying sovereign risk, and default needs to be fully endogeneized in order to get meaningful results.

Finally, I make a methodological contribution by presenting a new computational method for solving endogenous default models. It is shown to dramatically improve the existing speed-accuracy frontier.

Keywords: Sovereign debt; Strategic default; Lévy stochastic processes; Self-fulfilling crises; Endogenous grid method

Résumé

Cette thèse contribue à la littérature sur la dette souveraine et le risque de défaut, en se fondant notamment sur les récents développements de la littérature quantitative sur la dette souveraine.

La première contribution est une solution au problème suivant : la plupart des modèles de dette souveraine prédisent le défaut pour des valeurs très faibles du ratio dette sur PIB, en contradiction avec ce qui est observé dans les données. En partant de l'observation que les pays ne souhaitent généralement pas faire défaut mais y sont forcés par les marchés, je présente un modèle qui peut reproduire les principaux faits stylisés concernant le risque souverain.

J'établis ensuite une typologie des crises de dette en trois catégories : les crises qui sont la conséquence d'un choc exogène, celles qui sont des prophéties *auto-réalisatrices*, et les crises *auto-imposées* qui sont la conséquence d'une tendance rationnelle au surendettement lorsque le risque d'un choc négatif est élevé. La proportion de crises auto-réalisatrices et auto-imposées dans les données est estimée à environ 10% pour chacune de ces catégories.

J'étudie également comment le défaut souverain peut se comprendre dans les modèles de cycles réels en petite économie ouverte. Il ressort que ces modèles oscillent entre deux cas polaires : le défaut y est soit inexistant soit trop fréquent. Ces modèles sont donc peu adaptés à l'étude du risque de défaut, risque qui doit donc être endogénéisé pour obtenir des résultats utiles.

Enfin, je fais une contribution méthodologique en présentant une nouvelle méthode de résolution des modèles de défaut souverain endogène. Cette méthode améliore significativement la frontière vitesse-précision actuelle.

Mots clefs : Dette souveraine ; Défaut stratégique ; Processus stochastiques de Lévy ; Crises auto-réalisatrices ; Méthode des grilles endogènes

Contents

Chapter 1

Introduction

Preliminary remark on the terminology: In this thesis, I define *sovereign debt* as the debt that a sovereign government owes to creditors (whether public or private) outside the country.[1, 2] Sovereign debt such defined is therefore a component of *external debt*, which includes debt owed to foreign creditors by domestic private debtors as well.[3] Sovereign debt is also a component of *total public debt*, which includes debt owed by the government to domestic creditors as well. To put it shortly: I use *sovereign debt* as a synonymous for *public external debt*.

1.1 Selected facts on sovereign debt

1.1.1 A historical perspective

As the issue of sovereign debt is making the headlines and occupying the minds of many citizens in the United States and in the eurozone, one should remember that sovereign debt crises are by no means a new phenomenon in economic history, but on the contrary constitute a recurrent pattern. The first record of a sovereign default goes back to the 4$^{\text{th}}$ century B.C., when ten Greek municipalities defaulted on a loan from the Delos temple (Sturzenegger and Zettelmeyer, 2007, p. 3). In the modern era, Reinhart et al. (2003, Table 2) document that France and Spain defaulted many times during the 16$^{\text{th}}$, 17$^{\text{th}}$ and 18$^{\text{th}}$ centuries, followed by Portugal, Germany, Austria and Greece during the 19$^{\text{th}}$ century. The 20$^{\text{th}}$ century was also marked by many sovereign defaults, mainly by European, Latin American and African countries, culminating in the Latin America crisis of the 1980s, the Mexican crisis of 1994 and the Russian crisis of 1998. The 21$^{\text{th}}$ century started with one of the largest default in history with Argentina defaulting on \$82 billion.[4] Over the period 1820–2004, Tomz and Wright

1. Sovereign debt also includes debt owed by the private sector but guaranteed by the government.
2. Note that some authors alternatively define sovereign debt as the debt owned by a sovereign government to both domestic and foreign creditors.
3. Some authors use an alternative definition for the distinction between domestic and external debt. They consider that domestic debt is debt issued under domestic jurisdiction, while external debt is debt issued under foreign jurisdiction, no matter whether the creditor is a domestic or a foreign resident.
4. Source: Hatchondo et al. (2007a, p. 168).

(2007) document that 106 countries defaulted, making a total of 250 default episodes. Using a different methodology and restricting themselves to the recent era, Kraay and Nehru (2006) identify 94 episodes of debt distress over the period 1970–2001.[5]

It should be noted that there is no unique definition of a sovereign "default"—also called a "credit event"—and this explains why there is no canonical and universally accepted list of default episodes. However, the general consensus is that a default is characterized when a country does not fully meet its contractual obligations towards its creditors. The most obvious case of default is when a debtor fails to honor some scheduled interest or principal payment. Are also generally considered as defaults the events when syndicated loans or bonds are rescheduled or exchanged against new securities with less favorable terms (whether this is done unilaterally by the debtor country or through some multilateral agreement such as those negotiated at the Paris Club or the London Club). Some authors, such as Kraay and Nehru (2006), also consider as being in default those countries that benefit from balance of payments support from the International Monetary Fund (IMF) or other multilateral financial institutions; the idea is that a country in need of external financial support would have been unable to meet its external obligations in the absence of such a support and is therefore in near default. Characterizing such events as default events is nevertheless debatable since creditors are not effectively impacted in that case. Once default events are identified, another issue is to measure the duration of the default episode: the end of the episode can be either recorded when full payments are resumed, when an rescheduling agreement is reached, or when the country recovers its access to private financial markets.[6]

All countries are not equal with respect to sovereign default. The central thesis of Reinhart et al. (2003) is that countries fall in two broad categories: those which virtually never default (this includes most industrialized countries but also many emerging countries) and those that have repeatedly defaulted many times throughout history (most of them being Latin American countries). The latter are qualified by those authors of "serial defaulters" which suffer from some ontological "debt intolerance" disease, leading them to default even when their indebtment level is low by common standards. The cause of their repeated defaults would therefore not be a repeated occurrence of adverse exogenous shocks but rather a structural and persistent tendency to over-borrow and mismanage debt. This view is challenged by Cohen and Valadier (2011), at least for the recent era: they argue that, over the period 1970–2007, there is no such "serial defaulter" pattern among the 126 emerging countries they consider. I return to this issue in section 1.1.3 when considering the determinants of debt defaults.

With respect to the time dimension, some clear patterns emerge. Several authors document that defaults typically occur in clusters, suggesting that the world business cycle, the major trends in the international capital markets or other global events can cause sovereign defaults. In particular, Sturzenegger and Zettelmeyer (2007) show that waves of default usually follow lending booms: the most famous example of such a boom-bust sequence is the

5. Other large default databases include those of Benjamin and Wright (2009), Borensztein and Panizza (2009), Cohen and Valadier (2011), Detragiache and Spilimbergo (2001), Pescatori and Sy (2007) and Rose (2005).

6. Note that the latter occurs relatively fast on average, see section 1.1.2.

lending boom triggered by the African independences and the oil price shock of 1973—when oil exporting countries recycled their earnings by massively lending to developing countries—leading to the wave of defaults that started with the Mexican default in 1982. Again, Cohen and Valadier (2011) give a slightly different picture for the recent period: though they recognize that world economic shocks have an impact on the default risk and that some default peaks occurred in the early 1980s and 2000s, they argue that the "global shocks" theory does not have a strong explanatory power.

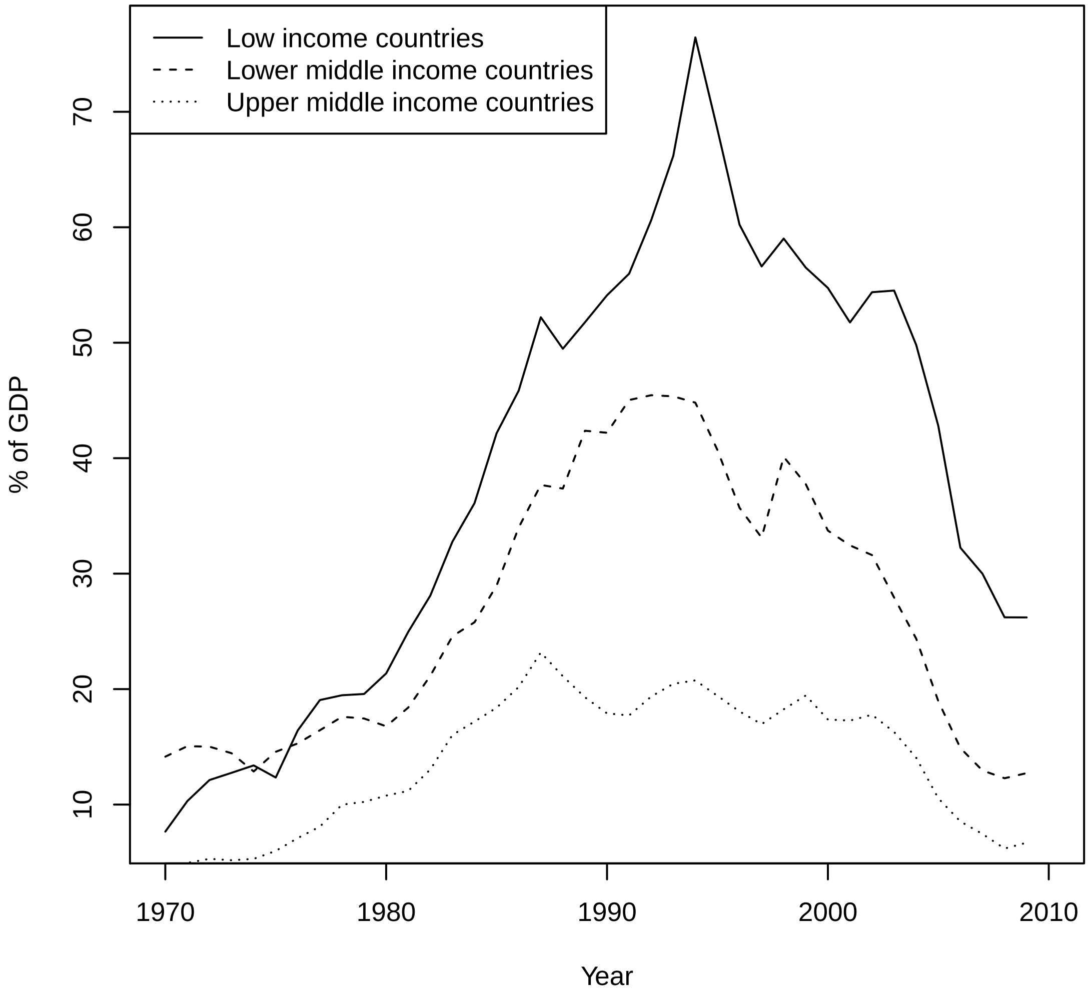
Over time, the characteristics of sovereign debt have also changed along several dimensions: the total amount of outstanding debt, the type of creditors (public versus private), the type of instruments (bonds, syndicated loans, concessional loans) and the currency denomination.

Figure 1.1 plots the evolution of long-term sovereign debt as a ratio of GDP for three groups of countries (low income, lower middle income and upper middle income) over the period 1970–2009. Two main facts emerge from this picture: first, the level of external indebtment decreases with the level of income per capita; second, there is a clear time pattern: broadly speaking, sovereign debt has been on the rise from 1970 to the mid-1990s, and has declined since. Several events can probably account for the recent decline: the burst of the 1980s crisis, the Heavily Indebted Poor Countries (HIPC) initiative for debt reduction (which started in 1996), and the development of domestic debt markets. This latter fact is documented by Reinhart et al. (2003) who show that the domestic market for government debt in emerging countries has significantly expanded during the 1980s and 1990s, so that in many countries domestic public debt is now higher than external public debt.

The share of sovereign debt in total external debt also varies across countries and over time, as shown in Table 1.1. The average pattern is that the poorer the country, the higher the share of sovereign debt in total external debt: this is probably the consequence of the underdevelopment of the private sector and financial system in poor countries. Between 1970 and 2009, the share of sovereign debt in total external debt has significantly declined, especially for middle income countries. The types of creditors for sovereign debt also exhibit a pattern across countries and over time: low-income countries have little access to private credit markets, and this has worsened over time; at the other extreme, as of 2009, almost two third of sovereign debt in upper middle income countries is held by private creditors.

Concerning the type of instruments, while only bonds were used before World War II, the creation of the World Bank and other development banks led to the development of *concessional loans* to the least developed countries, *i.e.* loans which incorporate a grant element in the form of a long grace period or low interest rate. In parallel, on the private market, the lending boom of the 1970s took the form of syndicated bank loans instead of the more traditional bonds but, contrarily to what was announced by some market participants, this change of instrument did not prevent the boom from being followed by a bust. Bond instruments also evolved over time: during the 2000s collective action clauses in bond contracts became increasingly popular, with the aim of facilitating the coordination among a large number of creditors and therefore favoring the orderly resolution of crises.

Figure 1.1: Long-term sovereign debt for three groups of countries (1970–2009)



Long-term public and publicly guaranteed external debt, at face value, as a ratio of GDP
Source: World Bank (2010)

1.1.2 The cost of defaulting

Of course, defaults are costly both to the creditors and the debtor. The costs to the creditors are relatively easy to quantify, since they are of a financial nature. Relevant data are widely available, at least for the recent period. Using different datasets and methodologies, Moody's (2008), Benjamin and Wright (2009) and Cruces and Trebesch (2011) all document that, following a default, the average recovery rate is about 60% over the period 1970–2010.[7]

7. Or, alternatively, the average haircut is 40%. The recovery rate can be defined either as the ratio of the post-default market price versus the face value, or as the ratio of the present value of cash flows effectively received after restructuring divided by the present value of cash flows initially promised. The two ways of computing the recovery rate do not give the same exact figures, but are close in most cases.

Table 1.1: Composition of external debt for groups of countries

	Low income			Lower middle income			Upper middle income		
	1970	1990	2009	1970	1990	2009	1970	1990	2009
Short term	4.0	7.0	9.1	9.7	12.4	14.1	15.2	17.3	23.5
Private long term	2.5	1.5	1.9	7.4	4.3	31.2	33.3	6.0	42.2
Sovereign long term	92.0	87.5	84.5	81.0	81.0	52.0	51.0	73.7	33.4
— *Official creditors*	*75.1*	*79.8*	*81.3*	*68.9*	*59.4*	*41.5*	*29.5*	*26.7*	*11.9*
— *Private creditors*	*16.9*	*7.7*	*3.2*	*12.1*	*21.6*	*10.5*	*21.5*	*47.0*	*21.4*
IMF credit	1.4	4.0	4.5	1.9	2.3	2.7	0.5	3.0	0.9

Averages across countries, expressed as a percentage of total external debt. Note that short term debt aggregates both private and sovereign short term, which are not disentangled for lack of available data. *Source:* World Bank (2010)

Though this means that outright defaults on the totality of the outstanding debt are quite rare, the cost that lenders have to bear after a credit event is nevertheless very significant. As discussed by Sturzenegger and Zettelmeyer (2007), this high cost to creditors in bad times is more than offset, on average, by high returns in good times: the average historical spread of emerging market debt relative to sovereign debt in industrialized countries is positive, though not so big (about 150 basis points).

More difficult to identify and quantify are the costs incurred by the defaulting country. This is however a crucial task because theoretical sovereign debt models (see section 1.2) show that the magnitude of default costs have a direct impact on the amount of debt that a country can borrow: in the extreme case, if there is no cost to default, the country has no incentive to repay at all and will therefore be rationed by the markets *ex ante*.

There is clear evidence that sovereign defaults have a direct impact on growth. Of course, in order to measure this impact, the econometrician has to acknowledge the existence of a strong endogeneity problem in the relationship between default and growth: if defaults may cause a slowdown in growth, the reverse causality is also at work (Tomz and Wright, 2007). Taking this into account, Chuan and Sturzenegger (2005) estimate that default episodes cause a reduction of growth of approximately 0.6 percentage points per year; moreover, if the default coincides with a banking crisis, then the negative effect is much more important and is of the magnitude of 2.2 percentage points of growth. Similarly Borensztein and Panizza (2009) find that, on average, a default is associated with a decrease in growth between 1 and 1.2 percentage points per year, depending on the way the endogeneity problems are dealt with; also, this negative impact is relatively short-lived (not significant after one year).

The literature identifies three types of costs than can explain this negative growth impact: the exclusion from financial markets and other reputational costs; direct sanctions from creditors; and domestic political and financial costs.

The threat of exclusion from financial markets is a cost that is incorporated in most models of sovereign defaults (see section 1.2 and Hatchondo et al., 2007b). This is the penalty that creditors can most easily impose on the defaulting debtor country, but there are theoretical reasons why this exclusion can be difficult to implement in practice: it is a time-inconsistent policy, and coordination problems can arise in the presence of a large number of creditors. In an empirical analysis on market access, Gelos et al. (2011) document that the exclusion penalty is actually enforced, in the sense that defaults are consistently followed by a period of market exclusion. However, the authors show that this penalty is rather short-lived (the exclusion lasts 4.5 years on average following a default), that this exclusion period has become shorter in the 1990s than in the 1980s, and that the probability of reentering markets is only marginally affected by previous default decisions. These facts tend to show that the exclusion penalty, though real, is not very strong. Using a different dataset, Alessandro et al. (2011) find similar results on average, but show that there is a great deal of heterogeneity across countries: market access either resumes within the first six years after a default, or is lost for a much longer period of time. The literature also studies the impact of defaults on borrowing costs, once the country has recovered market access. For example, Dell'Ariccia et al. (2006) show that countries that have participated to the Brady restructurings in the 1980s faced an increase in borrowing costs of about 15–50 basis points in the early 1990s, and of 50–100 basis points in the late 1990s following the Russian crisis. Borensztein and Panizza (2009) show that defaults have a direct impact on credit ratings (negative effect of about one notch in the three years following the default) and on borrowing costs (250 to 400-basis points increase in the two years after the default), but that these effects are rather short-lived (not statistically significant after two or three years). Cruces and Trebesch (2011) show that defaulting countries are punished by investors after they have recovered market access: the interest rate spreads they face are directly correlated with the size of the haircut that investors had to accept during the default episode. There is however no broad consensus on this issue in the literature, some other papers finding no significant effect of default on borrowing costs (see Borensztein and Panizza, 2009, pp. 701–703 for a review).

Creditors can also impose direct sanctions of various types to defaulting debtors: diplomatic pressures, trade sanctions, legal actions, foreign assets withholdings, threat of military interventions and, exceptionally, actual interventions. Sanctions through the trade channel are studied by Rose (2005) who finds an econometrically significant effect of debt renegotiation on bilateral trade between the debtor and its creditors, using a standard gravity trade model on panel data for the period 1948–1997; this decline is about 8% of the total bilateral trade, and lasts 15 years. Evidence provided by Borensztein and Panizza (2009) suggests that this trade impact is essentially caused by a decline in trade credit during the four years following the default. Going further back in the past, Mitchener and Weidenmier (2005) study default costs over the period 1870–1913: they find no evidence of trade sanctions after a default over this period; rather, they show that defaulting countries were subject to "super-sanctions," *i.e.* gunboat diplomacy or loss of fiscal sovereignty.

On the overall, while the literature agrees on the existence of a variety of different default costs,[8] there is so far no consensus regarding the relative quantitative importance of these in

8. See Borensztein and Panizza (2009, Table 13, p. 723) for a summary of the various costs identified by the empirical literature.

the total default costs.

1.1.3 Determinants of crises

There is a large body of literature trying to empirically identify the determinants of sovereign defaults. The general methodology consists in estimating regressions where the dependent variable is an indicator of debt crises and the explanatory variables consist of several macroeconomic and institutional indicators. There is however a potential endogeneity bias in this methodology since some of the explanatory variables might as well be influenced by defaults, or some common factor could cause both defaults and the explanatory variables. The literature tries to deal with this endogeneity issue but, despite these efforts, the results can hardly be given a causal interpretation: these exercises are rather identifying the risk factors associated with sovereign defaults.

Such an analysis is carried by Kraay and Nehru (2006) who construct a panel dataset of default episodes across most low- and middle-income countries over the period 1970–2001. They define a default as an occurrence of at least one of the three following conditions: substantial principal or interest payment arrears, debt relief received in the form of debt reduction or rescheduling by the Paris Club, or non-concessional balance of payment support by the IMF. With such a definition, the authors identify a total of 94 distress and 286 "normal times" episodes. Using a probit regression, they find that crises are more likely if the debt level is high (measured as the external debt-to-exports or external debt service-to-exports ratio), institutions (measured by the CPIA index [9]) are of poor quality and real GDP growth is low. [10] Between the two measures of debt burden—debt- to-exports ratio and debt service-to-exports ratio—, the latter has a more important explanatory power: this fact suggests that crises are more often triggered by liquidity than by solvency problems. To assess the significance of these variables, the authors also explore the out of sample predictive power of their model: when the model is estimated using the three aforementioned explanatory variables (and only these three) in the pre-1990 sample, it is able to correctly forecast 84% of the episodes of the post-1990 sample. Moreover, the authors show that depreciations in the real exchange rate, changes in the terms of trade, level of development (measured as log real per capita GDP at purchasing power parity—PPP) and a known history of bad policies are all statistically insignificant. Finally, building on these results, the authors try to quantify the indebtment levels at which countries can be considered "safe," keeping in mind that the level of growth and the quality of institutions play their part in the determination of that threshold: for example, if a debt distress probability of 25% is aimed at, a country with poor institutions and average growth can sustain a 100% external debt to exports ratio, while a similar country but with good institutions can sustain a 300% ratio.

Manasse et al. (2003) conduct a study similar in spirit and reach broadly consistent conclusions: the risk of crisis is explained by measures of solvency (external debt-to-GDP ratio),

9. The World Bank's Country Policy and Institutions Assessment index.
10. The fact that low growth is associated with defaults is consistent with the findings of Tomz and Wright (2007).

measures of illiquidity (in particular the importance of short term debt in overall external debt), low growth, current account imbalances, political uncertainty. In a companion paper, Manasse and Roubini (2009) derive a set of simple rules (based on thresholds for external debt to GDP, inflation, growth, and some other macroeconomic indicators) that can be used to easily determine if a country is in a "safe zone" or a "danger zone."

Instead of studying the determinants of actual crises, some authors look at the determinants of market perceptions of default risk. This perception can be embodied either in some non-market indicator (*e.g.* investors surveys) or in market indicators (*e.g.* bond prices on the secondary market). For example, Reinhart et al. (2003) examine the *Institutional Investor* ratings, a panel of economists and sovereign risk analysts who rate countries according to their perception of a risk of default; according to the authors, two factors explain 75% of the cross-country variance of the rating: the debt-to-GNP ratio, and the history of bad policies (hyperinflation, previous episodes of default or restructurings). The authors argue that the fact that institutions and history matters in determining crises is a proof of their theory of "debt intolerance," *i.e.* the idea that some countries have a structural tendency to default, independently of other economic or financial factors. The authors also study debt thresholds above which the risk of default raises significantly; taking a purely descriptive approach, they study the distribution of the debt-to-GNP ratio in the first year of a default, and show that there is a great variance of this ratio across countries: for the serial defaulters, the default risk significantly increases at debt-to-GNP ratios as low as 30–35%, while for the typical country the threshold is in the range of 41–60%, and ratios in excess of 100% are not uncommon. Catão and Kapur (2004) provide a different explanation for the behavior of serial defaulters: using a logit regression, they show that defaults are well predicted by the external debt-to-exports ratio and by the volatility of output, the latter variable being very significant. The authors also estimate regressions including the credit history variable of Reinhart et al. (2003) as explanatory variable: this variable is significant if the macroeconomic volatility variable is not included, and becomes statistically insignificant in the other case. This suggests that the credit history variable is actually a proxy for the magnitude of macroeconomic volatility. Of course, this does not tell what are the causes of this volatility, and it is well possible that bad policies are one of the causes, but at least this explanation gives a more practical policy answer to the problem of serial defaulters than does the vague "debt intolerance" concept. Another alternative explanation of the serial defaulters pattern is given by Kohlscheen (2007): he shows that presidential regimes are 5 times more likely to default than parliamentary regimes, *ceteris paribus*, and rationalizes this fact using political economy considerations.

The determinants of market perceptions of risk has also been studied from the angle of the dynamic relationship between emerging country spreads, domestic business cycle indicators and global macroeconomic indicators. The difficult point is that there is probably double causality at work here: the domestic macroeconomic situation impacts spreads via the perceived default risk, but the reverse causality also exists since external financing difficulties can have a direct and adverse effect on economic conditions. Uribe and Yue (2006) tackle this endogeneity issue and show that country spreads are influenced by domestic output, domestic investment, the current account and the US real interest rate. More precisely, in response to an increase in US interest rates, country spreads first fall, then display a large and delayed overshooting. Note that the authors do not find that the debt-to-GDP ratio has an impact on country spreads in this specification, a fact which seems at odds with other evidence; however, when they remove the current account variable, then the significance of the external debt-to-GDP ratio is restored: this is probably the consequence of the long run relationship between financial flows and stock of debt. Also, the authors show the existence of a significant though modest reverse causality from country spreads to domestic activity, but this is out of the scope of the present discussion.

Political factors also play a role in sovereign defaults. This is discussed for instance by Hatchondo et al. (2009) who study a composite index of political risk published by the International Country Risk Guide. This index is intended to measure the risk of default related to political risk, independently of economic and financial risks: to some extent, it measures the probability of a "political default," *i.e.* the risk of having a creditor-unfriendly government coming in power and repudiating the debt. Focusing on the Argentina default of 2001, the index suggests that the government in Argentina was more creditor-friendly before the default than after: the authors take this as an indication that this specific default episode was triggered (at least partially) by a political change rather than by economic or financial conditions. Conversely, the authors cite the examples of Russia in 1998 and Uruguay in 2003 as default episodes that are likely not political ones.

Finally, there is an abundant literature on the simultaneous occurrence of banking and currency crises (the so-called "twin crises"). But the literature on simultaneous banking and debt crises, on one hand, and currency and debt crises, on the other hand, is much less abundant. For instance, Reinhart (2002) shows that most sovereign defaults are associated with currency crises (but the reverse is only true in half of the cases), and preliminary evidence suggest that the causality goes from currency crises to debt crises. Reinhart and Rogoff (2011b) show that banking crises often precede or accompany sovereign debt crises. They suggest the following explanation for this simultaneity: banking crises often accompany currency crises (as documented by the twin crises literature), which in turn deteriorate the solvency indicators (as the latter are often expressed as the ratio of a debt stock in some foreign currency to GDP expressed in domestic currency). These balance sheet effects lead, through currency mismatches, to a default on sovereign debt. But the reverse causality may also be at work: Borensztein and Panizza (2009) document that sovereign defaults increase the probability of a banking crisis by 11%.

1.1.4 The debt overhang debate

So far the analysis has focused on the effects and determinants of sovereign debt crises, but there is also a literature on the effect of the excessive accumulation of external debt *per se*, independently of the occurrence of defaults and crises. This effect is known as the "debt overhang" effect, as coined by Sachs (1989): the accumulation of debt has a crowding out effect on investment as the service of the debt diverts resources out of the country to foreign creditors. Also, heavily indebted countries do not have strong incentives to implement good policies, since a large share of the returns from these efforts will be captured by the creditors. Comparing debt repayments to a tax on the economy, Krugman (1988) speaks of a "debt Laffer curve" about the relationship between face value outstanding debt and market value of debt (*i.e.* the expected repayments). For low levels of debt, face value and market value are almost equal, since full repayment is expected; then, as face value increases, market value increases but not at parity, because a partial default is expected; and, past some threshold, market value becomes a *decreasing* function of face value, as the debt burden becomes a strong disincentive for the country to implement the right policies.

This theory has important policy implications. In particular, it is the rationale for the *Heavily Indebted Poor Countries* (HIPC) initiative, which has lead to significant external debt reductions for a panel of heavily indebted low-income countries.

Therefore, several papers have tried to empirically test the "debt overhang" theory. Cohen (1993) studies the link between debt and investment and finds that, in the 1980s, there is no direct relationship between the *stock* of debt and investment, but that the *service* of the debt crowds out investment (a 1% of GDP paid abroad reduces investment by 0.3% of GDP): this results seems to confirm the investment channel for the "debt overhang" effect.

More directly, a series of paper try to test the relationship between debt and growth, with the goal of identifying a debt threshold above which debt becomes detrimental to growth. As discussed below, the evidence for the "debt overhang" effect is mixed and subject to controversy.

On one hand, several papers claim to put in evidence such an effect. Pattillo et al. (2011) estimate the nonlinear relationship between debt and growth, and obtain a bell curve relationship as expected; their main result is that the overall effect of debt becomes negative around 35–40% of GDP, while the marginal effect of debt becomes negative at about half of this level. Ruiz-Arranz et al. (2005) find a negative effect of debt for intermediate levels of debt, but almost no effect for low levels and high levels (in the latter case, the authors talk about "debt irrelevance"); since most HIPC countries are in the debt irrelevance zone, where the marginal effect of debt is zero, they would only benefit from debt reductions big enough to bring them out of the debt irrelevance zone. Using non-parametric techniques, Imbs and Rancière (2005) provide evidence in support of a debt Laffer curve, with a negative growth effect of debt when its present value reaches 40% of GDP, and with negative effects on investment and economic policies.

These results are criticized by Depetris-Chauvin and Kraay (2005) who argue that they are contaminated by strong endogeneity problems; also, using a dataset of low-income countries who have benefited from debt relief, they cannot find any evidence of a positive impact of debt reductions on growth, casting doubt on the efficiency of the overall debt reduction initiative. Presbitero (2008) argues that the debt-growth relationship is not statistically robust to the inclusion of the quality of institutions in the analysis, and that debt is basically irrelevant for growth in countries with weak institutions. Another criticism that can be made to the aforementioned studies on the debt-growth relationship is that they don't test (except for Imbs and Rancière (2005)) for the impact of debt crises: one possible channel for the negative impact of debt on growth could very well be the costs associated to defaults, which are more likely to occur in highly indebted countries, as discussed in sections 1.1.2 and 1.1.3.

1.2 The economics of sovereign debt

In this section, I review the most important contributions of the literature to the theory of sovereign debt. First, I discuss the motivations for a sovereign to accumulate external debt; then I discuss the mechanisms which make debt sustainable in equilibrium; finally, I focus on those debt crises that are the result of a self-fulfilling prophecy.

1.2.1 The motivations for accumulating external debt

There are many possible ways of theorizing the motives for a sovereign government to accumulate debt in general, and external debt in particular. [11]

The first motive for debt is the *tax smoothing* theory, which basically states that, in the context of fluctuating public spending, a benevolent social planner should endeavor to keep the tax rate constant (in the simplest setup of a perfectly anticipated future), in order to minimize tax-related distortions. This implies that the government will run budget deficits and therefore accumulate debt when spending is high and, conversely, will pay back its debt by running fiscal surpluses when spending is low. This theory is partly confirmed by the data for industrialized countries but is clearly not sufficient, especially in the case of low- and lower middle-income countries whose external debt-to-GDP ratios exhibit low-frequency movements which seem incompatible with this theory (see Figure 1.1).

The tax smoothing model relies on the assumption of a representative, rational, fully informed agent existing in the countries. Other explanations for the presence of budget deficits rely on departures from this canonical assumption. For example, the theory of *fiscal illusion* postulates that voters have a systematic tendency to overstate the benefits of current government spending and understate the costs of debt. They are therefore willing to support politicians who consistently run fiscal deficits. Recent empirical studies seem however to raise doubts about the reality of this mechanism (Eslava, 2006). Debt has also been viewed as a weapon in *inter*-generational conflicts: if today's voters have no inter-generational altruism, they have an incitation to spend more today through borrowing and leave the resulting debt to the next generation which, by definition, has no voting rights today. Alternatively, debt can also be used in *intra*-generational conflicts: if several parties with different preferences over spending priorities alternate in power (*e.g.* one party prefers to finance the military, while the other prefers to finance social spending) then each party, when in power, has an

11. For an extensive review on this topic, see Alesina and Perotti (1995).

incentive to finance its priorities through borrowing so that the hands of the other party are tied when coming to power. Another form of intra-generational conflict which can lead to government debt is the *war of attrition* model of delayed fiscal adjustment: in the case of two competing parties simultaneously in power (typically in a coalition), if an exogenous shock occurs which creates a fiscal deficit, the negotiation over how to share the burden of fiscal adjustment between parties can rationally be protracted, thus creating fiscal deficits in the meantime.

As I discuss in section 1.3, most quantitative models of sovereign debt do not incorporate the political economy features that we have just described.[12] In order to nevertheless embed a structural tendency for the government to borrow, these models usually choose to calibrate the time preference rate of the social planner to a value greater than the world interest rate; as a consequence, the sovereign will want to accumulate debt up to some ceiling, above which it becomes rationed by the markets.

I have just reviewed some explanations given in the literature for the tendency of some governments to run fiscal deficits over a long period and therefore to accumulate debt. But this does not explain why a significant part of this debt comes from abroad, *i.e.* why government debt is for a large part external debt, at least in low- and middle-income countries.

One possible explanation of this fact may be the underdevelopment of the domestic financial system and the scarcity of domestic capital. This must be particularly true for low-income countries. The case of middle-income countries is a bit different: as documented by Reinhart et al. (2003, Table 14, p. 41), between the beginning of the 1980s and the end of the 1990s, the domestic debt market has dramatically expanded in many emerging countries (in part because of the massive public bailouts of the financial system in the aftermath of the 1997–98 crisis), with the external debt market still remaining strong. The hypothesis of the underdevelopment of domestic markets for middle-income countries is further challenged by Reinhart and Rogoff (2011a), who show that domestic debt has always accounted for a large part of the total public debt (with a peak in the 1950s, then a steady decrease until the 1990s, and more recently an upward move).

Another fundamental cause for the accumulation of external debt by poor countries is given by Lucas (1990): in a context of perfect international capital markets, and assuming standard production functions, one should observe a massive flow a capital from rich to poor countries in order to equalize marginal returns to capital. The difficulty is that for, decentralized market economies, this model predicts a surge in privately-owned external debt, not in publicly-owned external debt. But in practice, governments often give some degree of public guarantee on debt due by domestic private agents. And, in some cases, governments even take the place of the private sector when it is failing (for example in the case of infrastructure projects financed by concessional loans from international financial institutions). This model of international capital mobility can therefore easily be reinterpreted as an explanation for the accumulation of external sovereign debt by poor countries.

12. One exception is Cuadra and Sapriza (2008) who study the impact of alternating political parties in power on sovereign debt in a quantitative framework.

However, it is well-known that the central point in Lucas (1990) is that we do *not* observe as much capital flows from rich to poor countries as predicted by a back-of-the-envelope calculation on marginal returns. Lucas tries to rationalize this fact by taking into account the differences in human capital between rich and poor countries, but this is not quantitatively sufficient to resolve the paradox. The only explanation that is left to this apparent paradox is the difference in political and credit risks between rich and poor countries, the latter being generally riskier than the former. But Lucas dismisses this explanation by taking the example of pre-independence India: the English rule of law was in place by that time and, still, capital flows to India did not have the magnitude that the economic theory would predict. Taking the opposite stance, Reinhart and Rogoff (2004) argue that the political and credit risks in poor countries are the primary reason for the relative paucity of capital flows to them; in the case of serial defaulters, they even consider that the paradox is not that so little capital flows to them, but rather that *so much* capital flows to them, at least in the form of defaultable debt.

1.2.2 Sustaining debt in equilibrium

Sovereign debt is very different from corporate debt because of the lack of an effective enforcement mechanism for sovereign debt. In the case of corporate debt, a company cannot just decide not to repay and then continue its business as usual: it will be sued by its creditors and a court will force it to repay or—in the extreme case—liquidate the company and hand over its remaining assets to the creditors. In the case of sovereign debt, there is no supranational court that could enforce the repayment to creditors. Creditors can recover assets located in foreign jurisdictions by suing the country before the corresponding national jurisdictions, but quantitatively these foreign assets are generally small compared to the outstanding debts.

From a theoretical point of view, the absence of a strong enforcement mechanism leads to the question of why creditors are willing to lend in the first place, since the sovereign creditor cannot credibly commit to repay its debt. Of course, the answer lies in the costs to default that I have discussed in section 1.1.2 and that have been considered in several theoretical models.

The seminal paper in this literature was written by Eaton and Gersovitz (1981) who were the first to introduce a model of endogenous sovereign debt and default. The sovereign is modeled with a representative agent who receives an exogenous stream of endowment and makes consumption decisions. The possible gap between endowment and consumption is financed by debt borrowed from international private investors. If the country decides to default, it is permanently excluded from financial markets, and is therefore forced to live in autarky.[13] In this model, the country uses debt as a consumption smoothing device: it borrows in bad times and repays in good times. Defaulting on debt has therefore a welfare cost, equal to the opportunity cost of consumption smoothing, and the magnitude of this cost is decreasing with the elasticity of inter-temporal substitution and increasing with the volatility of income. At every period, the country balances the costs and benefits of repayment before making its decision; knowing this *ex ante*, the investors will impose an endogenous

13. For a detailed description of a generalized version of this model, see section 1.3.1.

credit ceiling (in a deterministic setup) or a decreasing credit supply curve (in a stochastic setup).[14]

Eaton and Gersovitz (1981) have therefore shown that a certain amount of sovereign debt is sustainable on the sole basis of reputational concerns and in the absence of any credible commitment device. This result is central to the sovereign debt literature and has stirred many further research efforts. It has also been criticized in two main directions. The first problem, raised by Kletzer (1994), concerns the time consistency of the permanent exclusion from financial markets: after a default, once the exclusion is in place, there is potentially a net social surplus to be gained by both parties from a new agreement involving positive lending. Since the possibility of such an agreement is foreseen by both parties before the default, this undermines the *ex ante* credibility of the exclusion penalty and therefore the possibility of sustaining positive lending. Note that the data seem to confirm that permanent exclusion is an unrealistic assumption, and that defaulting countries recover market access relatively quickly (see section 1.1.2). Another line of criticism is developed by Bulow and Rogoff (1989b): assuming that the country has also access to *cash-in-advance* contracts which let him buy insurance against future shocks (in the exchange of a down payment at the current period), those authors show that any positive level of debt is *un*sustainable if the only default penalty is the exclusion from financial markets. The intuition is that, after having borrowed, there will always be a moment where it will be optimal for the country to default and to invest in insurance contracts instead of repaying the existing debt. As a consequence, there is no such thing as reputational debt. This result is however dependent on the existence of contingent cash-in-advance contracts, which is a not so general hypothesis.

In order to take into account these criticisms which pose a serious challenge to the explanation of sovereign debt by reputational concerns, the literature has tried to explore alternative explanations and has added new modelling ingredients.

The most commonly adopted solution is the introduction of direct punishments by the creditors in case of default. These penalties can for example take the form of seizure of assets located outside the borders of the country or of denial of trade credit, as discussed in section 1.1.2.[15] Among others, Sachs and Cohen (1982) and Bulow and Rogoff (1989a) studied the theoretical foundations and implications of direct punishments in sovereign debt models. Such punishments are now routinely incorporated in quantitative debt models such as those surveyed in section 1.3 as well as those that I present in chapters 2 and 3. Other theoretical answers to the theoretical challenge posed by reputational debt are reviewed, for example, by Sturzenegger and Zettelmeyer (2007, chapter 2).

14. In a deterministic setup, defaults never happen in equilibrium and spreads are therefore zero. The possibility of a default has nevertheless an impact since it induces credit rationing above a certain level.

15. Rose and Spiegel (2004) have empirically confirmed a link between international trade patterns and lending patterns. This is consistent with the hypothesis that trade channels are a vehicle for punishment in case of default and therefore make debt sustainable in equilibrium. See section 1.1.2 for more evidence on the existence of direct sanctions.

1.2.3 Self-fulfilling crises

The literature has also studied the theoretical possibility of *self-fulfilling* debt crises, *i.e.* crises that are triggered by self-fulfilling pessimistic beliefs about the ability or willingness of the sovereign debtor to reimburse its debts. The typical situation is that of a debtor whose fundamentals are sane enough to make it solvent if confronted to good market conditions, but who rather faces suspicious lenders who ask for a high risk premium or refuse to lend in sufficient amounts, thus precipitating a default by their very action.

Technically, a model of self-fulfilling crises is a model with *multiple equilibria*: typically the model will sustain both a "good" equilibrium where lenders offer credit at low rates and in large enough quantities—so that the debtor can rollover its debts and follow its reimbursement schedule—, and a "bad" equilibrium where lenders are suspicious about the country's willingness to repay—so that the country actually defaults because it faces market conditions which make it optimal to act so. It is important to note that both equilibria are compatible with rational expectations: in both cases, market expectations, whether bad or good, are realized in equilibrium. In general the model will not explain how the equilibrium on which agents coordinate is selected: it is assumed to be the outcome of some random variable unrelated to economic fundamentals called a *sunspot*.[16]

The first paper to examine self-fulfilling crises in the context of government debt is Calvo (1988). For some values of the fundamentals, he shows the theoretical possibility of multiple equilibria in the interest rate: if lenders ask for the riskless interest rate, then the country is indeed safe and reimburses; and if lenders ask for a risk premium, the interest payments accumulate up to the point where debt becomes an unsustainable burden for the debtor who then partially defaults. This effect can be qualified of a *snowball* effect, since the fear that debt becomes unmanageable directly contributes to the buildup which makes it actually unmanageable. Several papers have tried to exhibit conditions under which the snowball effect can be neutralized. Cohen and Portes (2006) show that, in a simple two-period setup, the snowball effect cannot operate if lenders and debtors are capable of an efficient *ex post* restructuring in case of default, *i.e.* a restructuring in which all the costs of defaulting are captured by lenders so that nothing is socially lost. Also, as noted by Chamon (2007, footnote 7), multiple equilibria in the interest rate can be avoided if the rules of the borrowing game are slightly modified. Multiple equilibria can arise if the country announces the *amount that it wants to borrow today*, and the investors reply with the interest rate (or equivalently, with the amount they want to be repaid tomorrow). If instead the country commits on the *amount to be repaid tomorrow*, and the investors reply with the interest rate (or equivalently, with the amount they lend today), then multiple equilibria in the interest rate are impossible by construction.

Another type of self-fulfilling crises is studied by Cole and Kehoe (1996, 2000) who con-

16. This name comes from William Stanley Jevons's theory according to which economic cycles are correlated with the cyclic appearance of spots on the surface of the Sun every 11 years. Since it is doubtful (though not totally impossible) that sunspots have a direct impact on economic fundamentals, this term has been applied by extension to any non-fundamental variable that influences economic outcomes only through its effect on expectations.

struct a model which essentially focuses on *liquidity crises*. Their model is in many ways similar to the model of Diamond and Dybvig (1983) for bank runs. In this setup, a government borrows from foreign investors and makes at every period a decision to repay or default. The debt has a maturity of one period so that, for high levels of indebtment, the country cannot repay without being first refinanced: debt is essentially rolled over at every period (possibly with small adjustments to the level of borrowing). Therefore, if the country is excluded from financial markets at some period, it is likely to default because of a lack of liquidity even if it is fundamentally solvent. The authors show that, in such a setup and for some values of the parameters, there is a self-fulfilling zone where the outcome will be entirely determined by the investors' expectations: if they expect the country to default, then they refuse to refinance the debt and the country is indeed forced to default. Technically, the authors construct a continuum of sunspot equilibria where default in the self-fulfilling zone is triggered by a sunspot variable: the probability of default in that zone can therefore be chosen arbitrarily. The problem of self-fulfilling liquidity crises is tightly linked to the average debt maturity: Cole and Kehoe (2000) show that the self-fulfilling zone disappears if the maturity is increased enough; moreover, Cole and Kehoe (1996) argue that the Mexican crisis of 1994 (the so-called "Tequila crisis") was a liquidity crisis and could have been avoided if the average maturity had been 16 months instead of 9 months. An alternative cure for liquidity crises is given by Chamon (2007). He argues that these crises essentially stem from a coordination failure among lenders, and he suggests an easy solution for improving the coordination: the idea is to add a clause to bond contracts saying that the contract will be canceled if the number of bond subscribers is below some threshold. In this way, the bond issuer (the country) is able to always select the "good" equilibrium by issuing contingent bonds with a participation constraint high enough to ensure that the country does not default: investors will be willing to subscribe to this bond, since they will lose nothing whatever the outcome of the bid. Chamon (2007) concludes that the existence of simple mechanisms to cope with the above type of coordination failures casts doubts on the relevance of the debt run explanation for sovereign debt crises.

In the end of this analysis, I consider another possible type of self-fulfilling debt crisis: it is a crisis triggered by a confidence shock which has *on its own* the potential to damage the fundamentals of the sovereign. Think for example of capital flows or speculative attacks on the currency which can happen if markets expect the country to default in the short run. The key point is that the confidence shock has the potential to destroy the fundamentals *even if the country does not default* in the end: this is what differentiates this destruction of fundamentals from the penalty applied by debtors in case of default.[17] In this setup, a self-fulfilling crisis can happen even if the coordination mechanisms described by Chamon (2007) are implemented. I develop and analyze this idea in chapter 3.

17. Of course, in equilibrium, this destruction of fundamentals will only happen if the country actually defaults. But the fact that potentially the destruction of fundamentals could occur independently of a default critically changes the setup of the model.

1.3 Quantitative models of sovereign debt

1.3.1 The canonical framework

In this section I describe a "canonical" model of sovereign debt. Most recent papers in the quantitative sovereign literature are essentially variations or extensions of this model. The main characteristics of this model were already described in Eaton and Gersovitz (1981) and Sachs and Cohen (1982). The exact version that I present here is model II of Aguiar and Gopinath (2006) (up to a change in notations).

The economy

A sovereign country is inhabited by a representative consumer, who is able to tilt consumption away from output by borrowing or lending on the international financial markets. Output produced at time t is exogenous and given by the random variable Q_t, which follows a Markovian process. More precisely, let's assume the following non stationary process:

$$Q_t = g_t Q_{t-1} \tag{1.1}$$

$$g_t = e^{y_t} \tag{1.2}$$

$$y_t - \mu_y = \rho_y(y_{t-1} - \mu_y) + \varepsilon_t^y \tag{1.3}$$

where $\rho_y \in [0, 1)$, $\varepsilon_t^y \rightsquigarrow \mathcal{N}(0, \sigma_y^2)$, and $\mu_y = \log(\mu_g) - \frac{\sigma_y^2}{2(1-\rho_y^2)}$ (so that $\mathbb{E}(g_t) = \mu_g$).

The world financial markets are characterized by a constant riskless rate of interest r. Lenders are risk-neutral and subject to a zero-profit condition by competition. Let's further suppose that debt is short-term and needs to be refinanced at every period.

At any time t, the country has incurred a debt obligation D_t and may decide to default upon it (only if $D_t > 0$). When it does so, I assume that the country suffers forever after a negative productivity shock. One can say that default creates a panic that destroys capital either through an exchange-rate or a banking crisis. Post default output is therefore assume to be:

$$Q_t^d = (1 - \lambda) Q_t$$

where the d superscript stands for "default," and $\lambda \in [0, 1)$ captures the magnitude of the default penalty on output. As another cost, let's assume that the country is temporarily constrained to financial autarky; at every period, the country is redeemed with probability x and then recovers access to financial markets with its previous debts cancelled.

Financial markets

The timing of events is as follows. First assume that the country has incurred a debt obligation falling due at time t (if $D_t > 0$), or has accumulated assets (if $D_t < 0$), and is currently not excluded from financial markets. At the beginning of period t the country learns the value of its output Q_t. Then it decides to default or to reimburse its debt.

If the debt is reimbursed in full, the country can continue to trade bonds and chooses a new amount of debt D_{t+1} which must be repaid at time $t+1$. Given the demand D_{t+1} of the country for debt due tomorrow, the supply function of the international investors is the amount $\tilde{L}(Q_t, D_{t+1})$ that they are willing to lend today. The implicit interest rate associated to this supply function is $\frac{D_{t+1}}{\tilde{L}(Q_t, D_{t+1})} - 1$; it will be equal to r if the bonds are considered riskless by the investors, and greater to that if there is a risk of default.[18] The risk premium Δ_t can therefore be expressed as:

$$\Delta_t = \frac{D_{t+1}}{\tilde{L}(Q_t, D_{t+1})} - (1+r) \tag{1.4}$$

Such financial agreements being concluded, the country eventually consumes, in the event it services its debt in full:

$$C_t^r = Q_t - D_t + \tilde{L}(Q_t, D_{t+1}) \tag{1.5}$$

where the r superscript stands for "repayment."

Alternatively, in the event of a debt crisis the country's consumption is nailed down to:

$$C_t^d = Q_t^d = (1-\lambda) Q_t$$

Preferences

The decision to default or to stay current on the financial markets involves a comparison of two paths that implies expectations over the entire future. Let's call β the discount factor and u the instantaneous utility function of the representative agent of the country (assumed to be a constant relative risk aversion (CRRA) function):

$$u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$$

where γ is the coefficient of relative risk aversion.

Let's call J^r (resp. J^d) the country's payoff conditional to repayment (resp. to default), and J^* the country's unconditional payoff. In recursive form, those functions satisfy:

$$J^*(D_t, Q_t) = \max\{J^r(D_t, Q_t), J^d(Q_t)\} \tag{1.6}$$

$$J^r(D_t, Q_t) = \max_{D_{t+1}} \left\{ u(Q_t - D_t + \tilde{L}(Q_t, D_{t+1})) + \beta \mathbb{E}_t J^*(D_{t+1}, Q_{t+1}) \right\} \tag{1.7}$$

18. Note that, by construction, this formulation avoids multiple equilibria in the interest rate, as discussed in section 1.2.3.

$$J^d(Q_t) = u((1-\lambda)Q_t) + \beta\,\mathbb{E}_t\left[(1-x)J^d(Q_{t+1}) + x\,J^*(0, Q_{t+1})\right] \tag{1.8}$$

The default decision function:

$$\tilde{\delta}'(D_t, Q_t) = \mathbb{1}_{J^r(D_t, Q_t) < J^d(Q_t)} \tag{1.9}$$

is equal to 1 in case of default and 0 in case of repayment.

Finally, the investors are assumed to be risk neutral and in perfect competition, which implies that their credit supply function must satisfy the following zero profit condition:

$$(1+r)\tilde{L}(Q_t, D_{t+1}) = \mathbb{E}_t\left[1 - \tilde{\delta}'(D_{t+1}, Q_{t+1})\right] D_{t+1} \tag{1.10}$$

Equilibrium and basic properties

In the following, let's drop time subscripts. A quote on a variable name (like D') designates a next period variable.

Definition 1.1 (Recursive equilibrium)**.** *A recursive equilibrium for this economy is given by a credit supply function $\tilde{L}(Q, D')$, value functions $J^*(D, Q)$, $J^r(D, Q)$, $J^d(Q)$, and a default decision function $\tilde{\delta}'(D, Q)$ such that:*

- *Given the credit supply function, the value functions and the default decision function satisfy the government optimization problem* (1.6)–(1.9)*;*
- *Given the default decision function, the credit supply function satisfies the zero profit condition* (1.10)*.*

The choice function for tomorrow's debt (conditionally to repayment today) is denoted $\tilde{D}'(D, Q)$.

Proposition 1.2 (Existence and unicity)**.** *There exists a unique recursive equilibrium for this economy.*

Proof. See the appendix of Aguiar and Gopinath (2004). □

Note that since the model exhibits a growth trend, it is necessary to normalize some variables (GDP, debt levels, value functions) in order to compute the numerical solution. To detrend, I normalize all variables at date t by the following factor:

$$\Gamma_t = \mu_g Q_{t-1} \tag{1.11}$$

The detrended variables are denoted with a "hat." For example the detrended debt is $\hat{D}_t = \frac{D_t}{\Gamma_t}$; note that the detrended debt level is almost equal to the debt-to-GDP ratio (up to the deviation of current growth rate to its mean).[19] The detrended equations of the model are given in appendix 1.4.

Business cycle properties

I replicate here the main results of Aguiar and Gopinath (2006, model II). The calibration of the model is given in Table 1.2. The time unit is the quarter. The goal of the exercise is to replicate Argentina's business cycle statistics over the period 1983–2000. The parameters of the output process are set in order to match Argentina's. The preference parameters are standard, except for the discount factor (0.8) which is rather low, especially for a quarterly model: Aguiar and Gopinath (2006) argue this low value is necessary in order to have a reasonable rate of default in this model. The probability of redemption (10%) matches the average stay in autarky (2.5 years) exhibited by Gelos et al. (2011). Finally, the default penalty on output is set to a standard value (2%).

Table 1.2: Calibration of the canonical model

Parameter	Symbol	Value
Mean gross growth rate	μ_g	1.006
Auto-correlation of the growth rate	ρ_y	0.17
Innovation variance of the growth rate	σ_y	3%
Loss of output in autarky (% of GDP)	λ	2%
Probability of settlement after default	x	10%
World riskless interest rate	r	1%
Discount factor	β	0.8
Risk aversion	γ	2

As is well known, a model like the present one cannot be solved analytically. It is not possible to derive an exact formula for the policy and value functions and one therefore has to resort to numerical approximation techniques in order to compute an approximate solution. The chapter 5 is precisely dedicated to the solution methods for sovereign default models such as the present one (and more generally for all models based on the Eaton and Gersovitz (1981) framework). The contribution of that chapter is twofold. First it presents a new solution algorithm for sovereign debt models, based on the so-called endogenous grid method, and shows that this new technique dramatically improves the existing speed-accuracy frontier for the resolution of such models. Second, it provides a systematic accuracy comparison of several solution algorithms—including the new proposed one—using Euler equation-based error.

Table 1.3 reports various business statistics obtained by simulating the canonical sovereign debt model. Note that these results are different from those of Aguiar and Gopinath (2006), because the resolution method used in that paper was very imprecise (as shown by Hatchondo

19. The fact that the current growth g_t does not enter Γ_t guarantees that if X_t is in the information set at date $t-1$, then so is $\hat{X}_t$.

et al. (2010)) and instead I used the more precise technique presented in chapter 5.[20]

Table 1.3: Business statistics of the canonical model

	Data	Model
Rate of default (%, per year)	3.00	0.86
Mean debt output ratio (%, annualized)	45.99	4.68
$\sigma(Q)$ (%)	4.08	4.40
$\sigma(C)$ (%)	4.85	4.64
$\sigma(TB/Q)$ (%)	1.36	0.92
$\sigma(\Delta)$ (%)	3.17	0.06
$\rho(C, Q)$	0.96	0.98
$\rho(TB/Q, Q)$	-0.89	-0.18
$\rho(\Delta, Q)$	-0.59	0.05
$\rho(\Delta, TB/Q)$	0.68	0.53

σ = standard deviation, ρ = correlation, TB = trade balance, Δ = spread over riskless rate. The data facts are for Argentina over the period 1983–2000 and come from Aguiar and Gopinath (2006) (except for the debt output ratio computed by myself using World Bank (2010)). The simulations results reported are my own; they are averages over 500 simulations each of length 500. The standard deviations and correlations are obtained after detrending the series with an HP filter of parameter 1600.

The model does a good job at replicating well-known stylized facts regarding output, consumption and the trade balance in emerging countries; in particular, as in the data, consumption is more volatile than output and the trade balance is (weakly) counter-cyclical. The model is also able to replicate the positive correlation between spreads and the trade balance. Showing that a sovereign debt model to replicate key business cycle statistics of emerging countries is the outstanding contribution of Aguiar and Gopinath (2006) and also of Arellano (2008) (who uses a similar but slightly different model). However note that the model fails in some other important dimensions: the volatility of spreads is far too low, the spreads are weakly pro-cyclical (they are counter-cyclical in the data), the default rate is three times lower than in the data, and the mean debt to output ratio is too low by a factor of 10.

In chapter 2 I propose a way of addressing the main problem of this canonical sovereign debt model (and also of most models based on this framework), which is its inability to quantitatively match the facts for the default probabilities and average debt-to-output ratio, as evidenced by Table 1.3. The improvement that I suggest is based on two key ideas: first, the understanding that the stochastic process assumed for output is fundamental from a theoretical point of view and that only a process with discrete jumps (like a Poisson process) can truly generate defaults; second, the observation that in most default episodes, the decision to default is not a purely strategic decision but rather something that is imposed on the country by the markets. Incorporating these two ideas into a sovereign debt model leads to a

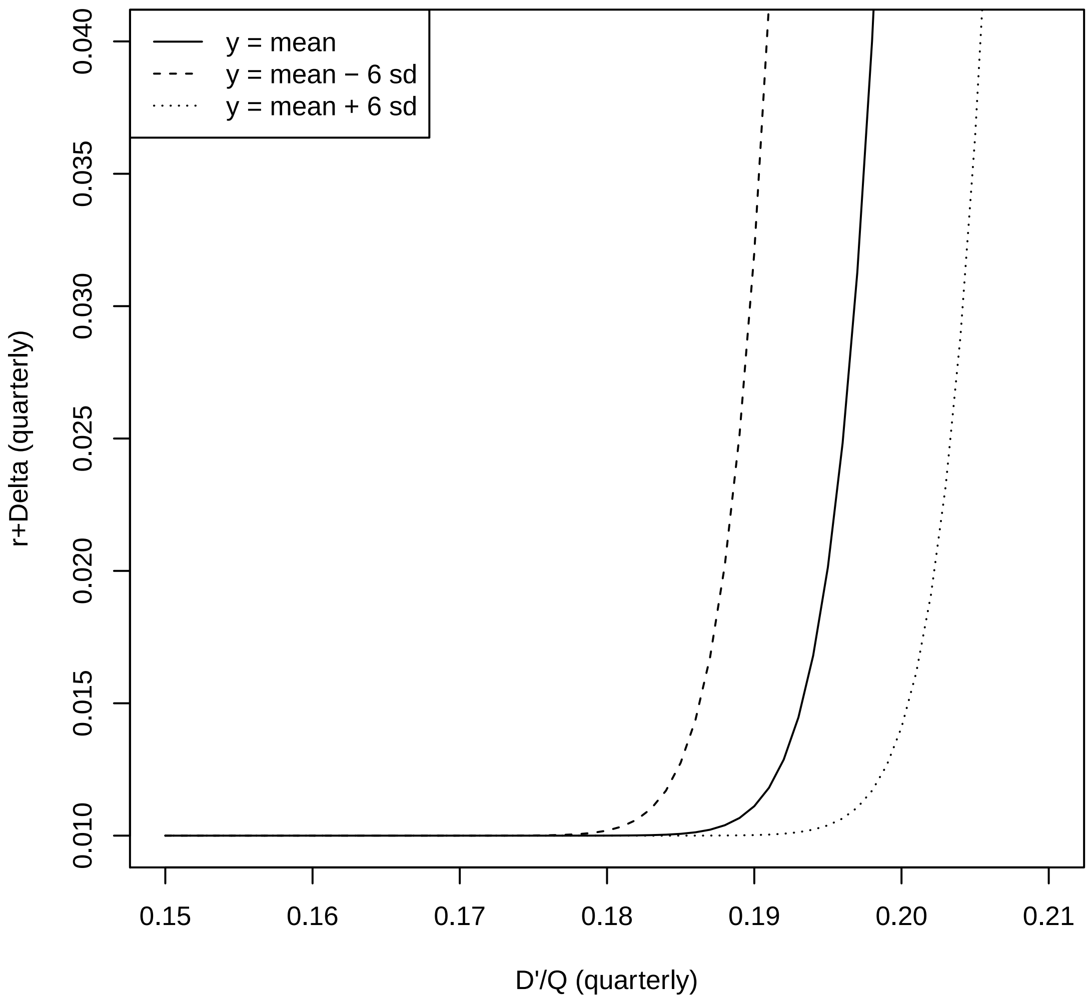
20. For solving this model, I used a technique very similar to the "spline" method of Hatchondo et al. (2010). More precisely, I used: value function iteration; spline interpolation over a grid of 30 points for D and 15 points for Q; Gauss-Legendre quadrature with 16 points for the expectation terms. More details on solution techniques are given in chapter 5.

realistic probability of default and a realistic average debt-to-GDP ratio at the same time, as I show in chapter 2. Other ways of addressing the shortcomings of the canonical model have been proposed in the literature. In the following subsections, I give a brief and incomplete overview of recent developments of quantitative sovereign debt models; for a more extensive review, one can refer to Stähler (2011).

In the remaining of the present subsection, I give a few more insights about the properties of the canonical model.

Figure 1.2 plots the (implicit) interest rate schedule faced by the country. This interest rate is defined, for a given level of debt D' to be repaid tomorrow, as $r + \Delta = \frac{D'}{\tilde{L}(Q,D')} - 1$ where r is the riskless rate and Δ is the risk premium (see equation (1.4)).

Figure 1.2: Interest rate schedule faced by the country

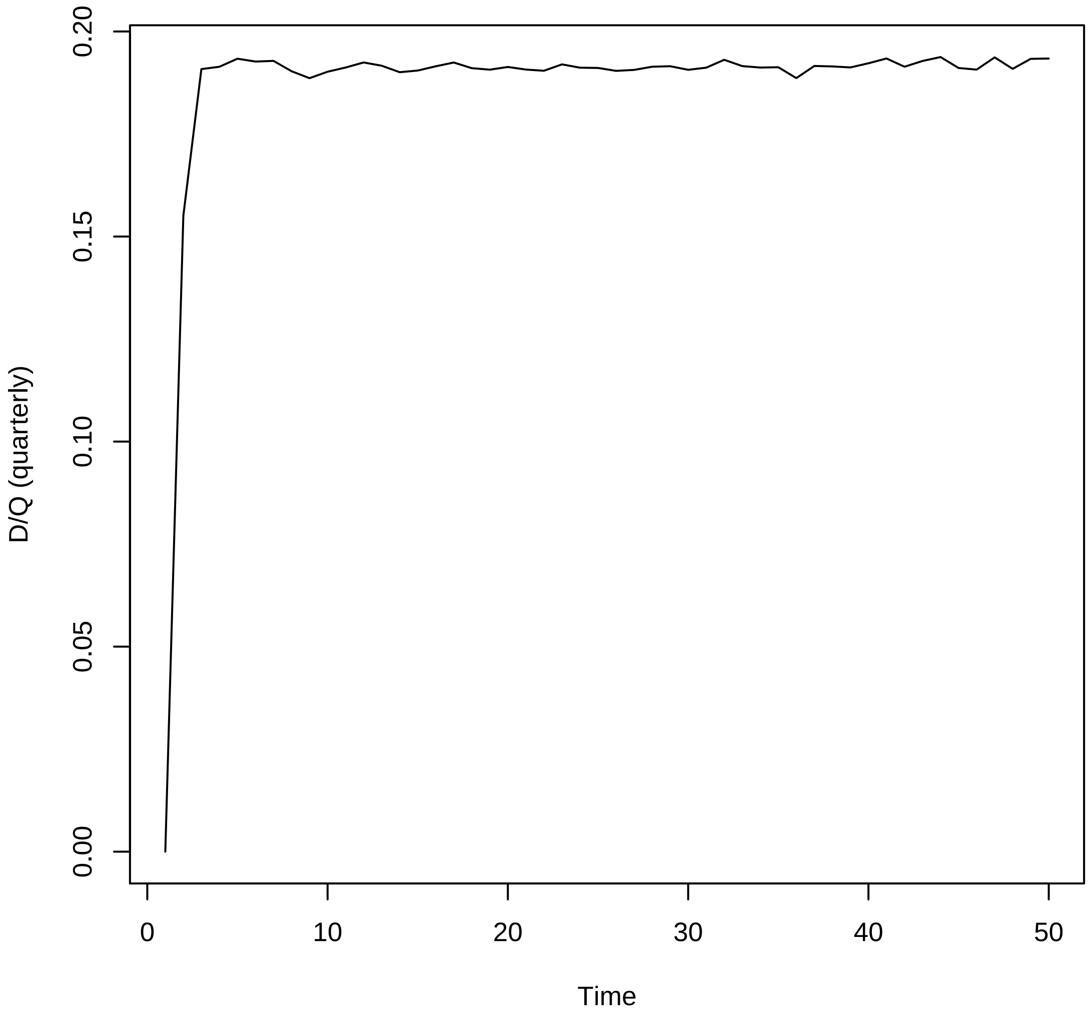


For a given demand of debt D', the interest rate is computed as $\frac{D'}{\tilde{L}(Q,D')} - 1$.

The graph clearly shows that the risk premium is zero as long as the debt-to-GDP ratio is sufficiently small; then as the debt-to-GDP ratio increases, the risk premium suddenly becomes positive with a very steep slope; then for still higher debt-to-GDP ratios, the country becomes rationed by the international investors who anticipate a certain default at next period. What is striking is that the transition zone between a zero risk and a complete rationing is very small: its width is only about 1.5 percentage points of quarterly GDP. Also, as expected, the interest rate schedule asked by investors is a decreasing function of current GDP, because GDP is serially auto-correlated.

The shape of the interest rate schedule has a direct impact on the profile of simulated series, such as the one reported on Figure 1.3.

Figure 1.3: Sample simulated series of debt-to-GDP ratio

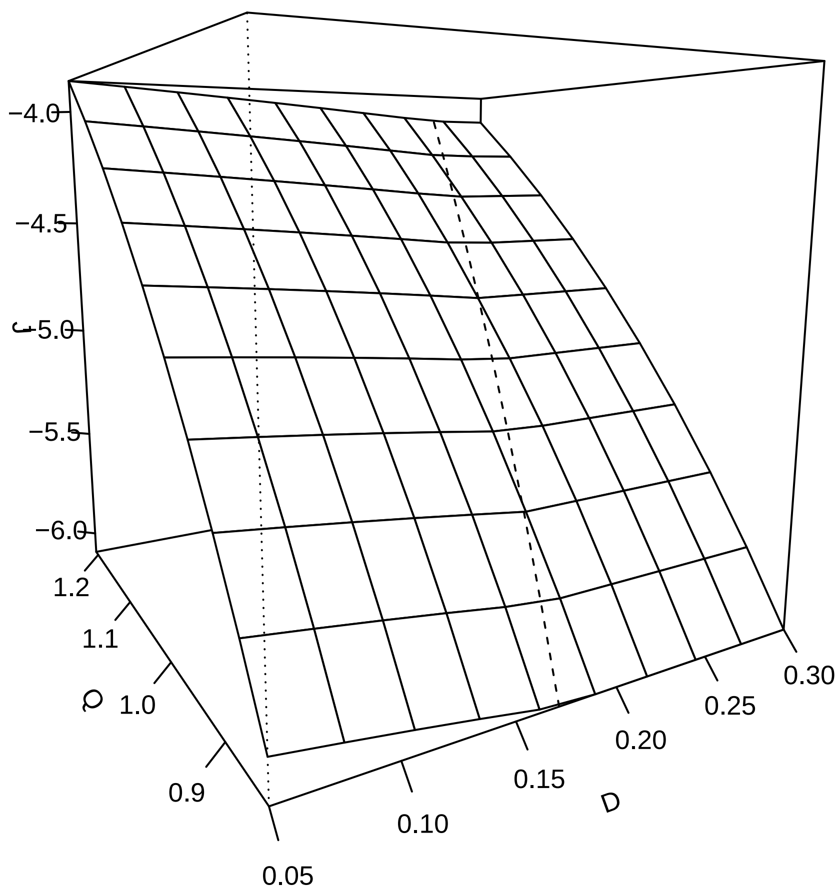


Initial state is $D/Q = 0$ and $y = 0$.

This series start with an initial indebtment level equal to zero. One sees that the country very quickly jumps (in two periods) to a level of debt-to-GDP around which it tries to stabilize itself thereafter, while it faces a sequence of productivity shocks. The level around which the stabilization is done is precisely in the zone identified on Figure 1.2 where the risk premium is non zero but still small enough. The simulated series presented on Figure 1.3 does not exhibit a default episode. Defaults typically occur when a bad productivity shock suddenly moves the debt-to-GDP ratio above the default threshold (which is actually close to the average debt-to-GDP ratio as one can see from Figures 1.2 and 1.3).

Finally, Figure 1.4 plots the value function of the country, as a function of detrended debt $\hat{D}$ and output $\hat{Q}$; the frontier between default and repayment is also made apparent on the graph. As expected, welfare is a non-increasing function of debt and an increasing function of output (more precisely, welfare is constant with respect to debt in the default zone, and decreasing in the repayment zone). When output is higher, the country can sustain a higher level of debt without defaulting.

Figure 1.4: Value function in the canonical model



The dotted line on the surface represents the frontier between default and repayment.

1.3.2 Debt maturity

One extension of the canonical model that has been studied in the literature is the introduction of long-duration bonds. Indeed, the canonical model considers only short term bonds that have to be repaid and rolled over at every period. Since the typical calibration uses a quarterly frequency, which therefore implies a maturity of one quarter, it is natural to wonder how the model would behave if the maturity was closer to what is observed in the data.

Hatchondo and Martinez (2009) introduce longer maturities in the canonical model by assuming that the sovereign issues bonds which promise an infinite stream of coupons. The value of the coupons decline geometrically with time. The rate of decline of the coupons is inversely related to the average maturity of the coupons (as shown by the Macaulay formula). The beauty of this approach is that it is analytically simple, since it does not add new state variables compared to case with one-quarter bonds. The authors show that longer maturities increase the tendency to over-borrow because the sovereign has an incentive to dilute the debt issued in previous periods. The authors compare the one-quarter maturity with a maturity of 4 years (which corresponds to the average observed maturity in emerging countries) and show that the long maturity model delivers a higher mean and standard deviation for the interest rate spread, more in line with the data. However, longer maturities implemented in this way do not help to increase the average debt-to-GDP ratio to more realistic values.

Chatterjee and Eyigungor (2011) explore another setup where bonds mature probabilistically at each period (maturation is essentially a pure Poisson process). From an analytical point of view, this assumption leads to a tractable model similar to that of Hatchondo et al. (2009). The authors claim that their model is able to quantitatively match both the default frequency and average debt-to-output ratio historically observed for Argentina; however this is achieved using a very high output cost in case of default. [21]

1.3.3 Renegotiation

The canonical model makes two strong assumptions about the negotiation process following a default. First, it assumes that the probability of a settlement between the debtor and its creditors is exogenous and constant across periods (in other words, the settlement is triggered by a pure Poisson process). Second, it assumes that this settlement is particularly unfavorable to creditors, since they recover nothing out on the defaulted debt. The model that I present in chapter 2 relaxes the second assumption by taking into account the possibility that creditors recover some of their claims after a default.

Some papers in the literature have relaxed these two assumptions in more radical ways. Yue (2010) presents a model of endogenous default where the creditors and the sovereign debtor enter a Nash bargaining after a default. Either they reach an agreement over the sharing of the negotiation surplus, or alternatively the debtor remains in permanent autarky and the creditors are entirely wiped out. Both the probability of reentry on the markets and the haircut that investors have to concede are therefore be made endogenous. The model is able to replicate key business cycle statistics along with realistic default probabilities and

21. More precisely, the output cost in this model is of the form $d_0 y + d_1 y^2$ where y is output, $d_0 = -0.19$ and $d_1 = 0.25$. For the average value of y (which is slightly above 1), the corresponding output cost is around of 6% of GDP, which is very high. And the cost is even higher for values of y in the upper part of the distribution (it actually increases quadratically).

recovery rates. However it sustains debt-to-GDP ratios which are even lower than those of the canonical model.

Two other papers by Bi (2008) and Benjamin and Wright (2009) go a little further in the modelling of the negotiation process in the sense that they also make endogenous the *delay* after which a post-default settlement takes place. This process takes the form of a repeated Nash bargaining where, at every period, one of the two parties makes an offer which is either accepted or refused by the other party; in case the offer is rejected, the roles of the two parties are swapped at the next period. In the specific framework adopted by Bi (2008), it turns out that it is sometimes optimal for both parties to delay an agreement until the "cake" to be shared is bigger (*i.e.* the productivity level in the debtor country is higher); the resulting model is capable of replicating historical average renegotiation delays. Benjamin and Wright (2009) propose a similar model, with a slightly different negotiation process (to put it shortly, they include the possibility that a defaulting country contracts a new loan instead of (or in addition to) making a down payment to its creditors). This paper is very promising because, despite its technical complexity, it is able to replicate many stylized facts, including a substantial debt-to-GDP ratio (while keeping the output cost of default at a low level, contrary to Chatterjee and Eyigungor (2011)—see section 1.3.2).

1.3.4 Incorporating the RBC/DSGE paradigm

As all models derived from the Eaton and Gersovitz (1981) tradition, the canonical model that I have presented above endogeneizes the decision to default (and also the interest rates spread as a by-product), but it takes the output process as entirely exogenous.

There is another strand in the literature which makes exactly opposite choices when trying to model small open emerging economies: it is the so-called real business cycle (RBC) school, which has given birth to the more complex dynamic stochastic general equilibrium (DSGE) models during the last decade. In this literature, the production of the small open economy (SOE) is done using capital, labor and possibly other factors, so that optimal and decentralized decisions are made in the productive sphere. On the other hand, even though the country trades with the rest of the world and therefore can potentially accumulate an external debt, default is not an option and its possibility is not endogeneized. Some models will incorporate an interest rate risk premium, but it will necessarily be *ad hoc* and model-inconsistent since by construction default is not possible.

So far, the endogenous default models and the SOE-RBC models strands have been pursued in largely independent ways, with relatively little cross-fertilization between the two paradigms. In chapter 4 I try to contribute to the filling of this gap by looking at SOE-RBC models from the perspective of endogenous default models. More precisely, I compute the default probabilities that would be implied in an SOE-RBC model if the country was given the option to default in a similar fashion as in the canonical model. This is done without modifying the core of the RBC model (*i.e.* the possibility of a default is still not endogeneized *ex ante*); I rather compute an out-of-model default value function corresponding to what the country would get if it were to default; then I test whether the value function of the RBC model is greater or smaller than the default value function. Of course this way of computing default probabilities is not fully model consistent, but it is the furthest one can go while remaining within the RBC framework.

A more consistent and ambitious approach for reconciling the RBC/DSGE models with the endogenous default paradigm has been undertaken by Mendoza and Yue (2012). Their model shares the core features of the endogenous default models—*i.e.* the sovereign endogenously decides to default or repay by choosing the welfare maximizing option—but at the same time it incorporates elements of the RBC/DSGE literature—since it features capital accumulation, labor participation decisions and financing constraints for entrepreneurs. This approach is very appealing and gives good results with respect to replicating key stylized facts of emerging countries, even though there are still some shortcomings (in particular on the level of average debt ratios). Nevertheless this approach seems very promising for future research and could lead to more feature complete models of emerging countries. Many challenges still remain to be solved, especially on the computational front, since the models which will be build along this research direction are likely to have a state space of higher dimension; they may benefit from the methodological contribution that I present in chapter 5.

1.4 Appendix: the canonical model expressed in detrended form

The policy and value functions are expressed in terms of y_t rather than $\hat{Q}_t$, recalling that $g_t = e^{y_t}$ (see equation (1.2)) and that $\hat{Q}_t = \frac{g_t}{\mu_g} = \frac{e^{y_t}}{\mu_g}$.
The value functions are:

$$J^*(\hat{D}_t, y_t) = \max\{J^r(\hat{D}_t, y_t), J^d(y_t)\}$$

$$J^r(\hat{D}_t, y_t) = \max_{\hat{D}_{t+1}} \left\{ u(\hat{Q}_t - \hat{D}_t + \tilde{L}(y_t, \hat{D}_{t+1})) + \beta\, g_t^{1-\gamma}\, \mathbb{E}_t J^*(\hat{D}_{t+1}, y_{t+1}) \right\}$$

$$J^d(y_t) = u((1-\lambda)\hat{Q}_t) + \beta\, g_t^{1-\gamma}\, \mathbb{E}_t \left[(1-x) J^d(y_{t+1}) + x\, J^*(0, y_{t+1}) \right]$$

The lending supply function verifies the zero profit condition:

$$(1+r)\tilde{L}(y_t, \hat{D}_{t+1}) = \mathbb{E}_t \left[1 - \tilde{\delta}'(\hat{D}_{t+1}, y_{t+1}) \right] g_t \hat{D}_{t+1}$$

where the default decision function is:

$$\tilde{\delta}'(\hat{D}_t, y_t) = \mathbb{1}_{J^r(\hat{D}_t, y_t) < J^d(y_t)}$$

Chapter 2

The sovereign default puzzle: Modelling issues and lessons for Europe

2.1 Introduction

Europe has recently been hit by a sovereign debt crisis which has caused three of its members to be ousted from financial markets. Those three countries, Greece, Ireland and Portugal, had to ask for the support of the other eurozone countries to refinance their debt. Additionally, in the case of Greece the eventual implementation of a nominal haircut of more than 50% was decided. In response to this unexpected crisis, Europe decided to impose a much stricter budgetary discipline, aiming for a near zero deficit rule. How did the eurozone suddenly become so vulnerable to sovereign risk? Is Europe overreacting by imposing budget constraints that are too restrictive?

Sovereign debt crisis specialists have been asked for answers. Trying to understand why some countries default is the theme of a large body of literature, as overviewed in chapter 1. In particular, Reinhart et al. (2003) have introduced the notion of "serial defaulters," and Greece is certainly one of these, having already defaulted many times over the past two centuries. The key paradox of the academic literature however is that, as already mentioned in section 1.3, it is actually very hard to satisfactorily fit the data on default probabilities and debt levels. Work by Aguiar and Gopinath (2006) or Arellano (2008) for instance struggled with the fact that a debt-to-GDP ratio in excess of only 5% could trigger a default within reasonably calibrated models. These papers have, on the surface, trivialized the problem, as almost any level of debt seems to create a risk of default.

These difficulties led Rogoff (2011) to argue that the narrative approach to debt default, as exemplified in Reinhart and Rogoff (2009), does a better job at understanding default than simulated models. This is clearly a provocative statement. Barring a calibrated model, how can one think about what the proper debt levels should be? Further, how can one rationalize the eurozone policymakers' attempts to set safe debt levels in order to avoid another crisis?

In previous models of sovereign risk, default is a costly decision that the country weighs against the alternative of repaying its debt. From a modeler's perspective, the following trade off arises. Either the cost of default is high, in which case high debt-to-GDP ratios can be sustained at the expense of a low frequency of default, as countries don't default when the costs are high. Or the cost of default is set at low levels, in which case the frequency of default can fit the data, but the sustainable debt levels become abnormally low; this is the outcome of most calibrated models today.

In this chapter, I revisit existing sovereign debt models and amend them in order to get predictions which better fit real world debt levels. The key motivation of this analysis comes from the following observation of which the Greek crisis is one illustration: countries usually do not want to default unilaterally. In fact, as well documented by the Inter-American Development Bank (2007) and Levy-Yeyati and Panizza (2011), in all cases of sovereign debt crises but one, the "decision" to default was never really a decision of the country: it came after the crisis already took place. The only case of a "strategic default" is Ecuador in 2009. This leads to the following new modelling assumption. In the model that I present, the sequencing of events is inverted: the crisis begins before the decision to default has been taken. Think of a bank panic or a temporary collapse of a key industrial sector. In these "trembling times," the cost of default becomes lower as the financial panic or the economic meltdown already happened. Default does add extra costs, but lower than those which would have be borne in "normal times." With this distinction, I show that the resulting model can simultaneously deliver high levels of debt with a high frequency of default.

This chapter is organized as follows. Section 2.2 provides a brief overview of recent debt models, then conveys an intuitive outline of the present contribution. In section 2.3, relying on the key insights of the theory of Lévy processes which allows one to split output into a Brownian and a Poisson process, I develop a simplified model. "Trembling times" are interpreted as shocks generated by Poisson jumps; in this model, these shocks are those which have the potential to generate default. I demonstrate that Brownian shocks instead do not have the property of triggering default events. In a continuous time setting, I show that an optimizing social planner should always absorb Brownian shocks so as to avoid default. This allows to discriminate among two key causes of debt crises. One is the failure to adjust in real time to a smooth shock, the solution to which being to have a more efficient monitoring of intra-annual deficit. The second is the challenge of a discontinuous shock, which is where the core problem comes from. I argue that previous models' difficulties in replicating default owe much to the lack of understanding of this distinction. In section 2.4, I present and simulate the full-fledged version of the model using the standard assumptions of emerging economies. In section 2.5, the model is recast in the European setting and some policy conclusions for the eurozone are drawn. Section 2.6 concludes.

2.2 Calibrating sovereign debt models

Calibrated models of sovereign debt owe much to the papers by Aguiar and Gopinath (2006), Arellano (2008) and Mendoza and Yue (2012), which followed in the (earlier) tradition of Eaton and Gersovitz (1981), Cohen and Sachs (1986) and Bulow and Rogoff (1989a). Their general framework have been presented in section 1.3.

As already mentioned, these models have successfully reproduced key business cycle correlations regarding aggregate spending and balance of payments in particular. The problem encountered by these models however, is that they meet great difficulties in calibrating reasonable debt thresholds and probabilities of default at the same time. Table 2.1 summarizes the key results obtained by several recent papers along these dimensions.

Table 2.1: Overview of mean debt-to-GDP ratios and default probabilities in the literature

Paper	Main feature	Debt-to-GDP mean ratio (%, annual)	Default probability (%, annual)
Arellano (2008)	Non-linear default cost	1	3.0
Aguiar and Gopinath (2006)	Shocks to GDP trend	5	0.9
Cuadra and Sapriza (2008)	Political uncertainty	2	4.8
Fink and Scholl (2011)	Bailouts and conditionality	1	5.0
Yue (2010)	Endogenous recovery	3	2.7
Mendoza and Yue (2012)	Endogenous default cost	6	2.8
Hatchondo and Martinez (2009)	Long-duration bonds	5	2.9
Benjamin and Wright (2009)	Endogenous recovery	16	4.4
Chatterjee and Eyigungor (2011)	Long-duration bonds	18	6.6

Most papers report the debt-to-GDP ratio using GDP measured at a quarterly frequency; here instead I choose to use GDP measured at an annual frequency, since this is the convention used by policymakers and in the policy debate. For Aguiar and Gopinath (2006), the reported results are for their model II (with shocks to GDP trend). For Arellano (2008) and Aguiar and Gopinath (2006), the reported values come from Hatchondo et al. (2010) who re-simulate these models using more precise numerical techniques. For Hatchondo and Martinez (2009), the reported values are those obtained for their λ parameter equal to 20%.

Before discussing the results of these papers, one should note the improbably high discount factor that some models have to rely on to sustain their equilibrium. For example, Yue (2010) and Aguiar and Gopinath (2006) set respective values of 0.72 and 0.8 for the (quarterly!) discount factor. This high impatience helps to generate frequent defaults and a desire to hold debt, but it is unrealistically high, even when accounting for political instability. Others, like Arellano (2008), Benjamin and Wright (2009) or Chatterjee and Eyigungor (2011) use values close to 0.95, which is more realistic. I will use this last value in the simulations presented in section 2.4.4.

In order to fit the conventional wisdom of markets and international financial institutions, one would want a model that could predict:

– *Threshold debt levels in the vicinity of 40% of yearly GDP.* The mean debt-to-GDP ratio in 2009 was 42% across countries, according to World Bank (2010). Note that World Bank

(2004) classifies as "severely indebted" countries with a debt-to-GNI ratio above 80%, and as "moderately indebted" countries with a ratio above 48%: the target of 40% is therefore in the lower end of the range of interest; in section 2.5.3 I show how to reach higher levels of debts.

– *Annual default probabilities in the range of 3%.* Yue (2010) reports that the average default rate of Argentina since 1824 is 2.7%. Benjamin and Wright (2009) estimate an average default rate across countries of 4.4% for the period 1989–2006. In the data collected by Cohen and Valadier (2011) over the period 1970–2007, which includes "soft defaults" such as IMF loans, an even higher probability of default of about 7% is documented. I stick to the 3% target preferred by most papers, to make the comparison easier.

Even though many papers listed in Table 2.1 reach the target in terms of default probabilities, they all fail with respect to the sustainable debt ratios; the two best results along this dimension are Benjamin and Wright (2009) and Chatterjee and Eyigungor (2011) who respectively reach debt levels of 16% and 18% of yearly GDP.[1]

I now turn to the task of proposing a quantitative sovereign debt model that matches the two stylized facts regarding debt levels and default probabilities, with a minimal departure from the canonical model. The proposed modifications hinge on the following arguments:

1. The cost of default in the models found in the literature is too low to be true. Based on historical averages they typically assume that it usually takes two and a half years of financial autarky to pay for the consequences of a default. I revise upwards this cost by adding one first trick: post default countries are rarely debt free. As documented by Sturzenegger and Zettelmeyer (2007) and Cruces and Trebesch (2011), creditors do capture a recovery value of debt after default. Cohen (1992) also showed that post-Brady recovery values were quite significant in the 1980s. Even in the most celebrated default incident, Argentina, creditors clawed back about one third of their claims. By taking into account the post-default recovery value, the upper limit of debt is significantly raised. Although the point is often acknowledged in the literature,[2] it has seldom been theorized or calibrated in previous models (there are some exceptions, see section 1.3.3).

2. Another critical ingredient is added, building on the theoretical inspiration of the Lévy stochastic processes. These processes can be roughly defined as the generalization of random walks to continuous time. More precisely, any stochastic process in continuous time with stationary and independent increments is a Lévy process. The Lévy-Itô decomposition states that any Lévy process is essentially the sum of two components: a Brownian process and a compound Poisson process. As I shall demonstrate, Brownian processes do not have the ability to generate defaults. Instead they function as in deterministic models; whatever the cost of default, the corresponding probability of

1. Note that Benjamin and Wright (2009) argue that the historical average of the yearly debt-to-GDP ratio over their data set is precisely 18%. They choose their calibration in order to match that target and are able to do so using a relatively low value for the output cost of default. Their model may therefore be able to reach higher levels of debt while still keeping the output cost at a reasonable value, but I did not check that.

2. See, for example, Hatchondo and Martinez (2009, footnote 15).

default is zero. Default must depend on exogenous shocks, creating discrete jumps in the wealth of a nation. Such shocks are well-represented by the Poisson process.

3. This insight allows to add the critical change that I alluded to in the introduction, namely that crises almost always precede the decision to default, rather than the other way round, as usually assumed by the literature. The Poisson component is used for generating the "trembling times" during which a transitory crisis hits the country. They correspond to the episodes when default becomes possible.

2.3 A Lévy driven model of default

This section develops a very stylized model of sovereign default to demonstrate that the properties of these models dramatically change with the type of stochastic process assumed for output. The discussion is based on the theory of Lévy processes, that are briefly introduced below. Building on this intuition, section 2.4 will present a quantitative sovereign debt model designed to match the quantitative targets identified in section 2.2.

2.3.1 Lévy processes

Definition and key properties

A Lévy process is a stochastic process that has stationary and independent increments.[3] It is the generalization in continuous time of random walks in discrete time. As the Lévy-Itô decomposition shows, a Lévy process is the sum of three terms: a Brownian process with deterministic drift; a compound Poisson process; and a third term which intuitively represents an infinite sum of infinitesimally small jumps. I ignore the third term since it is more a mathematical curiosity, and thus consider a process which is simply the sum of a Brownian process with drift and of a compound Poisson process.

In order to simplify the presentation, a discrete time approximation of this process will be considered, calling h the length of the time period that shall be shrunk to zero in the analysis. The two limiting cases at hand are first examined.

The Brownian case

A first simple case is when the law of motion of (the log of) output corresponds (asymptotically) to a discrete time version of a Brownian process:

$$Q_{t+h} = \begin{cases} e^{\sigma\sqrt{h}}Q_t & \text{with probability } \frac{1}{2} + \frac{\mu}{2\sigma}\sqrt{h} \\ e^{-\sigma\sqrt{h}}Q_t & \text{with probability } \frac{1}{2} - \frac{\mu}{2\sigma}\sqrt{h} \end{cases}$$

3. In addition to these two basic properties, there are also technical regularity conditions. See for example Applebaum (2004) for more details.

As h goes to zero, this process converges towards a geometric Brownian process of "percentage drift" μ and "percentage volatility" σ.

The Poisson case

The second simple case is when the law of motion of (the log of) output corresponds (asymptotically) to a discrete time version of a compound Poisson process:

$$Q_{t+h} = \begin{cases} Q_t & \text{with probability } e^{-p_0 h} \\ k(h)\, \tilde{m}_t\, Q_t & \text{with probability } 1 - e^{-p_0 h} \end{cases}$$

where $\tilde{m}_t$ is a stationary process. For the purpose of the economic analysis, I will assume that the support of $\tilde{m}_t$ is included in the interval $(0, 1)$, and therefore represents a "malus:" with an infinitesimal probability, the country loses a non infinitesimal amount of output. The term $k(h) = \frac{p_0 h}{1 - e^{-p_0 h}}$ is a technical artifact of the discretization,[4] and it goes to 1 as h goes to 0.

As h goes to zero, Q_t converges towards a geometric compound Poisson process. More precisely, $\log Q_t$ converges towards a compound Poisson process whose rate is p_0 and whose jump size distribution equals the stationary distribution of $\tilde{m}_t$.

General form

A discrete time approximation of a Lévy process can be embedded in the following model:

$$Q_{t+h} = \begin{cases} e^{\sigma\sqrt{h}} Q_t & \text{with probability } \left(\frac{1}{2} + \frac{\mu}{2\sigma}\sqrt{h}\right) e^{-p_0 h} \\ e^{-\sigma\sqrt{h}} Q_t & \text{with probability } \left(\frac{1}{2} - \frac{\mu}{2\sigma}\sqrt{h}\right) e^{-p_0 h} \\ k(h)\, \tilde{m}_t\, Q_t & \text{with probability } 1 - e^{-p_0 h} \end{cases}$$

2.3.2 Financial markets

Financial environment

The world financial markets are characterized by an instantaneous, constant, riskless rate of interest r. Lenders are risk-neutral and subject to a zero-profit condition by competition. Debt is short-term and needs to be refinanced every year.

The timing of events is as follows. First assume that the country has incurred a debt obligation D_t due at time t, and has always serviced it in full in previous years. At the beginning of period t, the country learns the value of its output Q_t. It then either defaults on its debt or reimburses it. If the debt is reimbursed in full, the country can contract a new loan, borrowing L_t, which must be repaid at time $t + h$ in the amount of D_{t+h}. Note that the implicit instantaneous interest rate is equal to $\frac{\log(D_{t+h}/L_t)}{h}$. Such financial agreements being

4. To be precise, $k(h)$ corresponds to the expectation of the number of shocks of the continuous Poisson process during a period of length $p_0 h$, conditional on the fact that there is at least one shock in this interval.

concluded, the country eventually consumes, in the event it services its debt in full:

$$C_t^r = Q_t + L_t - D_t.$$

Alternatively, in the event of a debt crisis the country may default (see below). This occurs when output is too low to allow the country to service its debt. Call $\pi_{t+h|t}$ the probability of default at time $t+h$ from the perspective of date t.

The zero-profit condition for creditors may be written as:

$$L_t\, e^{rh} = D_{t+h}(1 - \pi_{t+h|t}) \tag{2.1}$$

Note that it is assumed that in case of default, the investors recover nothing. This assumption will be relaxed further in the paper.

Default

At any time t, the country that has accumulated a debt D_t may decide to default upon it. When it does so, it is assumed that the country suffers a penalty $\lambda \in [0, 1)$ on output as a consequence of the crisis. This penalty is captured by no one and is therefore a net social loss. Let's call Q_t^d the post-penalty value of income (which is different from output) and for the time being simply write:

$$Q_t^d = (1 - \lambda)Q_t.$$

As another cost, let's assume that the country is subject to financial autarky, being unable to borrow again later on.[5] Consumption can then be written as:

$$C_t^d = Q_t^d = (1 - \lambda)Q_t.$$

2.3.3 Preferences and equilibrium

Preferences

The decision to default or to stay on the financial markets involves a comparison of two paths that implies expectations over the entire future. In order to address this problem, let's assume that the country seeks to solve:

$$J^*(D_t, Q_t) = \max_{\{C_{t+jh}\}_{j\geq 0}} \mathbb{E}_t \left\{ \sum_{j=0}^{\infty} e^{-\omega jh} u(C_{t+jh}) \right\}$$

5. A milder form of a sanction would be, more realistically, that the country is barred from the financial market for some time only, as in the canonical model of section 1.3.1. This less demanding assumption is explored in the model of section 2.4.

where ω is the instantaneous rate of preference for the present. D_t can be negative if the country builds up foreign assets. Utility is isoelastic, of the form:

$$u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$$

Let's call:

$$J^d(Q_t) = \mathbb{E}_t \left\{ \sum_{j=0}^{\infty} e^{-\omega jh} u(Q^d_{t+jh}) \right\}$$

the post-default level of utility, which becomes by definition independent of debt, and to which the country is nailed down in case of servicing difficulties. If it were to stay current on its debt obligation, the country would obtain:

$$J^r(D_t, Q_t) = \max_{L_t, D_{t+h}} \left\{ u\left(Q_t + L_t - D_t\right) + e^{-\omega h} \mathbb{E}_t J^*(D_{t+h}, Q_{t+h}) \right\}$$

subject to the zero profit condition (2.1).

When comparing how much it can get by staying on the markets and the post-default level of welfare, the country chooses its optimum level:

$$J^*(D_t, Q_t) = \max\{J^r(D_t, Q_t), J^d(Q_t)\}$$

Recursive equilibrium

Let's define a recursive equilibrium in which the government does not have commitment and in which the various agents act sequentially.

The aggregate state of the model is $s = (\delta, D, Q)$, where δ is past credit history (equal to 1 if country is barred from financial markets, 0 otherwise), D is the stock of debt due in the current period (necessarily equal to zero if $\delta = 1$) and Q is current GDP.

Definition 2.1 (Recursive equilibrium in Lévy model). *The recursive equilibrium for this economy is defined as a set of policy functions for the (i) government's default decision $\tilde{\delta}'(s)$; (ii) government's decision for tomorrow's debt holding $\tilde{D}'(s)$; and (iii) investor's supply of borrowing $\tilde{L}(s, D')$ such that:*

– *taking as given the investor's policy function, the default decision $\tilde{\delta}'(s)$ and decision for tomorrow's debt holding $\tilde{D}'(s)$ satisfy the government optimization problem:*

$$\tilde{\delta}'(s) = \begin{cases} 1 & \text{if } \delta = 1 \text{ (default in the past) or } J^d(Q) > J^r(D, Q) \text{ (default now)} \\ 0 & \text{otherwise} \end{cases}$$

$$\tilde{D}'(s) = \begin{cases} \arg\max_{D'} \left\{ u(Q - D + \tilde{L}(s, D')) + e^{-\omega h} \mathbb{E}_Q J^*(D', Q') \right\} & \text{if } \tilde{\delta}'(s) = 0 \\ 0 & \text{otherwise} \end{cases}$$

where:

$$
\begin{aligned}
J^r(D,Q) &= \max_{D'} \left\{ u(Q - D + \tilde{L}(s,D')) + e^{-\omega h} \mathbb{E}_Q J^*(D',Q') \right\} \\
J^d(Q) &= u((1-\lambda)Q) + e^{-\omega h} \mathbb{E}_Q J^d(Q') \\
J^*(D,Q) &= \max\{J^r(D,Q), J^d(Q)\}
\end{aligned}
$$

– *taking as given the government's default decision function, the investor's policy function $\tilde{L}(s,D')$ satisfies the zero profit constraint:*

$$\tilde{L}(s,D') = e^{-rh} \left[1 - \mathbb{E}_Q \tilde{\delta}'(\delta, D', Q')\right] D'$$

Note that the formulation for lending decision by the investors prevents multiple equilibria in the interest rate, as noted in section 1.2.3.

Equilibrium in the Brownian case

I first investigate the nature of the equilibrium in the Brownian case. In this section output is supposed to follow a discretized version of the geometric Brownian motion, as described in section 2.3.1. The following result holds:

Proposition 2.2. *In the Brownian case, if $h < \frac{1}{\left(\frac{\mu}{\sigma}+4\sigma\right)^2}$, only two cases are possible (for a given initial value of the debt-to-GDP ratio):*

– *the country immediately defaults;*

– *the country never defaults (whatever the future path of output).*

Proof. See the appendix 2.7. □

In other words, either the debt is already too high and the country immediately defaults, or it will never do so. The intuition is straightforward: because of the continuous nature of growth, the country can always adjust to shocks and the creditors monitor it. Brownian noise is not different from deterministic fluctuations.

One empirical question that this result points to is whether, in the real world, decisions are indeed taken continuously. The Greek case provides an instructive example. When Prime Minister Papandreou took office, he realized that the deficit he inherited was much larger than he originally thought. Having been given the wrong information in the beginning inevitably delayed the right policy choices on his part. Perhaps, the lag in evaluating the situation is responsible for the crisis. I return to this question in the empirical analysis below. One can nevertheless compute the length of the time period h^* during which a policymaker can prevent crises triggered by Brownian shocks. For reasonable parametrizations of the model, one has a time window of about one quarter (this would be roughly the case when μ/σ is near one). For more volatile economies, say when $\mu = 2\%$ and $\sigma = 3\%$ in quarterly frequency, the time window is about 5 months. A paradox here is that the more volatile an economy is, the more time a policymaker has to react to the shocks.

Equilibrium in the Poisson case

I now investigate the nature of the equilibrium in the Poisson case. In this section output is supposed to follow a discretized version of the geometric compound Poisson process, as described in section 2.3.1. The following result holds:

Proposition 2.3. *In the Poisson case, the probability of default between dates t and* $t+1$ *is inferior to* $1-e^{-p_0}$.

Proof. By lemma 2.8 (see appendix 2.7), default never happens in the good state of nature. So the probability of not having a default between dates t and $t+h$ is superior to $e^{-p_0 h}$. Therefore the probability of not having a default between dates t and $t+1$ is superior to e^{-p_0} (using the independence of growth shocks between periods). □

There are cases in which the probability of not having a default at each period is exactly equal to e^{-p_0}. Consider the extreme case where the country totally ignores the future ($\omega = 0$). The default threshold (expressed as a debt-to-GDP ratio) is clearly $d^* = \lambda$. Since, by lemma 2.8 (see appendix 2.7), default never happens in the good state of nature, the country will always choose the maximum debt level conditional to not defaulting in the good state (*i.e.* $D_{t+1} = \lambda Q_t$). This means that the country will default in the bad state, and therefore the probability of default at each period is equal to the probability of moving to the bad state, *i.e.* e^{-p_0}.

The comparison between the Brownian and Poisson cases is straightforward. When the economy is smooth, countries can continuously adjust their debt levels and never default. Obviously, when the economy is disrupted by a Poisson shock, default becomes a possibility of probability p_0 (per unit of time).

Comparison with Aguiar and Gopinath (2006)

Aguiar and Gopinath (2006) suggested an interesting line of reasoning, which can be summarized as follows: in emerging countries, growth rates (not output) are highly volatile. When growth is expected to be high, this raises the willingness to borrow (as fast growth raises debt by raising the permanent income) and therefore it also raises the risk of a debt overhang. Yet the outcome is less satisfactory than expected as shown in Table 2.1: despite very high discount factors, the debt levels remain quite low. Aguiar and Gopinath (2006) fail to recognize that the volatility of the growth rate is not enough to trigger the risk of default; what is really needed is a discontinuous jump in the parameters that switch the probability of default.

2.3.4 Numerical results

In order to get a better understanding of the properties of the models presented in this section, numerical exercises are performed on calibrated versions of those models and the sensitivity of the results to the length of the time period h is analyzed. Table 2.2 shows the calibration used for this exercise.

Table 2.2: Calibration of discretized Lévy models

Risk aversion	γ	2
Discount rate	ω	$\log(0.8)$
Riskless interest rate	r	$\log(1.01)$
Loss of output in autarky (% of GDP)	λ	0.5%
Drift of Brownian process	μ	1%
Volatility of Brownian process	σ	2.2%
Period size for which Brownian and Poisson are observationally equivalent	h_0	4

Quarterly frequency.

For the Brownian case (section 2.3.1), I set $\sigma = 2.2\%$, $\mu = 1\%$. The corresponding threshold for h under which defaults are impossible in this model, as given by proposition 2.2, is $h^* \simeq 3.4$ (which is almost one year). For the Poisson case, I slightly modify the process described in section 2.3.1 to allow for a positive trend, so that:

$$Q_{t+h} = \begin{cases} g^+(h)Q_t & \text{with probability } e^{-p_0 h} \\ g^-(h)Q_t & \text{with probability } 1 - e^{-p_0 h} \end{cases}$$

where g^+ and g^- are functions of h which are defined below. Note that the jump size is not stochastic, since this is not needed for the purpose of this exercise. The Poisson process is calibrated so that:

- for some specific value $h = h_0$, the Brownian and Poisson are observationally equivalent (*i.e.* same probabilities for up and down moves, same jump sizes);
- for all values of h, the two processes have the same average growth;
- the magnitude of the output loss in case of a bad Poisson shock does not depend on h.

These constraints translate into the following relationships, which identify p_0, $g^+(h)$ and $g^-(h)$:

$$e^{-p_0 h_0} = \frac{1}{2} + \frac{\mu}{2\sigma}\sqrt{h_0}$$

$$\forall h,\ e^{-p_0 h} g^+(h) + (1 - e^{-p_0 h}) g^-(h) = \left(\frac{1}{2} + \frac{\mu}{2\sigma}\sqrt{h}\right) e^{\sigma\sqrt{h}} + \left(\frac{1}{2} - \frac{\mu}{2\sigma}\sqrt{h}\right) e^{-\sigma\sqrt{h}}$$

$$\forall h,\ g^-(h) \frac{p_0 h}{1 - e^{-p_0 h}} = e^{-\sigma\sqrt{h_0}} \frac{p_0 h_0}{1 - e^{-p_0 h_0}}$$

Finally, I set $h_0 = 4$, *i.e.* one year.

Table 2.3 reports the results from the numerical simulations of these two models for various values of h, which range approximately from one year to one week.

One can see that proposition 2.2 is verified empirically: for $h < h^*$, the Brownian model has zero default; conversely, the Poisson case still exhibits defaults as h goes to zero.

Table 2.3: Moments of discretized Lévy processes for various period durations

Length of time period (h, in quarters)	4	2	1	0.33
Discretized Brownian process				
Default threshold (debt-to-GDP, quarterly, %)	48.4	51.9	68.8	79.3
Default probability in 10 years (%)	35.7	0.0	0.0	0.0
Discretized Poisson process				
Default threshold (debt-to-GDP, quarterly, %)	48.4	47.7	47.6	47.5
Default probability in 10 years (%)	35.1	34.6	34.3	40.0

The processes are parametrized as described in section 2.3.4. The solution to the detrended model is approximated using value function iteration on a grid of 25 points for the debt-to-GDP ratio. Moments are obtained by averaging over 1,000 simulations of a length of 10 years.

A similar exercise is performed by Carré (2011) on the canonical model (*i.e.* model II of Aguiar and Gopinath (2006)). In that model, the growth rate follows an AR(1) process (see (1.3)): such a process can be obtained as the discretization of an Ornstein-Uhlenbeck process.[6] The results obtained by Carré (2011) are consistent with those of the present section: below a certain value for the discretization step h, defaults simply disappear. This shows that the defaults exhibited by the canonical model at the quarterly frequency (see Table 1.3) are essentially an artifact of the discretization process.

2.4 The full-fledged model

Building on the ideas presented in the previous section, I now construct a full-fledged model of sovereign debt. It shares the core features of the canonical model presented in section 1.3.1 and incorporates a few key ingredients which enable it to perform better with respect to default frequency and average debt-to-GDP ratio.

2.4.1 The stochastic process

Note that the model presented in this section has a time period of constant length, calibrated to one quarter, as in the canonical model (in terms of the notations of section 2.3, one has $h = 1$).

General form

Let's assume that the output of the country is described by the following stochastic process:

$$\frac{Q_t}{Q_{t-1}} = g_t = e^{y_t} + z_t$$

6. The Ornstein-Uhlenbeck process is a continuous time stochastic process with Brownian increments and a mean-reverting tendency. Note that it is not a Lévy process, because its increments are not independent.

where y_t and z_t are two stochastic processes. The y_t process is a standard auto-regressive process:

$$y_t - \mu_y = \rho_y(y_{t-1} - \mu_y) + \varepsilon_t^y \qquad \varepsilon_t^y \rightsquigarrow \mathcal{N}(0, \sigma_y^2) \qquad \mu_y = \log(\mu_g) - \frac{\sigma_y^2}{2(1-\sigma_y)^2}$$

It is such that $\mathbb{E}(e^{y_t}) = \mu_g$. This component of the growth process is identical to the process assumed for the canonical model. From the perspective of the previous section, it embodies the "Brownian" component of the growth process. Let's call it the *B component* of the process. The second term, z_t, embodies the "Poisson" component of the growth process. Let's call it the *P component*. It evolves according to the following law of motion:

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z$$

where the behavior of the innovation ε_t^z depends on the state of the economy, which is now described.

1. So long as the **country has not defaulted**, let's assume that the economy is in either of the following two states of nature: "normal times" or "trembling times."
   - In "normal times," the innovation follows:

$$\varepsilon_t^z = \begin{cases} 0 & \text{with probability } 1-p \\ -\mu_z & \text{with probability } p \end{cases} \tag{2.2}$$

     This variable embodies the risk of a low-probability but violent negative confidence shock, in the spirit of the Poisson process. I assume that when this shock occurs, the economy moves to the "trembling times" state: the markets have lost confidence, this lack of confidence has real negative consequences, but the markets are willing to revise their judgment if the country behaves responsibly and does not default.
   - The "trembling times" correspond to a situation where the markets have doubts concerning the economy's strength. These doubts have real negative consequences for the economy, but there is a possibility for confidence to be restored and for real negative effects to be reverted. In other words, the crisis is "reversible." The innovation follows:

$$\varepsilon_t^z = \begin{cases} 0 & \text{with probability } 1-q \\ \mu_z & \text{with probability } q \end{cases} \tag{2.3}$$

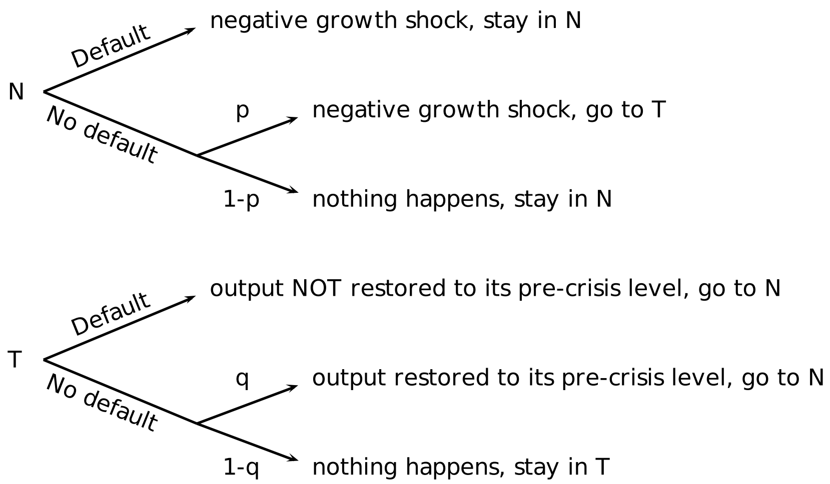
     The idea here is that the country can recover from the tremor of the confidence shock with a probability q (which is typically much higher than p). Once the country moves out of the confidence crisis, growth is "restored" to its pre-crisis level and the economy returns to "normal times."
2. **Following a default**, the economy evolves as follows:
   - If the country defaults during "normal times," then the country suffers the same

negative growth shock that it would have undergone in case of a confidence crisis (*i.e.* $\varepsilon^z_{t+1} = -\mu_z$). However after default it remains in "normal times," which means that the output loss is not reversible (since the country has already defaulted).

– If the country defaults during "trembling times," then it loses the ability to restore output to pre-crisis levels. The country does not suffer an additional negative growth shock (it has already undergone one), but the economy returns to "normal times," which means that it can no longer expect that positive news may end the crisis. Since the country has defaulted, the doubts of the investors about the strength of the economy have been confirmed and there is no reason to revert the shock.

In other words, a confidence shock acts like a "trembling hand" event: it shakes the economy for a while. If during such an episode the country defaults, then the trembling shock becomes permanent and no recovery takes place. When instead the country defaults while being in good times, the default creates on its own a confidence shock from which the economy does not recover (except for the fact that the growth loss dies out naturally over time, since z_t is a mean reverting process). The whole process is summarized in Figure 2.1.

Figure 2.1: Law of motion of the economy



N is "normal times", T is "trembling times".

It should be noted here that the process assumed for the output shares some features with the Markov-switching model for GDP introduced by Hamilton (1989). Like Hamilton's, the present model has an underlying state variable corresponding to the current regime, and the mean of the growth rate is different across regimes. But there are two critical differences. First, in the present model, the growth rate is only temporarily lowered in the "trembling" state, even if the economy stays in that state for a long time (because z_t is a mean reverting process), while in Hamilton's model, the growth rate is permanently lowered as long as the economy is in the bad state. More importantly, the switch between the two regimes is partly endogenous in the present model (it can be triggered by a default decision), while it is entirely exogenous in Hamilton's model.

2.4.2 The other costs of default

Beyond the output costs that were just described, a defaulting country is subject, as in the previous models in the literature, to the following costs. First, creditors manage to inflict, on top of the trembling shock, a penalty λ (which they do not monitor). Furthermore, they impose financial autarky on the debtor, at least for a while, so that:

$$C_t^d = Q_t^d = (1 - \lambda)Q_t$$

as long as the country is in default.

As in the canonical model of section 1.3.1, I assume that a defaulting country can return to financial markets after a while. Let's call x the probability of a settlement at a given period. When the settlement occurs, the penalty λ is lifted (but not the effect of the trembling shock). Once a settlement is reached, debt is not canceled; it is only written down to a level consistent with the post-default level of output and the historical data on post-default haircuts (see more on this in section 2.4.4). Let's call V the settlement value of post default debt.

The pair (x, V) (the duration of financial autarky after a default and the post-default recovery value) have been modeled by Yue (2010) and Benjamin and Wright (2009) as the endogenous outcome of a bargaining process following a default (see section 1.3.3). Contrary to these authors, I assume that the recovery value is not a function of past debt. The idea is simply that, after a default, prior commitments become irrelevant.[7]

Also note that if the recovery value V were too high (for example greater or equal to the debt threshold above which the country defaults), then the resulting model would be conceptually equivalent to a model where no settlement is ever reached after a default (*i.e.* where $x = 0$).[8]

2.4.3 The equilibrium

Let's define a recursive equilibrium in a similar fashion as in section 2.3.3. The government does not make decisions under commitment, and the various agents act sequentially in response to a state as defined below.

Definition 2.4 (State of the economy)**.** *The state of the economy is:*

$$s = (\delta, \theta, D, y, z, Q)$$

where δ is past credit history (equal to 1 *if the country is barred from financial markets,* 0 *otherwise), θ*

7. Indeed, Yue (2010) demonstrates that in her model the recovery value is indeed independent of the level of pre-default debt (see Theorem 2). Note that the recovery value is still a function of the productivity level.

8. Note that in the present framework the two options are not strictly equivalent because, in the case of a high V, the country will repeatedly pay the negative growth shock that occurs after a default. In the $x = 0$ case, the country pays the negative growth shock only once.

is equal to N in "normal times" and T in "trembling times," D is the stock of debt due in the current period (necessarily equal to zero if $\delta = 1$), y and z characterize current GDP growth as described in section 2.4.1, and Q is current GDP level.

Starting from a state $s = (\delta, \theta, D, y, z, Q)$, the next state $s' = (\delta', \theta', D', y', z', Q')$ is defined by the following law of motion:

- The default history evolves according to the corresponding policy function (see definition 2.5):

$$\delta' = \tilde{\delta}'(s)$$

- θ' and z' are jointly determined according to the following table (which is the mathematical reformulation of Figure 2.1):

	$\tilde{\delta}'(s) = 0$ (Repayment)	$\tilde{\delta}'(s) = 1$ (Default)
$\theta = N$ (Normal)	$\begin{cases} \theta' = N, z' = \rho_z z & \text{with prob. } 1-p \\ \theta' = T, z' = \rho_z z - \mu_z & \text{with prob. } p \end{cases}$	$\theta' = N, z' = \rho_z z - \mu_z$
$\theta = T$ (Trembling)	$\begin{cases} \theta' = T, z' = \rho_z z & \text{with prob. } 1-q \\ \theta' = N, z' = \rho_z z + \mu_z & \text{with prob. } q \end{cases}$	$\theta' = N, z' = \rho_z z$

- The B component of growth evolves according to the law described in section 2.4.1:

$$y' = \mu_y + \rho_y(y - \mu_y) + \varepsilon'^y \qquad \varepsilon'^y \rightsquigarrow \mathcal{N}(0, \sigma_y^2)$$

- GDP is augmented by its growth rate, as described in section 2.4.1:

$$Q' = Q(e^{y'} + z')$$

- The level of debt evolves according to the corresponding policy function (see definition 2.5):

$$D' = \tilde{D}'(s)$$

Definition 2.5 (Recursive equilibrium). *The recursive equilibrium for this economy is defined as a set of policy functions for (i) the government's default decision $\tilde{\delta}'(s)$, (ii) the government's decision for tomorrow's debt holding $\tilde{D}'(s)$, and (iii) the investor's supply of lending $\tilde{L}(s, D')$ such that:*

- *Taking as given the investor's policy function, the default policy function $\tilde{\delta}'(s)$ and the decision for tomorrow's debt holding $\tilde{D}'(s)$ satisfy the government optimization problem:*

$$\tilde{\delta}'(s) = \begin{cases} 1 & \text{if } \delta = 1 \text{ (default in the past) and no redemption} \\ 1 & \text{if } \delta = 0 \text{ and } J^{d,\theta}(y, z, Q) > J^{r,\theta}(D, y, z, Q) \text{ (default now)} \\ 0 & \text{otherwise} \end{cases}$$

$$\tilde{D}'(s) = \begin{cases} \arg\max_{D'} \{u(Q - D + \tilde{L}(s, D')) & \textit{if } \delta = 0 \textit{ and } \tilde{\delta}'(s) = 0 \\ \quad + \beta\,\mathbb{E}_y \left[(1-p) J^{*,N}(D', y', \rho_z z, Q') \right. & \textit{and } \theta = N \\ \quad\quad \left. + p\, J^{*,T}(D', y', \rho_z z - \mu_z, Q') \right] \} & \\ \arg\max_{D'} \{u(Q - D + \tilde{L}(s, D')) & \textit{if } \delta = 0 \textit{ and } \tilde{\delta}'(s) = 0 \\ \quad + \beta\,\mathbb{E}_y \left[(1-q) J^{*,T}(D', y', \rho_z z, Q') \right. & \textit{and } \theta = T \\ \quad\quad \left. + q\, J^{*,N}(D', y', \rho_z z + \mu_z, Q') \right] \} & \\ V(Q') & \textit{if } \delta = 1 \textit{ and } \tilde{\delta}'(s) = 0 \textit{ (redemption)} \\ 0 & \textit{if } \tilde{\delta}'(s) = 1 \textit{ (default now)} \end{cases}$$

where β is the subjective discount factor and:

$$\begin{aligned} J^{r,N}(D, y, z, Q) &= \max_{D'} \{u(Q - D + \tilde{L}(s, D')) + \beta\,\mathbb{E}_y \left[(1-p) J^{*,N}(D', y', \rho_z z, Q') \right. \\ &\qquad \left. + p\, J^{*,T}(D', y', \rho_z z - \mu_z, Q') \right] \} \\ J^{r,T}(D, y, z, Q) &= \max_{D'} \{u(Q - D + \tilde{L}(s, D')) + \beta\,\mathbb{E}_y \left[(1-q) J^{*,T}(D', y', \rho_z z, Q') \right. \\ &\qquad \left. + q\, J^{*,N}(D', y', \rho_z z + \mu_z, Q') \right] \} \\ J^{d,T}(y, z, Q) &= u((1-\lambda)Q) + \beta\,\mathbb{E}_y \left[(1-x) J^{d,T}(y', \rho_z z, Q') + x\, J^{r,N}(V(Q'), y', \rho_z z, Q') \right] \\ J^{d,N}(y, z, Q) &= u((1-\lambda)Q) + \beta\,\mathbb{E}_y \left[(1-x) J^{d,T}(y', \rho_z z - \mu_z, Q') \right. \\ &\qquad \left. + x\, J^{r,N}(V(Q'), y', \rho_z z - \mu_z, Q') \right] \\ J^{*,\theta}(D, y, z, Q) &= \max\{J^{r,\theta}(D, y, z, Q), J^{d,\theta}(y, z, Q)\} \end{aligned}$$

where $V(Q')$ is the recovery value after a default (as a function of GDP)

– *Taking as given the government's default policy function, the investor's policy function $\tilde{L}(s, D')$ satisfies the zero profit constraint:*

$$(1+r)\tilde{L}(s, D') = D' + \mathbb{E}_{y,\theta} \{\tilde{\delta}'(0, \theta', D', y', z', Q')(V(Q') - D')\}$$

where θ' and z' evolve according to the law of motion outlined above.

2.4.4 Calibration and simulation results

Benchmark calibration

Table 2.4 shows the benchmark calibration for the model. A quarterly frequency is chosen. Several parameters are set to the same value as in the canonical model of section 1.3.1: the risk aversion γ, the world riskless interest rate r, the loss of output in autarky λ, the probability of settlement after a default x, and the parameters of the B component of the growth process μ_g, σ_y, ρ_y. For the discount factor β, I use a substantially higher value than in the canonical model; the chosen value still amounts to a 20% time rate preference in annualized terms, but such a level is arguably plausible for an impatient and debt hungry country.

The other parameters are more difficult to calibrate, since they do not directly appear in other models of the literature. The closest analogs come from regime switching models à la Hamilton (1989). I discuss this literature below when analyzing the European responses to the crisis (section 2.5). But clearly one does not want to map business cycles into risks of default. The "trembling times" that are embedded in the model are certainly less frequent than mere recessions as they are associated with significant disruptions of economic activity. Bearing this in mind, the probability p of entering the "trembling times" is set to 6% in annualized terms. This is still above the unconditional historical probability of a sovereign default, since it is anticipated that in some cases the country will successfully go through the crisis without defaulting. The value of 6% is also below the probability of entering into a recession in regime switching based models.[9] The probability q of exogenously leaving the "trembling times" is calibrated so that, on average, being hit by a trembling shock leads to a default half of the time; in section 2.4.4 sensitivity analysis exercises regarding this critical parameter are performed. The P component of growth is calibrated so that, when the shock hits, the annualized growth rate goes down by 4 percentage points, and the effect of the shock is halved after 3 quarters. The debt recovery value after a default is calibrated so that the average haircut is 40%, as in the historical data (see section 1.1.2).

Table 2.4: Benchmark calibration of the "trembling times" model

Risk aversion	γ	2
Discount factor	β	0.95
World riskless interest rate	r	1%
Probability of settlement after default	x	10%
Loss of output in autarky (% of GDP)	λ	2%
Probability of entering "trembling times"	p	1.5%
Probability of exiting "trembling times"	q	5%
Recovery value (% of yearly GDP)	V	25%
Size of trembling shock in the P component of growth	μ_z	1%
Auto-correlation of the P component of growth	ρ_z	0.8
Standard deviation of the B component of growth	σ_y	3%
Auto-correlation of the B component of growth	ρ_y	0.17
Mean gross growth rate (ignoring the P component)	μ_g	1.006

Quarterly frequency.

The model is solved using the endogenous grid method described in chapter 5. Table 2.5 reports the moments of the model simulated with the benchmark calibration. Like most models in the quantitative sovereign debt literature, the present model is able to replicate some

9. For example, Altug and Bildirici (2010) estimate that the annualized probability of entering a low-growth state between 1982 and 2009 is 22% for Mexico and 45% for Argentina. Over a longer period these estimates would have probably been lower, since the 1980s and 1990s were particularly difficult times for these countries.

important stylized facts of the business cycle in emerging countries, such as a counter-cyclical current account, counter-cyclical spreads and a more volatile consumption than output. But unlike most models in the literature, it is able to sustain both a realistic frequency of defaults and a realistic indebtment level (close to 40% of *annual* GDP).

Table 2.5: Moments of the benchmark "trembling times" model

	Benchmark	With no Poisson ($p = 0$)
Rate of default (%, per year)	2.50	0.26
Mean debt output ratio (%, annualized)	38.17	46.82
$\sigma(Q)$ (%)	4.45	4.42
$\sigma(C)$ (%)	6.04	6.89
$\sigma(TB/Q)$ (%)	2.63	3.47
$\sigma(\Delta)$ (%)	0.57	0.18
$\rho(C, Q)$	0.92	0.89
$\rho(TB/Q, Q)$	−0.41	−0.49
$\rho(\Delta, Q)$	−0.60	−0.41
$\rho(\Delta, TB/Q)$	0.64	0.90

Parameters of the model are set to their benchmark values as in Table 2.4. The solution to the detrended model is computed using the endogenous grid method described in chapter 5. The policy functions are interpolated using a cubic spline on a 3-dimensional grid of 10 points for y, z and the debt-to-GDP ratio. Moments are obtained by averaging over 500 simulated series of 1,500 points each, the first 1,000 of which are discarded. Q is GDP, C is consumption, TB/Q is trade balance over GDP, Δ is the spread. GDP, consumption, trade balance and spread are detrended with an HP filter of parameter 1600.

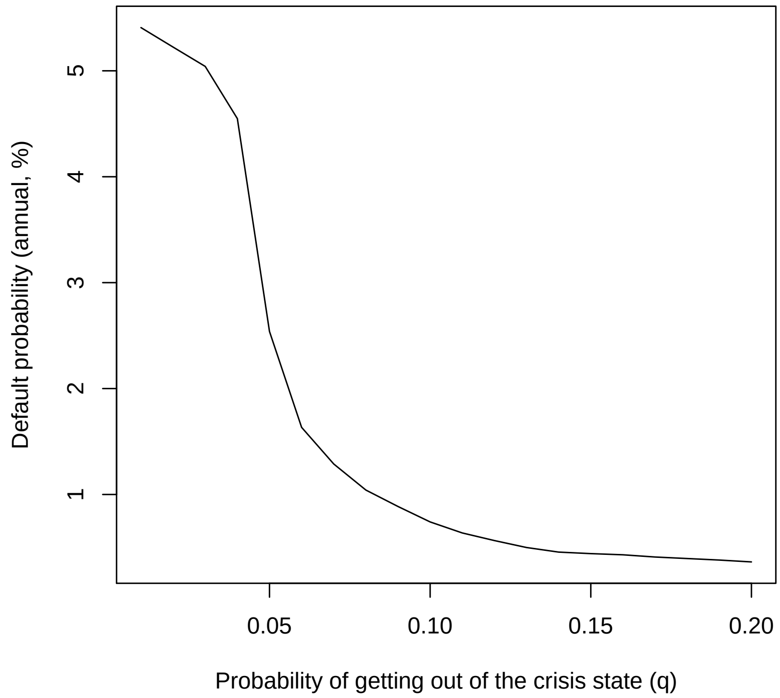
In the last column of this table I also report the moments of the model when all parameters are set to their benchmark values except for the probability p of a trembling shock which is set to zero. One can see that in this configuration defaults almost disappear. This shows the importance of the Poisson shock in this class of models. As another consequence, the volatility of spreads almost goes to zero since there is virtually no risk of default.

Sensitivity analysis

I first investigate the sensitivity of the results to the probability q of exiting the "trembling times." As Figure 2.2 shows, three ranges appear. When q is high, no default ever takes place: the trembling shock is expected to be short lived, the country will not destroy the recovery with a default. At the other extreme, when q is low, the shock "pre-pays for the default." Although the country would not do it on its own, the default now becomes the cheap option. In the intermediate case, the choice being made depends on when and how the shock occurs; when the economy is on a positive streak, default can be avoided; when instead the economy is already down, then default becomes more palatable.

Figure 2.3 shows the sensitivity of the mean debt-to-GDP ratio to the recovery value V. The mean debt-to-GDP ratio is an increasing function of the recovery value: this was to be expected since a higher recovery value means that default is more costly, and therefore a higher level of debt can be sustained by the sovereign country. This graph also plots the line

Figure 2.2: Default probability as a function of probability q of exiting the "trembling times"



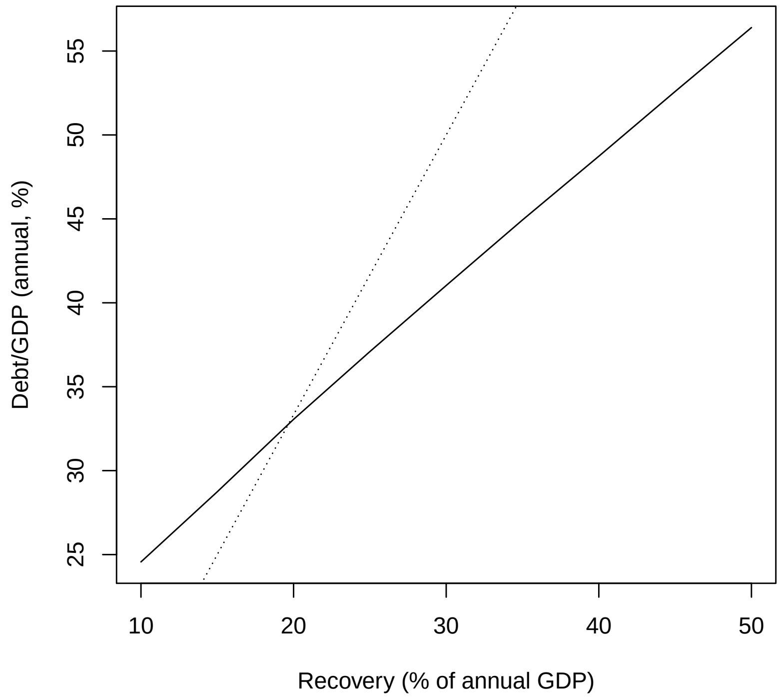
corresponding to a fixed 40% haircut: one can see that the recovery level consistent with this observed historical haircut is close to the 25% debt-to-GDP ratio that has been chosen for the benchmark calibration.

A self-fulfilling re-interpretation

When q is low enough, it is possible to reinterpret the model in the spirit of the self-fulfilling models overviewed in section 1.2.3. Taking into account the possibility of a self-fulfilling effect is important since, as I show in chapter 3, this effect plays a role in a significant minority of crises (around 10%). In the cases when q is low enough, the trembling shock always triggers a default: this is so because "the default is pre-paid" through the crisis. A self-fulfilling reinterpretation becomes possible, as outlined below.

Assume that markets fear a default anytime they see a sunspot. When markets starts anticipating a default, assume that they create a negative wave which is expected to be long lasting, corresponding to low values of q. The country then defaults with probability one. The shock is self-fulfilling. The difference with the model that is presented above is that, in the self-fulfilling re-interpretation, the shock is triggered because of the fear of a default rather than for reasons independent of the fear of default. But for low values of q the two are observationally equivalent.

Figure 2.3: Mean debt-to-GDP as a function of the recovery value V



The dotted line indicates the debt-to-GDP value corresponding to a 40% haircut.

2.5 Eurozone policies

2.5.1 Analysis at business cycle frequencies

As already mentioned, the modelling strategy used in the previous section for the output process is close to that of Hamilton (1989). Let us now see what the consequences would be of plugging the parameter values estimated by the Markov-switching literature into the model of section 2.4.

The original model of Hamilton (1989) estimated on US data for the period 1952–1984 gives $p = 9.5\%$ and $q = 24.5\%$. Later, Goodwin (1993) has estimated a similar model on 8 advanced economies from the late 1950s/early 1960s to the late 1990s, and came up with values for p ranging from 1% to 9%, and for q ranging from 21% to 49%.[10] Table 2.6 reports default probabilities and mean debt-to-GDP ratios obtained with the model of section 2.4 for values of p and q lying in the estimated range for business cycles of advanced countries.

As can be seen from this table, the most prominent fact exhibited by this exercise is that the risk of default at business cycle frequencies of advanced economies is negligible. The "trembling times" that are embedded in the model are therefore events which are less frequent and more severe downturns than are business cycles downturns; think of a banking crisis or a very severe recession. This is why I chose parameter values such that crises are less

10. I disregard the result for Italy, since the author labels it a pathological case and explains that a 3-state specification would probably better fit the data for that country.

Table 2.6: Model simulations using business cycle frequencies in advanced economies

Probability of entering "trembling times" (p, per quarter)	1%	1%	10%	10%
Probability of exiting "trembling times" (q, per quarter)	20%	50%	20%	50%
Rate of default (per year)	0.38%	0.27%	0.32%	0.29%
Mean debt output ratio (annualized)	45%	47%	43%	46%

The simulations are done using the model presented in section 2.4. The values tested for p and q correspond approximately to the extreme values estimated by Hamilton (1989) and Goodwin (1993) on 7 advanced economies over the postwar period. Other parameters of the model are set to their benchmark values as in Table 2.4.

frequent (lower p) and longer lasting (lower q) than mere recessions.

2.5.2 Credit ceilings

Following the Greek crisis, the eurozone imposed a new set of stringent rules to avoid future crises. The idea of policymakers is to impose a tougher credit ceiling on eurozone countries in order to protect the zone from any risk of default. The following questions are therefore of major importance from a policy perspective: how low should the debt ceilings be to avert any risk of crisis? What are the welfare implications of these constraints?

In order to address these questions, I have computed the levels of debt that are consistent with no default in either "normal" or "trembling" times in the model. Figure 2.4 summarizes the results as a function of the key parameter q. The solid line represents the mean debt-to-GDP ratio, the dotted line represents the maximum debt-to-GDP level under which there is no default in "normal times," and the dashed line represents the maximum debt-to-GDP level under which there is no default in "trembling times."

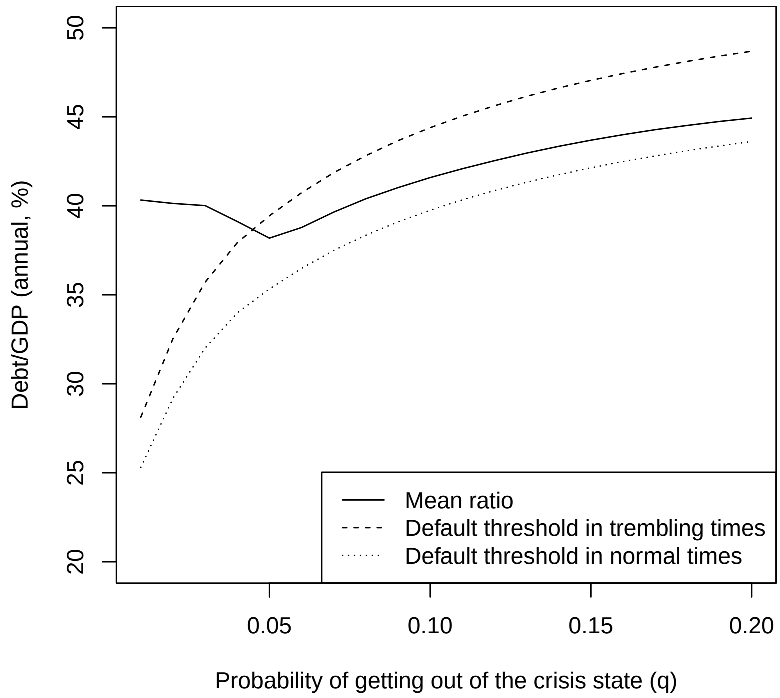
It is interesting to note that, for values of q less than 5%, the mean debt-to-GDP ratio is a decreasing function of q; the country becomes less prudent and is willing to take on more debt as the risk of default becomes higher because it knows that it will not repay its debt in bad states of nature. This is the "Panglossian effect" described in chapter 3.

One should also note that the proper way to introduce a relevant credit ceiling is to allow for two different levels: one pertaining to "normal times" and one pertaining to "trembling times." Imposing one only ceiling for both states of nature would be quite an inefficient way to avoid default. Extraordinary times call at extraordinary debt ceilings. This is a key distinction that tends to be lost in current policy debates.

Figure 2.4 shows, as expected, that debt ceilings needed to avoid default are an increasing function of the parameter q. For a short-lived crisis episode (q around 20%), countries take care of themselves as they do not default and are able to stabilize their debt. Thus, no debt ceiling is needed in this situation. This is in line with the business cycle properties that were examined in section 2.5.1.

However, in the range when q is around or below 5%, the risk of default rises and stringent debt ceilings are needed if default is to be avoided in all circumstances. In the cases when q

Figure 2.4: Mean debt-to-GDP and credit ceilings as a function of q



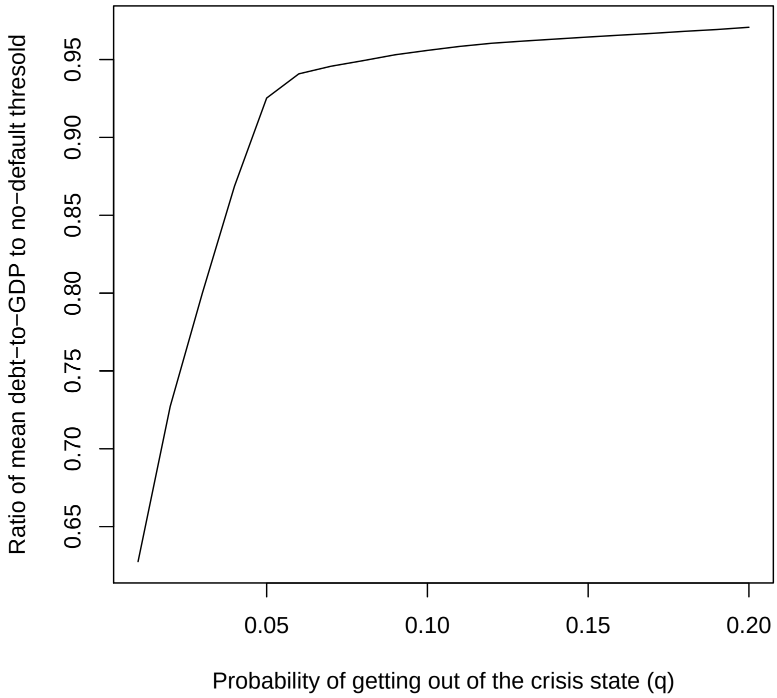
becomes very low, the constraint of a debt ceiling becomes quite strong; the ratio of the credit ceiling in normal times to the natural level of debt drops below two-thirds. For the critical value of 5%, the ratio is closer to one, slightly above 90%. These results are shown in Figure 2.5 which plots the ratio of mean debt-to-GDP to no-default threshold, as a function of q.

I also measure, in welfare terms, the impact of enforcing the credit ceiling. From a modelling perspective, I compute alternative solutions of the model where the credit ceiling is enforced,[11] and I compare the welfare obtained within the constrained model with the welfare obtained without constraints. Results are reported in Table 2.7.

As expected, the welfare cost is insignificant in the region of large q, and becomes quite significant for the low q zone. As a mean of comparison, Lucas (2003) estimates that the welfare cost of fluctuations in the US is about 0.1% of GDP (assuming the same value for risk aversion than the present calibration). In the median range for values of q (between 5% and 10%), the welfare cost of debt ceilings appears to be moderate and commensurate with Lucas's estimate for the cost of fluctuations. If avoiding a sovereign default is of systemic importance, this may be worth a try. In the lower end of q's values however, the cost is much higher (about 15 times Lucas's numbers) and it would be clearly inefficient to target a zero default equilibrium. In the benchmark calibration, which sets a 5% value for q, the constrained equilibrium remains reasonably close to the unconstrained one.

11. Technically I restrict the state space for the debt level by setting a lower bound equal to the ceiling that is to be imposed. The ceiling that is used corresponds to the lower line of Figure 2.3. As expected, the constrained models exhibit a zero probability of default.

Figure 2.5: Credit ceilings as a fraction of equilibrium levels in normal times



This graph shows the ratio of the credit ceiling to avoid default (in "normal times") to the mean debt-to-GDP ratio, as presented on Figure 2.4.

Table 2.7: Welfare cost of imposing credit ceilings

Probability of exiting "trembling times" (q, per quarter)	1%	5%	10%	20%
Unconstrained welfare	−18.273	−18.510	−18.524	−18.570
Constrained welfare	−18.573	−18.581	−18.578	−18.573
Cost of ceiling (as a permanent loss of GDP)	1.64%	0.39%	0.30%	0.02%

Welfare is computed in both models for a level of debt equal to the ceiling, at the mean productivity level ($y = \mu_y$, $z = 0$) and in "normal times" ($\theta = N$). The cost of imposing the ceiling as a percentage of GDP is computed using a Lucas (1987) type calculation; I first consider an economy with the same preferences but no fluctuations $\left(\textit{i.e.}\right.$ an economy where the welfare is $W = \frac{u((1-\lambda)\bar{Q})}{1-\beta}\Big)$, and then I report the value of λ that generates the same drop in welfare as the one observed between the unconstrained and constrained economies.

2.5.3 Further insights

When investigating the policy implications of the present model for the eurozone, the value of the parameter governing the magnitude of the trembling shock (μ_z) was kept at the same value as for the emerging country benchmark, *i.e.* at 1 percentage point of the GDP growth rate. Quantitatively, this means that following a trembling shock, the GDP level will be permanently lowered by 3.8% relative to the pre-shock trend.[12] This is a sizable shock, but not so big compared to what Greece is currently undergoing.[13] When it comes to the eurozone, it could therefore make sense to use greater values for the magnitude of the trembling shock and for the two other parameters governing the cost of default (the direct penalty on output λ and the recovery value V).[14] This would reflect the fact that in a monetary union with a highly integrated banking system, eurozone countries face a higher cost for default. For example, if these three parameters are increased by a factor of 1.5 (*i.e.* $\mu_z = 0.015$, $\lambda = 3\%$ and $V = 37.5\%$ of GDP), and with other parameters being held at the benchmark values given in Table 2.4, the mean debt-to-GDP ratio jumps to 58.8% (which is an increase by a factor of 1.5 compared to the benchmark case given in Table 2.5). Only the default frequency remains relatively unchanged at 2.4%. The model is therefore capable of delivering an arbitrarily high level of debt-to-GDP through a homothetic re-scaling of the three parameters μ_z, λ and V, while keeping the default probabilities at a constant level.

Another point worth mentioning is that in the framework under which the eurozone operates, up to 50% of public debt is held by foreigners. In fact the three countries which have been most vulnerable to the recent crisis (Greece, Portugal and Ireland) all had more than 70% of their public debt held by foreigners. A straightforward policy lesson to avoid default risk on sovereign issuers could be to make sure that sovereign debt is primarily held by domestic institutions or individuals, as is the case in Japan for instance. Of course, for a given path of current accounts, this would imply that other (private) debt would be held outside the country. This is irrelevant in the present model, since it doesn't distinguish between public and private external debt.[15] However, in a more thoughtful model where the distinction is introduced, this could have interesting policy implications.

12. If g_t is the growth rate without trembling shock, and $\tilde{g}_t$ is the growth rate following a trembling shock occurring at $t = 0$, then the long-term ratio of the two corresponding GDP levels is:

$$\frac{\prod_{t=0}^{+\infty} g_t}{\prod_{t=0}^{+\infty} \tilde{g}_t} = \prod_{t=0}^{+\infty} \left(1 + \frac{\rho_z^t \mu_z}{e^{\mu_y}}\right) \simeq 0.962$$

13. According to some estimates, Greece GDP level could be more than 10% below its pre-crisis trend.

14. Note that the size of the trembling shock μ_z influences the cost of default, because upon default the country loses the possibility of having its output restored to the pre-crisis level; since the size of the output restoration depends on μ_z as shown in (2.3), so does the cost of default. If one was to distinguish the size of the trembling shock in (2.2) from the size of the restoration in (2.3), then only the latter would have an impact on the default cost.

15. The rationale for not distinguishing between public and private debt is that private external loans generally come with an explicit or implicit public guarantee, as documented by Reinhart and Rogoff (2011b). This is confirmed by a positive correlation between sovereign default and access of the private sector to foreign credit, as documented by Arteta and Hale (2008). These considerations have led Mendoza and Yue (2012) to present a model where private and public external debt bear the same interest rate spread and the same credit risk.

2.6 Conclusion

I have analyzed a model in which countries usually do not like to default but rather are forced into it when the economy turns sour, arguing that this modelling choice better fits the historical reality. Based on this postulate, the key message of this chapter is simply that in order to avoid default, the critical parameter to analyze is the speed at which economies can move out of these "trembling times" (the parameter q in the model). Clearly, this is a lesson that European policymakers should understand, as the more protracted the economic crisis (and, hence, the perception that countries entering into "trembling times" will stay there for a while), the higher the risk of default. In the worst case scenario when a crisis is expected to be very long lasting, the debt ceiling needed to avoid default may become very low. Building institutions that avoid default risk should not only rely on debt ceilings, but also on mechanisms that limit the duration of the "trembling times." One key distinction between advanced and poor countries is the supposedly superior ability of advanced economies to recover from crisis (rather than sheer recessions), as documented by Hausmann et al. (2006). The mess created in Europe by the management of the sovereign crisis has certainly shifted the perception of the European ability to exit "trembling times," making the risk of a default much higher for all sovereigns within the eurozone. This is where the debate on the macro-management of the crisis would certainly need to be addressed. Reassuring investors of the policymakers' ability to address trembling episodes is perhaps more important than imposing credit ceilings that are too stringent.

2.7 Appendix: Extra results and proofs

2.7.1 General case

Proposition 2.6 (Eaton and Gersovitz (1981))**.** *Default incentives are stronger the higher the debt*

Proposition 2.7. *Default occurs if and only if the debt-to-GDP ratio is higher than a given threshold d^*.*

Proof. This is a straightforward implication of the isoelasticity of preferences and of the Markovian nature of the stochastic process driving output and recovery. □

2.7.2 Brownian or Poisson case

This section establishes a lemma valid for both the Brownian case (section 2.3.1) and the Poisson case (section 2.3.1).

Lemma 2.8. *For both the Brownian and the Poisson cases, the country never chooses an indebtment level such that default is sure tomorrow (i.e. a level for which $\tilde{L}'(s, \tilde{D}'(s)) = 0$).*

Proof. By contradiction.

Let's denote by g^+ the growth rate in the good case and g^- in the bad case,[16] and p is the probability of a being in the bad case.

Suppose that for some state s, the optimal choice is to repay and have $\tilde{L}'(s, \tilde{D}'(s)) = 0$. Since it means that default is sure tomorrow, one has:

$$d^* < \frac{\tilde{D}'(s)}{g^+ Q}$$

One also has:

$$J^r(s) = u(Q - D) + (1-p)e^{-\omega h} J^d(g^+ Q) + p\, e^{-\omega h} J^d(g^- Q)$$

Now let $D'_2 = d^* g^- Q$. It is clear that this level of indebtment is in the safe zone, so that $\tilde{L}(s, D'_2) = e^{-rh} D'_2$. If the country was choosing that level of indebtment, it would get:

$$J^r_2(s) = u(Q - D + e^{-rh} D'_2) + (1-p)e^{-\omega h} J^r(D'_2, g^+ Q) + p\, e^{-\omega h} J^r(D'_2, g^- Q)$$

But since $J^r(D'_2, g^+ Q) \geq J^d(g^+ Q)$ and $J^r(D'_2, g^- Q) \geq J^d(g^- Q)$ by construction of D'_2, one therefore has:

$$J^r_2(s) > J^r(s)$$

This is in contradiction with the optimality of $\tilde{D}'(s)$. □

2.7.3 Brownian case

In this section, let's denote by p the probability that output is low tomorrow, *i.e.* $p = \frac{1}{2} - \frac{\mu}{2\sigma}\sqrt{h}$.

In order to demonstrate proposition 2.2, I begin by establishing the following lemma:

Lemma 2.9. *In the Brownian case, if* $h \leq \frac{1}{\left(4\sigma + \frac{\mu}{\sigma}\right)^2}$*, the risky interest rate (*$r + p$ *in first order approximation) does not happen in equilibrium.*

Proof. By contraposition. Assume that for some state s, the optimal choice is to repay and $\tilde{L}(s, \tilde{D}'(s))$ is equal to $e^{-rh}(1-p)\tilde{D}'(s)$: this is the risky case where the country will repay next period in the good state of nature, but default in the bad state (the other two possible values for $\tilde{L}(s, \tilde{D}'(s))$ are $e^{-rh}\tilde{D}'(s)$ and 0, since output can take only two values). This implies that:

$$\frac{\tilde{D}'(s)}{e^{\sigma\sqrt{h}} Q} \leq d^* < \frac{\tilde{D}'(s)}{e^{-\sigma\sqrt{h}} Q}$$

16. In the Poisson case, g^- can actually be a random variable (see section 2.3.1). The demonstration still applies in this case, with the following modifications:

- in the definition of D'_2, replace g^- by the minimum value that growth can take in the case of a bad shock;
- add conditional expectation operators in the last terms defining $J^r(s)$ and $J^r_2(s)$.

One also has:

$$J^r(s) = u(Q - D + e^{-rh}(1-p)\tilde{D}'(s)) + (1-p)e^{-\omega h}J^r(\tilde{D}'(s), e^{\sigma\sqrt{h}}Q) + p\,e^{-\omega h}J^d(e^{-\sigma\sqrt{h}}Q)$$

Let $D'_2 = e^{-2\sigma\sqrt{h}}\tilde{D}'(s)$. One therefore has $\frac{D'_2}{e^{-\sigma\sqrt{h}}Q} \leq d^*$, which means that this level of indebtment is in the safe zone, and that $\tilde{L}(s, D'_2) = e^{-rh}D'_2$. If the country was choosing that level of indebtment, it would get:

$$J^r_2(s) = u(Q - D + e^{-rh}D'_2) + (1-p)e^{-\omega h}J^r(D'_2, e^{\sigma\sqrt{h}}Q) + p\,e^{-\omega h}J^r(D'_2, e^{-\sigma\sqrt{h}}Q)$$

By optimality of $\tilde{D}'(s)$, one has $J^r(s) > J^r_2(s)$. And since one has $J^r(D'_2, e^{\sigma\sqrt{h}}Q) > J^r(\tilde{D}'(s), e^{\sigma\sqrt{h}}Q)$ (by proposition 2.6) and $J^r(D'_2, e^{-\sigma\sqrt{h}}Q) > J^d(e^{-\sigma\sqrt{h}}Q)$ (by construction of D'_2), this implies that $u(Q - D + e^{-rh}(1-p)\tilde{D}'(s)) > u(Q - D + e^{-rh}D'_2)$. In turn, this implies that $(1-p) > e^{-2\sigma\sqrt{h}}$, or $\log(1-p) > -2\sigma\sqrt{h}$. Using the concavity of the logarithm, this implies $p < 2\sigma\sqrt{h}$, which is equivalent to $h > \frac{1}{\left(4\sigma + \frac{\mu}{\sigma}\right)^2}$. □

Intuitively, when h is small, then the variance of next period output conditionally to today's output is small. The country will prefer to borrow a little less, in order to be in the safe zone, since the effort to be done is small and tends towards zero, while the cost of a default remains high.

Proof of proposition 2.2. Assume that the country decides to repay. Given that next period output can take only two values, three cases are possible for tomorrow:

1. the country will repay in both states,
2. the country will repay in the good state of nature and default in the bad state,
3. the country will default in both states.

The second and third cases are excluded by lemmas 2.8 and 2.9.
By forward recursion, it is clear that the country will always repay in the future. □

Chapter 3

Endogenous debt crises

3.1 Introduction

International debt crises are (very) costly, as discussed in section 1.1.2. Why do we observe that so many countries fall into their trap? Should we not expect more prudent behavior from such countries? The theoretical answer in fact is: it depends. Take the simplest form of financial crisis driven by an exogenous shock. Spreads on sovereign bonds are high because the country is expected to be vulnerable to an earthquake or to a long-lasting commodity shock that is beyond its control. The country should then indeed behave with increased prudence: the greater the debt the country might have to repay, the heavier the cost of the earthquake relative to a favorable state of the nature. Yet, on the other hand, if the expected earthquake is so large that the country knows that it will actually default on its debt, then a "Panglossian attitude" (as Krugman has coined it) may become rational: the debt will lose all value after the earthquake, and it would then be absurd not to have borrowed more beforehand. The country behaves as if the risk of unfavorable shocks can be ignored. Following Dr. Pangloss, the character of Voltaire's book *Candide*, the country acts as if only "the best of all possible worlds" will occur. In this case, debt endogenously leads to debt; let's call this the *self-enforcing* case.

Let us now consider the case when crises are driven by the lack of confidence of financial markets towards a given country, making the country financially fragile through self-fulfilling behavior. Self-fulfilling debt crises have been analyzed in different forms, as seen in section 1.2.3.

In the model of Cole and Kehoe (1996, 2000), self-fulfilling crises are a variant of a liquidity crisis, by which a lack of coordination among creditors leads a solvent country to default. Such crises can be introduced in the canonical model of section 1.3.1 by allowing for strategic behaviors of the international investors, who will then make their lending decision conditional on the decision of other investors. As argued by Chamon (2007), however, such coordination crises can readily be avoided when lenders manage to offer contingent loans of the kind organized by venture capitalists. If any individual creditor offers a line of credit, conditionally on other creditors following suit, then liquidity crises can be easily avoided.

Self-fulfilling crises have also been analyzed as the perverse outcome of a snowball effect through which the buildup of debt becomes unmanageable, out of the endogenous fear that it can indeed become unmanageable (Calvo, 1988). Such crises can be introduced in the canonical model by changing the borrowing game so that the country announces the *amount that it wants to borrow today*, instead of the *amount to be repaid tomorrow*; such a variation is introduced in the model of section 3.2. Relying on an intuition developed in a simpler model in Cohen and Portes (2006), I then show that snowball spirals can only occur in cases where a debt crisis has the potential of damaging the fundamentals of the indebted country. If a crisis reduces the GDP of a country by say 10%, then it is clear that the lack of confidence towards a country can degenerate into a self-fulfilling crisis. If instead the fundamentals are not altered by the crisis, one can show that self-fulfilling crises of the Calvo type are (theoretically) impossible.

At the end of this argument, in this chapter I choose to focus on a simple characterization of a self-fulfilling debt crisis already given in sections 1.2.3 and 2.4.4: it is a crisis that is the outcome of an endogenous weakening of the country's fundamentals. Such crises can be introduced in the canonical model by adding an exogenous sunspot shock which has the potential of destroying output, along the lines of section 2.4.4. In the self-fulfilling case so defined, it is the crisis that reduces the GDP, originating from the various disruptions that a weakening of the confidence in a country may bring about (capital flight, exchange rate crisis...). In the "earthquake case," the sequence of causation is inverted: the fundamentals are first destroyed, then the crisis occurs.

From the theoretical model that is presented below, a simple typology of cases is obtained. Below a critical level of debt, a country tends to act prudently, aiming for instance to reduce its debt in response to a permanent adverse shock. Past a critical level of the debt-to-GDP ratio, which can be the outcome of a sequence of repeated unfavorable exogenous shocks, a country will begin to behave in the Panglossian mode, rationally ignoring the bad news, increasing the level of debt to its upper limit in a self-enforcing process. A crisis may then occur either because of the occurrence of another adverse exogenous shock or because of a self-fulfilling shock, *i.e.* one that endogenously weakens the ability of a country to service its debt. It should be noted that these three type of crises (exogenously driven, self-fulfilling, self-enforcing) are possible in the model presented in section 2.4: the typical crisis in that model is an exogenously driven one, but a self-enforcing crisis is nevertheless possible in some parameter ranges as noted in section 2.5.2; and the model can easily be given a self-fulfilling reinterpretation as explained in section 2.4.4.

The data is analyzed with this type of typology in mind. I use a slightly modified version of the database that has been compiled by Kraay and Nehru (2006), which is updated to cover all debt crises that have occurred until 2004. Following and adapting the work of these authors, it can be shown that the likelihood of a debt crisis is well explained by three factors: the debt-to-GDP ratio, the level of real income per capita, and a measure of overvaluation of the domestic currency.

In order to estimate the risk of a self-fulfilling debt crisis, the law of motion of the debt-to-GDP ratio in normal times is distinguished from the motion triggered by the onset of the crisis. I define a self-fulfilling crisis as one that would not have happened, had debt-to-GDP simply been driven along the pre-crisis path. Self-fulfilling crises, so defined, can be shown to correspond to a small minority of cases. On average, between 6% and 12% of crises (depending on the methodology) appear to be self-fulfilling. This proportion is clearly not negligible, however, and deserves to be taken seriously.

The strength of the Panglossian effect is also calibrated. Countries appear to have behaved as if the distribution of the risk was truncated, leading them to ignore risk. The influence of this mechanism on the debt buildup is tested through Monte-Carlo simulation. The Panglossian mechanism is shown to be substantial and representing about 12% of the cases (see Arellano (2008) for similar insights applied to the the case of Argentina).

This chapter proceeds as follows: section 3.2 presents an infinite-horizon model that is solved in section 3.3 and then used to analyze the logic of each crisis. Section 3.4 presents the dataset supporting the econometric analyses. In section 3.5 an econometric model is built and estimated in order to quantify the importance of both self-fulfilling and self-enforcing crises. Section 3.6 concludes.

3.2 A Panglossian theory of debt

In this section I develop a modeling framework which shares many features with the canonical sovereign debt model of section 1.3.1. The present model mainly differs in the way the stochastic process for output is specified: instead of assuming that it is totally exogenous, I suppose that there is a feedback effect of a default upon output (in addition to the conventional penalty imposed by creditors upon the defaulting country). The joint determination of default and output is a feature shared with the model of Mendoza and Yue (2012), who take into account the negative effect of high interest rate spreads on the domestic economy in a general equilibrium framework. In the present model, the effect of a default upon output is modeled in a rather *ad hoc* manner, since the goal is not to focus on a specific channel of transmission: beside the channel exhibited by Mendoza and Yue (2012), this feedback effect can also be understood as a proxy for other potential channels such as exchange rate crises, capital flights, political instability...

3.2.1 The economy

Let's consider a one-good exchange economy. The country is inhabited by a representative consumer who can tilt consumption away from autarky by borrowing or lending on the international financial markets.

Output produced at time t is a random variable Q_t, driven by a Markovian process. More precisely, the (gross) growth rate of output $g_t = \frac{Q_t}{Q_{t-1}}$ is assumed to be an *i.i.d.* variable, with a cumulative density function $\mathcal{F}(g)$. In other words, $\log Q_t$ is a random walk. For the sake of simplicity, the support of g is supposed to lie in an interval of the form $(0, g^{\max}]$.

The world financial markets are characterized by a constant riskless rate of interest r. Lenders are risk-neutral and subject to a zero-profit condition by competition. Debt is short-term and needs to be refinanced at every period.

In order to ensure that the wealth of the country is finite, the average growth rate $\mathbb{E}(g)$ is supposed less than the gross interest rate $1+r$.

At any time t, the country that has accumulated a debt D_t may decide to default upon it. When it does so, it is assumed that the country suffers forever after a negative productivity shock. One can say that default creates a panic that destroys capital either through an exchange-rate or a banking crisis. Post-default output can therefore be written:

$$Q_t^d = (1-\lambda)Q_t$$

in which $\lambda \in [0,1)$. As another cost of default, the country is subject to financial autarky, being unable to borrow again later on (a milder form of a sanction would be, more realistically, that the country is barred from the financial market for some time only; analytically, the outcome is formally equivalent).

Once the country has defaulted, creditors will attempt to recover some of their losses. In order to do so, they further reduce the resources of the country, in a way which, is assumed to be socially efficient: the fraction that they grab is simply subtracted, one for one, from the country's post-default output. Call Λ_t the fraction so reduced. I assume that Λ_t is itself an *i.i.d.* stochastic variable, in the domain $[0,1)$ and independent of g_t, which varies with the (legal) strength of the international financial community. Let's denote by $\mathcal{G}(\Lambda_t)$ the cumulative density function of Λ_t. Creditors therefore capture:

$$P_t = \Lambda_t Q_t^d = \Lambda_t(1-\lambda)Q_t,$$

while the country consequently consumes (given financial autarky):

$$C_t^d = (1-\Lambda_t)Q_t^d = (1-\Lambda_t)(1-\lambda)Q_t. \tag{3.1}$$

In the case when λ is equal to zero, the outcome may be characterized as an efficient restructuring of the debt, at least from a static point of view (I return to this issue below): creditors are able to capture a fraction of output, which is less than what they are owed, but without imposing a social cost to the economy. When instead, at the other extreme, Λ_t is nil or very low and $\lambda > 0$, then the implication is that default is socially costly and almost no fraction of output can be captured by the creditors.

3.2.2 Financial markets

The timing of events is as follows. First assume that the country has incurred a debt obligation D_t, falling due at time t, and has always serviced it in full in previous periods. At the beginning of period t, the country learns the value of its output Q_t and the fraction of post-default output Λ_t that it would lose were it to default. After observing these variables, the country decides to default or to reimburse its debt.

If the debt is reimbursed in full, the country can contract a new loan, borrowing L_t, which must be repaid at time $t+1$, in the amount of D_{t+1}. In order to avoid coordination problems, let's assume, following Chamon (2007), that creditors can commit on L_t and D_{t+1} before the decision to service the debt is known, conditionally on the decision to service the debt being made.

Such financial agreements being concluded, the country eventually consumes, in the event it services its debt in full:

$$C_t^r = Q_t + L_t - D_t$$

Alternatively, in the event of a debt crisis the country's consumption is nailed down to the expression given in (3.1).

Let's denote by $\mathscr{D}(D_{t+1}, Q_t)$ the default set, *i.e.* the set consisting of all realizations (g_{t+1}, Λ_{t+1}) for which the country will decide to default in $t+1$, conditionally on the level of tomorrow's debt D_{t+1} and today's output Q_t. Let's denote by $\mathscr{R}(D_{t+1}, Q_t)$ the repayment set, *i.e.* the complementary to the default set.

One can then define the risk of a debt crisis in $t+1$ as it is perceived from the perspective of date t:

$$\pi_{t+1|t} = \mathbb{P}(\mathscr{D}(D_{t+1}, Q_t)).$$

The zero-profit condition for creditors may be written as:

$$L_t(1+r) = D_{t+1}(1-\pi_{t+1|t}) + \int_{\mathscr{D}(D_{t+1}, Q_t)} V_{t+1}(g\, Q_t, \Lambda) \mathrm{d}\mathcal{F}(g) \mathrm{d}\mathcal{G}(\Lambda) \tag{3.2}$$

in which $V_{t+1}(Q_{t+1}, \Lambda_{t+1})$ is the discounted present value of all cash-flows that the creditors will be able to extract from the country, as they expect to receive forever after $t+1$ an amount $P_{t+1+T} = \Lambda_{t+1+T} Q^d_{t+1+T}$ in every period.

Finally, the usual no-Ponzi game condition is supposed to hold so that, at all periods t:

$$\lim_{T\to+\infty} \mathbb{E}_t \frac{D_{t+T}}{(1+r)^{t+T}} = 0.$$

3.2.3 Preferences

The decision to default or to stay current on the financial markets involves a comparison of two paths that implies expectations over the entire future. The country seeks to solve:

$$J^*(D_t, Q_t, \Lambda_t) = \max_{\{C_{t+T}\}_{T\geq 0}} \mathbb{E}_t \left\{ \sum_{T=0}^{\infty} \beta^T u(C_{t+T}) \right\}$$

where β is the discount factor, $C_t > 0$. Note that D_t can be negative if the country builds up foreign assets. The instantaneous utility function is isoelastic, of the form:

$$u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$$

where $\frac{1}{\gamma}$ is the inter-temporal elasticity of substitution.
Let's call:

$$J^d(Q_t, \Lambda_t) = \mathbb{E}_t \left\{ \sum_{T=0}^{\infty} \beta^T u((1-\Lambda_{t+T})Q^d_{t+T}) \right\}$$

the post-default level of utility, which becomes by definition independent of debt, and to which the country is nailed down in case of servicing difficulties. If it were to stay current on its debt obligation, it would obtain:

$$\begin{aligned} J^r(D_t, Q_t) = \max_{L_t, D_{t+1}} \Bigg\{ & u(Q_t - D_t + L_t) + \beta \int_{\mathscr{D}(D_{t+1}, Q_t)} J^d(g_{t+1}\, Q_t, \Lambda_{t+1}) \mathrm{d}\mathcal{F}(g_{t+1}) \mathrm{d}\mathcal{G}(\Lambda_{t+1}) \\ & + \beta \int_{\mathscr{R}(D_{t+1}, Q_t)} J^r(D_{t+1}, g_{t+1}\, Q_t) \mathrm{d}\mathcal{F}(g_{t+1}) \mathrm{d}\mathcal{G}(\Lambda_{t+1}) \Bigg\} \end{aligned}$$

subject to the zero-profit condition (3.2). Note that $J^r(D_t, Q_t)$ does not depend on the current value of Λ_t.

When comparing how much it can get by staying on the markets and the post-default level of welfare, the country picks up its optimum level:

$$J^*(D_t, Q_t, \Lambda_t) = \max \left\{ J^r(D_t, Q_t), J^d(Q_t, \Lambda_t) \right\}$$

Note that $J^*(D_t, Q_t, \Lambda_t)$ is a function of the current value Λ_t through the influence of J^d.

3.3 Recursive equilibrium

3.3.1 Definition and basic properties

Let's now enunciate the formal definition of a recursive equilibrium in this model. Such an equilibrium consists of a set of policy functions for the country and the investors. Agents act sequentially and the government cannot commit to its future actions.

I also make the assumption that in the process of negotiating debt contracts, the country first announces the amount L that it wants to borrow today, and the investors reply with the amount D' that they ask tomorrow for that loan. This is a significant change from the canonical sovereign debt model of section 1.3.1, where the negotiation process is inverted. The purpose of this change is to allow for multiple equilibria, as discussed in sections 1.2.3 and 3.3.2.

Definition 3.1 (Recursive equilibrium)**.** *A recursive equilibrium is defined by default and repayment sets $\mathscr{D}$ and $\mathscr{R}$ and value functions J^r, J^d, J^* for the country, a policy function $\tilde{D}'$ and a default value function V for the investors, such as:*

– *The value function J^d in case of default satisfies:*

$$J^d(Q,\Lambda) = u((1-\Lambda)(1-\lambda)Q) + \beta \int J^d(g' Q,\Lambda')\mathrm{d}\mathcal{F}(g')\mathrm{d}\mathcal{G}(\Lambda') \tag{3.3}$$

– *Given default and repayment sets $\mathscr{D}$ and $\mathscr{R}$ and the investors' policy function $\tilde{D}'$, the value function J^r in case of repayment satisfies:*

$$J^r(D,Q) = \max_{L\in\mathscr{L}(Q),\,L\geq D-Q} \Bigg\{ u(Q-D+L) + \beta \int_{\mathscr{D}(\tilde{D}'(L,Q),Q)} J^d(g' Q,\Lambda')\mathrm{d}\mathcal{F}(g')\mathrm{d}\mathcal{G}(\Lambda') + \beta \int_{\mathscr{R}(\tilde{D}'(L,Q),Q)} J^r(\tilde{D}'(L,Q),g' Q)\mathrm{d}\mathcal{F}(g')\mathrm{d}\mathcal{G}(\Lambda') \Bigg\} \tag{3.4}$$

where $\mathscr{L}(Q)$ characterizes the domain of definition of $\tilde{D}'$.

– *J^* is the maximum of J^r and J^d, and the default and repayment sets verify:*

$$(g',\Lambda') \in \mathscr{D}(D',Q) \;\Leftrightarrow\; (g',\Lambda') \notin \mathscr{R}(D',Q) \;\Leftrightarrow\; J^d(g' Q,\Lambda') > J^r(D',g' Q)$$

– *The value V that investors can extract in case of default satisfies:*

$$V(Q,\Lambda) = \Lambda(1-\lambda)Q + \frac{1}{1+r}\int V(g' Q,\Lambda')\mathrm{d}\mathcal{F}(g')\mathrm{d}\mathcal{G}(\Lambda') \tag{3.5}$$

– *Given the default and repayment sets $\mathscr{D}$ and $\mathscr{R}$, the policy function of investors satisfies the zero-profit condition for all $L \in \mathscr{L}(Q)$:*

$$L(1+r) = \tilde{D}'(L,Q)\mathbb{P}[\mathscr{R}(\tilde{D}'(L,Q),Q)] + \int_{\mathscr{D}(\tilde{D}'(L,Q),Q)} V(g' Q,\Lambda')\mathrm{d}\mathcal{F}(g')\mathrm{d}\mathcal{G}(\Lambda') \tag{3.6}$$

At this point, it is important to note that the present model is constructed in such a way that homogeneous equilibria, as defined below, are possible.

Definition 3.2 (Homogeneous recursive equilibrium)**.** *A recursive equilibrium is said* homogeneous *if it satisfies the following relationships for* $a > 0$*:*

$$\begin{aligned}
J^r(aD, aQ) &= a^{1-\gamma} J^r(D, Q) \\
J^d(aQ, \Lambda) &= a^{1-\gamma} J^d(Q, \Lambda) \\
J^*(aD, aQ, \Lambda) &= a^{1-\gamma} J^*(D, Q, \Lambda) \\
\tilde{D}'(aL, aQ) &= a\,\tilde{D}'(L, Q) \\
V(aQ, \Lambda) &= a\,V(Q, \Lambda) \\
\mathscr{L}(aQ) &= a\,\mathscr{L}(Q) \text{ (with obvious notation)}
\end{aligned}$$

The possibility of homogeneous recursive equilibria stems from three specific features of the model: the isoelasticity of the utility function, the specific form of the output process (*i.i.d.* in growth rates), and the proportionality of default costs.

It is theoretically possible that the model has recursive equilibria that are not homogeneous, but such equilibria are more of the nature of mathematical curiosities rather than economically relevant objects. In the following, I will therefore assume that there exists at least a homogeneous equilibria, which satisfies standard regularity conditions, and then establish several results that apply to these homogeneous recursive equilibria.

Lemma 3.3. *The following functions have a closed-form solution:*

$$\begin{aligned}
J^d(Q, \Lambda) &= \frac{u((1-\Lambda)(1-\lambda)Q)}{1 - \beta \mathbb{E}(g^{1-\gamma})} \\
V(Q, \Lambda) &= \frac{\Lambda(1-\lambda)Q}{1 - \frac{\mathbb{E}(g)}{1+r}}
\end{aligned}$$

Proof. Immediate using the homogeneity of the functions in equations (3.3) and (3.5). □

Lemma 3.4. *Default occurs if and only if debt-to-GDP ratio is higher than a given threshold* $d^*(\Lambda)$*,* i.e. *one has:*

$$(g', \Lambda') \in \mathscr{D}(D', Q) \;\Leftrightarrow\; \frac{D'}{g'Q} > d^*(\Lambda')$$

Proof. Immediate consequence of the homogeneity of value functions. □

Lemma 3.5. *Lenders will not lend today more than the present value of the wealth expected tomorrow,* i.e. *one has:*

$$\forall L \in \mathscr{L}(Q),\; (1+r)L \leq \frac{\mathbb{E}(g)Q}{1 - \frac{\mathbb{E}(g)}{1+r}}$$

Proof. This is the intuitive consequence of the fact that debt is repaid out of the country's GDP, that consumption must be positive and that Ponzi games are excluded. □

Corollary 3.6. *The country always defaults if its debt is higher than its wealth,* i.e. *one has:*

$$d^*(\Lambda) \leq \frac{1}{1 - \frac{\mathbb{E}(g)}{1+r}}$$

Proof. See the appendix 3.7.2. □

Lemma 3.7. *The country does not default if it has access to a contract which gives him a higher* current *consumption level than in case of default. Formally, given D, Q and Λ, if there exists $L \in \mathscr{L}(Q)$ such that $L - D \geq -\Lambda(1-\lambda)Q - \lambda Q$, then default is not optimal,* i.e. $J^r(D,Q) \geq J^d(Q,\Lambda)$.

Proof. See the appendix 3.7.2. □

This leads to a lower bound on the default threshold:

Proposition 3.8. *The country never defaults if debt is lower than what the investors can extract in case of default plus the one-period loss of output due to the negative productivity shock,* i.e. *one has:*

$$d^*(\Lambda) \geq V(1,\Lambda) + \lambda = \frac{\Lambda(1-\lambda)}{1 - \frac{\mathbb{E}(g)}{1+r}} + \lambda$$

Proof. See the appendix 3.7.2. □

Finally here is a definition which will be useful for characterizing the case of multiple equilibria:

Definition 3.9 (Smooth default)**.** *Let's call the* smooth default *case the situation where the default threshold is equal to what the investors can extract in case of default,* i.e. *when $d^*(\Lambda) = V(1,\Lambda)$.*

As is clear from proposition 3.8, a smooth default is only possible when $\lambda = 0$, *i.e.* when a default leads to an efficient restructuring of the debt from a static point of view (there is still an inefficiency related to the loss of access to financial markets). The reciprocal is not true: it is possible to have statically efficient defaults which are not smooth. Think of a country with a low rate of time preference (lower than the riskless interest rate), and with a linear utility function. It is easy to show that, in that case, the country will be willing to repay a debt higher than what investors would extract in case of default (simply because the country has a lower discount rate than investors).

3.3.2 The risk of multiple equilibria

In a standard setup, the interest rate charged by investors is entirely determined by the probability of default, via the risk premium: the higher the risk, the higher the interest rate.

But the reverse causality can very well be also at work. One may have situations where two equilibria are possible: a "good equilibrium" where the investors ask for a low interest rate, leading to a low debt-to-GDP tomorrow and therefore a low risk of default (consistent with the low interest rate), and a "bad equilibrium" where investors ask for a high interest rate, consistently leading to a high level of risk.

This kind of multiple equilibria in the interest rate, also called the "snowball effect," have been studied by Calvo (1988).

Note that multiple equilibria in the interest rate are possible in the present model because the country announces L and the investors reply with some D' which satisfies the zero-profit condition; as noted by Chamon (2007, footnote 7), such multiple equilibria are impossible in the reverse setup, where the country announces D' and the investors reply with the corresponding L (as in the canonical sovereign debt model of section 1.3.1).

Formally, a multiple equilibrium in the interest rate is a situation where, for a given L_t, there are two values $D^1_{t+1} < D^2_{t+1}$ verifying the zero-profit condition, and such that $\mathscr{D}(D^1_{t+1}, Q_t) \subset \mathscr{D}(D^2_{t+1}, Q_t)$.

Proposition 3.10. *Multiple equilibria in the interest rate are impossible in the smooth default case.*

Proof. See the appendix 3.7.2. □

This result is the generalization of the result obtained by Cohen and Portes (2006) in a simpler model, who show that multiple equilibria are ruled out when default is statically efficient. Their intuition is simple: for a given set of fundamentals there can only be one equilibrium, in the simplest settings at least. What drives the multiple equilibrium case is the fact that the crisis endogenously destroys part of the fundamentals upon which the debt is repaid (since after default, in the previous case, creditors receive nothing). This may be the key reason why corporate self-fulfilling debt crises are a curiosity. To the extent that an appropriate bankruptcy procedure exists, the risk that a financial crisis can—out of its own making—endanger the value of a firm is much reduced.

3.3.3 Dynamics for non-defaulters

Let's now derive the Euler equation of non defaulters. Let's call

$$\Omega(D, Q) = -\frac{\partial J^r}{\partial D}(D, Q)$$

the marginal utility of one additional unit of net foreign assets.
Using the envelope theorem in equation (3.4), one has:

$$\Omega(D, Q) = u'(Q - D + L^*)$$

where L^* is the optimal level of borrowing in case of repayment. The first order condition of the maximization in (3.4) leads to:

$$u'(Q - D + L^*) = \beta \frac{\partial \tilde{D}'}{\partial L}(L^*, Q) \int_{\mathscr{R}(\tilde{D}'(L^*,Q),Q)} \Omega(\tilde{D}'(L^*, Q), g'\, Q) \mathrm{d}\mathcal{F}(g') \mathrm{d}\mathcal{G}(\Lambda') \tag{3.7}$$

(using the fact that J^r and J^d are equal at the default threshold).

The derivative of the investors' decision rule $\tilde{D}'$ can be obtained from equation (3.6), using the implicit function theorem: [1]

$$\frac{\partial \tilde{D}'(L,Q)}{\partial L} = \frac{1+r}{\mathbb{P}[\mathscr{R}(\tilde{D}'(L,Q),Q)] - \xi(\tilde{D}'(L,Q),Q)} \tag{3.8}$$

where:

$$\xi(D',Q) = \frac{D'}{Q} \int \frac{d^*(\Lambda') - V(1,\Lambda')}{d^*(\Lambda')^2} \mathrm{d}\mathcal{G}(\Lambda')$$

Note that $\xi(D',Q) \geq 0$ because $d^*(\Lambda') \geq V(1,\Lambda')$ (direct consequence of proposition 3.8). Also note that $\xi(D',Q) = 0$ in the smooth default case.

In equation (3.8), the term $\mathbb{P}[\mathscr{R}(\tilde{D}'(L,Q),Q)] - \xi(\tilde{D}'(L,Q),Q)$ is the marginal price of debt, in the sense of Bulow and Rogoff (1988): it is the total value to creditors of having the face value of the country's debt raised by one dollar. In this case, it is equal to the probability of repayment, minus an extra loss incurred by the investors when the default is not smooth.

In order to get the intuition behind this result, let's temporarily assume that Λ is a constant. With obvious notations, the zero-profit condition for creditors may be written as:

$$L_t(1+r) = D_{t+1}(1-\pi_{t+1|t}) + \int_0^{g^*_{t+1}} V(Q_{t+1}) d\mathcal{F}(g_{t+1})$$

in which $Q_{t+1} = g_{t+1}Q_t$, $g^*_{t+1} = \frac{D_{t+1}}{d^* Q_t}$ is the minimal growth rate to ensure repayment tomorrow, and $V(Q_{t+1})$ is the discounted present value of all cash-flows that the banks will be able to extract from the country, when they expect to receive forever an amount $P_{t+T} = \Lambda Q^d_{t+T}$ for every $T \geq 1$.

One can then write:

$$(1+r)\frac{\partial L_t}{\partial D_{t+1}} = (1-\pi_{t+1|t}) - \frac{\partial \pi_{t+1|t}}{\partial D_{t+1}} D_{t+1} + \frac{V(g^*_{t+1}Q_t)}{d^* Q_t} = 1 - \pi_{t+1|t} - \frac{D_{t+1} - V(g^*_{t+1}Q_t)}{d^* Q_t}$$

i.e.

$$\frac{\partial L_t}{\partial D_{t+1}} = \frac{1}{1+r}(1 - \pi_{t+1|t} - \xi_{t+1})$$

which corresponds to the more general solution obtained above. In the smooth repayment case ($\xi = 0$), this simply means that the marginal price of debt is equal to the probability of default.

Returning to the general case of a stochastic Λ, one can rewrite the Euler equation (3.7)

1. See the proof of proposition 3.10 in appendix 3.7.2 for some elements of the computation.

as:

$$\Omega(D,Q) = \frac{\beta(1+r)}{\mathbb{P}[\mathscr{R}(\tilde{D}'(L,Q),Q)] - \xi(\tilde{D}'(L,Q),Q)} \int_{\mathscr{R}(\tilde{D}'(L^*,Q),Q)} \Omega(\tilde{D}'(L^*,Q),g'\,Q)\mathrm{d}\mathcal{F}(g')\mathrm{d}\mathcal{G}(\Lambda')$$

Along an equilibrium path, this means that one has (switching back to the notation using time subscripts):

$$\Omega_t = \beta(1+r)\left(\frac{1-\pi_{t+1|t}}{1-\pi_{t+1|t}-\xi_{t+1|t}}\right)\mathbb{E}_t\left[\Omega_{t+1}\,|\mathscr{R}(D_{t+1},Q_t)\right] \tag{3.9}$$

where $\Omega_t = u'(C_t)$, $\xi_{t+1|t} = \xi(D_{t+1},Q_t)$, $\pi_{t+1|t}$ is the probability of default in $t+1$ from the perspective of date t, and the term $\mathbb{E}_t\left[\Omega_{t+1}\,|\mathscr{R}(D_{t+1},Q_t)\right]$ stands for the expectation of Ω_{t+1}, from the perspective of date t, *conditionally on the decision to repay at date* $t+1$.

This equation reveals the core of the Panglossian theory. First consider the smooth default case where $\xi_{t+1|t} = 0$. In that case, equation (3.9) boils down to:

$$\Omega_t = \beta(1+r)\mathbb{E}_t\left[\Omega_{t+1}\,|\mathscr{R}(D_{t+1},Q_t)\right]$$

When its decides its level of indebtment, the country only takes into account the consequences of its decision for the subset of events where growth is high and makes default non-optimal. It then rationally ignores risk: this is the Panglossian effect.

In the general case where $\xi_{t+1|t}$ is positive, the Panglossian motive is reduced. Taking a linear approximation of the term $\frac{1-\pi_{t+1|t}}{1-\pi_{t+1|t}-\xi_{t+1|t}}$, one can rewrite (3.9) as:

$$\Omega_t = \beta(1+r)(1+\xi_{t+1|t})\mathbb{E}_t\left[\Omega_{t+1}\,|\mathscr{R}(D_{t+1},Q_t)\right] \tag{3.10}$$

It is indeed evident that the term $\xi_{t+1|t}$ tends to raise the marginal utility of consumption at time t and consequently reduces the propensity to borrow. The intuition is straightforward: to the extent that default entails a social loss, the benefit of borrowing against future risk is reduced, decreasing the desirability of debt in consequence.

It should here be noted that the Panglossian or self-enforcing effect is not specific to the present model. It is actually present in most sovereign debt model, and in particular in the canonical model of section 1.3.1. This is apparent from the Euler equation (5.9) of that model (page 120), where one can see that the expectancy over tomorrow is only computed for states in which the country repays.

3.3.4 A linear approximation

In this section I analyze a first-order linear approximation of the model presented so far. Let us note: $\Omega_t = -\frac{\partial J^r}{\partial D}(D_t,Q_t) = a_0 + a_1\,Q_t - a_2\,D_t$

One may then write the Euler equation (3.10) as:

$$a_0 + a_1\, Q_t - a_2\, D_t = \frac{1}{a_3}\left[a_0 + a_1\, Q^+_{t+1|t} - a_2\, D^*_{t+1}\right]$$

in which $Q^+_{t+1|t} = \mathbb{E}_t\left[Q_{t+1}\left|\mathscr{R}(D^*_{t+1}, Q_t)\right.\right]$ is the expected output conditional to repayment, $a_3 = (\beta(1+r)(1+\xi))^{-1}$ (neglecting here the variability of the factor ξ), and D^*_{t+1} is the corresponding first best decision regarding debt. Let's denote:

$$\Xi_{t+1|t} = Q^+_{t+1|t} - \mathbb{E}_t Q_{t+1} = \pi_{t+1|t}(Q^+_{t+1|t} - Q^-_{t+1|t}),$$

in which $Q^-_{t+1|t} = \mathbb{E}_t\left[Q_{t+1}\left|\mathscr{D}(D^*_{t+1}, Q_t)\right.\right]$. The Euler equation can then be written as:

$$a_0 + a_1\, Q_t - a_2\, D_t = \frac{1}{a_3}\left[a_0 + a_1\, \mathbb{E}_t Q_{t+1} - a_2\, D^*_{t+1}\right] + \frac{a_1}{a_3}\, \Xi_{t+1|t}$$

or again as:

$$D^*_{t+1} = a_4 + a_3\, D_t + a_5\, \Xi_{t+1|t} + a_5\left[\mathbb{E}_t Q_{t+1} - a_3\, Q_t\right]$$

where $a_4 = \frac{a_0}{a_2}(1-a_3)$ and $a_5 = \frac{a_1}{a_2}$.

The term $\mathbb{E}_t Q_{t+1} - a_3\, Q_t$ may be interpreted as a business-cycle component of the debt buildup. When output is low compared to the expected mean of next period's output, it borrows in order to smooth out consumption. The term is neglected in the Markovian model presented above, and show not much empirical relevance below, so we ignore it from now on.

The term $a_5\, \Xi_{t+1|t}$ is the Panglossian term, which measures the way creditors truncate their forecasting set.

For practical matters, I shall also assume that the level of debt is not carefully derived from this first order equation. As shown by Campos et al. (2006), there is a lot of extrinsic noise in the level of debt, due to either unforeseen contingencies debt, or unpredicted valuation effects. In other words, I simply assume that D^*_{t+1} differs from actual debt by a noisy term, and write:

$$D_{t+1} = D^*_{t+1} + \varepsilon^d_{t+1} Q_{t+1}$$

where ε^d_{t+1} is an *i.i.d.* shock.

Let us then write:

$$\frac{Q_{t+1}}{Q_t} = 1 + g_{t+1}$$

the growth rate of the economy

With an obvious change of notation, one can redefine the Panglossian effect as:

$$\frac{\Xi_{t+1|t}}{Q_t} = \pi_{t+1|t}(g^+_{t+1|t} - g^-_{t+1|t}). \tag{3.11}$$

One can then finally write:

$$d_{t+1} = a_4 + a_3\, d_t - g_{t+1}\, d_t + a_5\, \pi_{t+1|t}\, (g^+_{t+1|t} - g^-_{t+1|t}) + \varepsilon^d_{t+1} \tag{3.12}$$

Note importantly that the growth rate itself will be negatively affected by the occurrence of the crisis, which is the essence of the risk of self-fulfilling equilibria. It is such an equation that is now applied to the data.

3.4 Dataset

The empirical strategy relies on a dataset of "debt distress" and "normal times" episodes, following the methodology of Kraay and Nehru (2006).

More precisely, for a given year, a country is considered to be in debt crisis if at least one of the following three conditions holds:

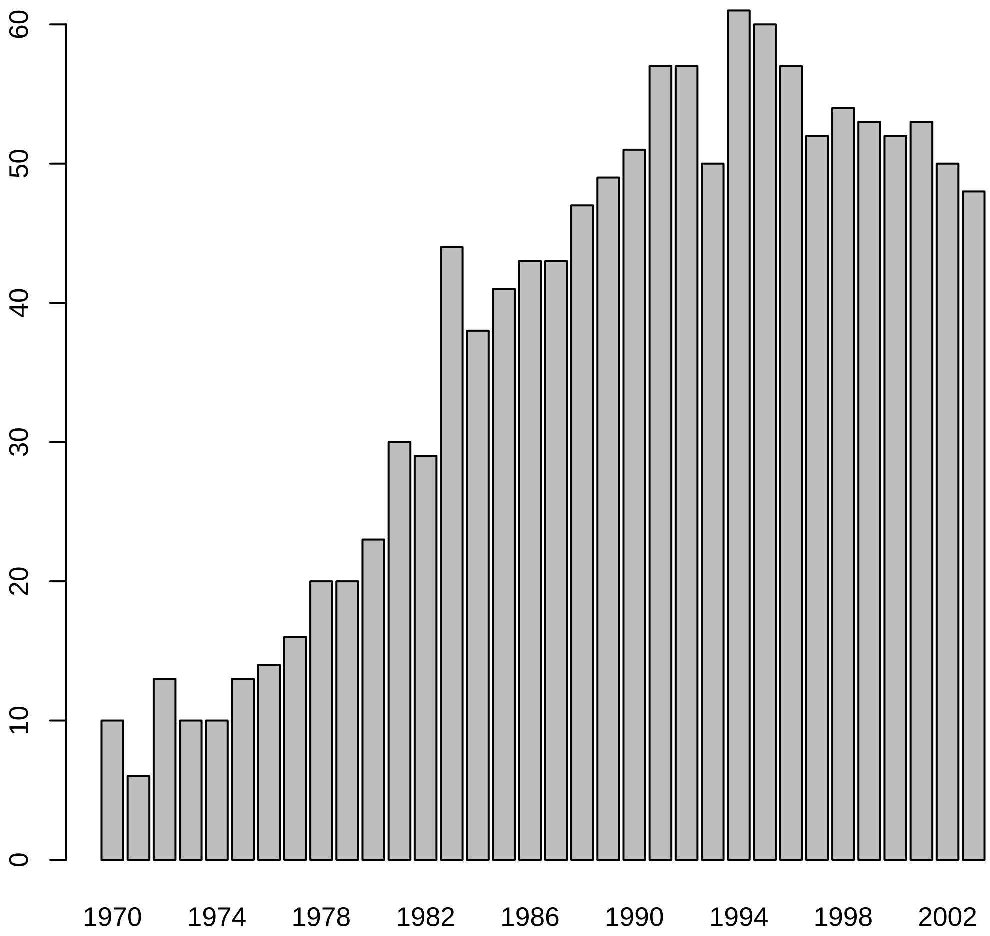
1. The country receives debt relief from the Paris Club in the form of a rescheduling and/or a debt reduction.
2. The sum of its principal and interest in arrears is large relative to the outstanding debt stock.
3. The country receives substantial balance of payments support from the IMF through a non-concessional Standby Arrangement (SBA) or an Extended Fund Facility (EFF).

For the last two conditions, I choose the same thresholds as do Kraay and Nehru (2006): a country is considered to be in crisis if its arrears are above 5% of the total stock of its outstanding debt, or if the total amount agreed to under SBA/EFF arrangements is above 50% of the country's IMF quota. Moreover, a country receiving Paris Club relief for a given year is also considered to be in crisis for the following two years since the relief decision is typically based on three-year balance of payments projections by the IMF.[2]

Figure 3.1 plots the number of countries in crisis for each year according to this definition. It is interesting to note that the time pattern is a steady increase from 1970 to the mid-1990s, then a steady decrease: this is very similar to the pattern of debt levels shown in Figure 1.1, suggesting that high debt levels are the main cause of defaults. Also, following the discussion in section 1.1.1, this presentation of the time profile of debt crises tends to reject the hypothesis that crises come in clusters and therefore confirms the conclusions of Cohen and Valadier (2011).

2. The following data sources are used for creating the dataset:
- the World Bank's *Global Development Finance* for data on debt levels and payment arrears (World Bank, 2006a);
- the Paris Club website for information on debt reliefs (`http://www.clubdeparis.org`);
- the IMF's *International Financial Statistics* for data on SBA/EFF commitments (International Monetary Fund, 2006);
- the World Bank's *World Development Indicators* for general macroeconomic variables (World Bank, 2006b);
- the *Penn World Table* for data on Purchasing Power Parity (PPP) variables (Heston et al., 2006).

Figure 3.1: Number of countries in crisis for a given year (1970–2004)



Computed by the author using a methodology similar to Kraay and Nehru (2006).
Covers all developing countries with market access.

Having defined when a country is considered to be in crisis or not, I then define "debt distress" episodes as periods of at least three consecutive years of crisis. Moreover, I impose the restriction that a distress episode should be preceded by at least three years without crisis, so that macroeconomic variables before a crisis episode can be considered as exogenous to the crisis.

Similarly, "normal times" episodes are defined as five consecutive years without any crisis (imposing no other restriction).

The set of countries over which are made the computations consists of the 135 developing countries defined by the World Bank, from which were removed the 38 countries that have absolutely no access to private financial markets.[3] I choose to remove them since their situation of indebtedness is somewhat different from that of the rest of the developing world (in particular, they have a much higher proportion of concessional lending). From the standpoint of the model, they probably fall into the category of countries that have no access to risky markets, and their debt dynamics must consequently be different.

The final sample therefore contains 97 countries. From the time angle, the data cover the period 1970–2004.

Prior to the elimination of certain observations in the econometric estimations (due to missing data), the sample of episodes consists of 70 distress episodes, and 223 normal times episodes. The median length of a crisis episode is 7.5 years, the mean is 11.2 years, and the standard deviation is 8.5 years: crises take a long time to settle on average, with a significant variance across countries. Figure 3.2 shows for each year the number of crisis episodes that start at that date: on this graph, there is a obvious peak in the 1980s; but there is no clear pattern for the 1990s and the 2000s, except that these years were less crisis prone than the 1970s and 1980s. The complete list of distress episodes can be found in appendix 3.7.1.

To summarize, the differences between the present dataset and that of Kraay and Nehru (2006) are twofold: first, their data is updated up to 2004, which is relatively minor but allows to include the Ecuadorian debt crisis of 2000 for instance; second, my analysis is restricted to the emerging countries that have access to private credit markets.

The interested reader can refer to Cohen and Valadier (2011) for additional insights on a very close dataset and for various descriptive statistics and econometric results extracted from it.

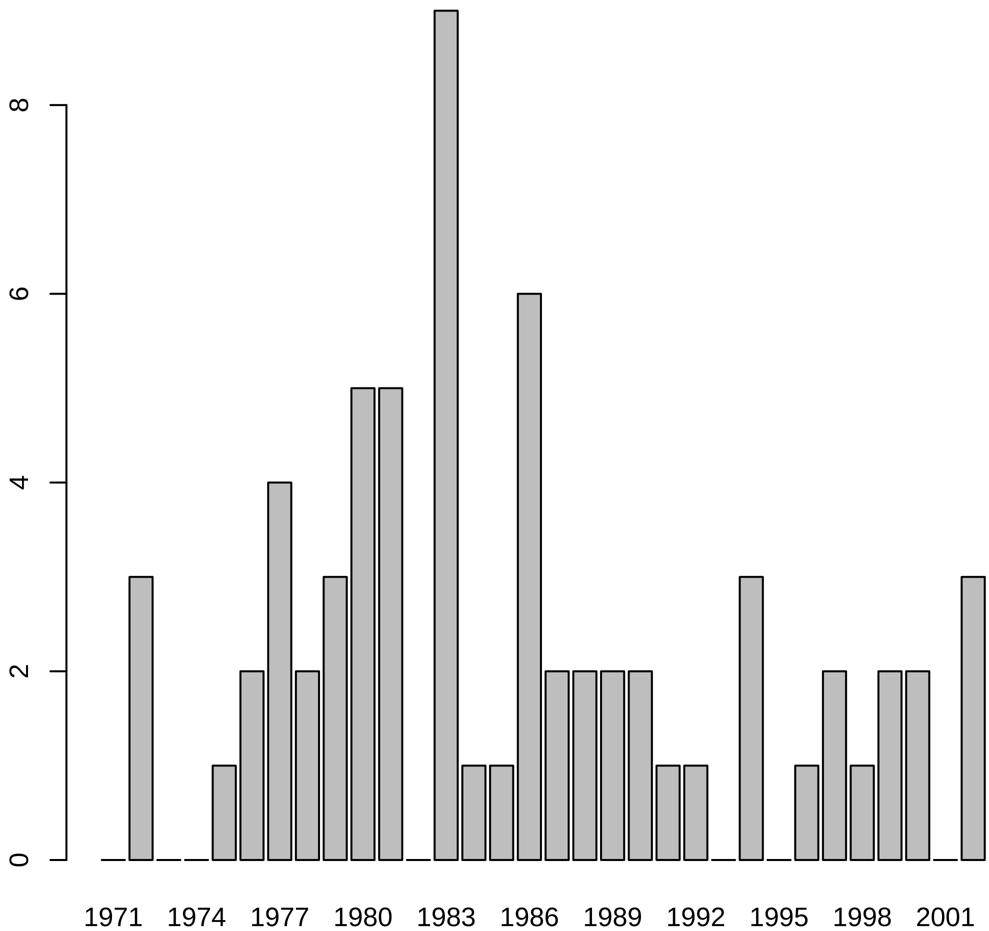
3.5 The econometric model

3.5.1 The estimated equations

The empirical framework is given by the following system of three simultaneous equations. Since these three equations exhibit a circular dependency, there is an identification

3. Market access is defined as in Gelos et al. (2004, 2011). The countries that were removed are those that never accessed international credit markets between 1980 and 2000, in accordance with the authors' definition. The complete country list of those countries can be found on page 29 of Gelos et al. (2004).

Figure 3.2: Number of crisis episodes, by starting year (1971–2004)



Computed by the author using a methodology similar to Kraay and Nehru (2006).
Covers all developing countries with market access.

issue, which is dealt with in the following section.

$$d_{it} = X^{d}_{i,t-1}\,\eta^{d} + g_{it}\,X^{d,g}_{i,t-1}\,\eta^{d,g} + \varepsilon^{d}_{it} \tag{3.13}$$

$$g_{it} = X^{g}_{i,t-1}\,\eta^{g} + \delta_{it}\,X^{g,\delta}_{i,t-1}\,\eta^{g,\delta} + \varepsilon^{g}_{it} \tag{3.14}$$

$$\delta_{it} = \mathbb{1}_{\{X^{\delta}_{i,t-1}\,\eta^{\delta} + d_{it}\,X^{\delta,d}_{i,t-1}\,\eta^{\delta,d} + \varepsilon^{\delta}_{it} > 0\}} \tag{3.15}$$

where i indexes countries, t indexes time, d_{it} is the debt-to-GDP ratio, g_{it} is the percentage year-on-year growth rate of nominal US$ GDP, δ_{it} is a dummy indicating a debt crisis; the various components of $X_{i,t-1} = (X^{d}_{i,t-1}, X^{d,g}_{i,t-1}, X^{g}_{i,t-1}, X^{g,\delta}_{i,t-1}, X^{\delta}_{i,t-1}, X^{\delta,d}_{i,t-1})$ are row-vectors of exogenous variables and the various components of $\eta = (\eta^{d}, \eta^{d,g}, \eta^{g}, \eta^{g,\delta}, \eta^{\delta}, \eta^{\delta,d})$ are column-vectors of parameters of corresponding sizes; ε^{d}_{it}, ε^{g}_{it}, and $\varepsilon^{\delta}_{it}$ are stochastic exogenous shocks.

Equation (3.13) reflects the theoretical debt dynamics equation (3.12), and the shock ε^{d}_{it} is therefore interpreted as a deviation from the Euler equation, for the reasons explained in section 3.3.4. In the growth equation (3.14), the shock ε^{g}_{it} is the driver of the country's growth exogenous uncertainty. Depending on the occurrence of a debt crisis, growth can be endogenously reduced, as captured by the incidence of the δ_{it} variable on growth. Finally, in the debt crisis equation (3.15), the shock $\varepsilon^{\delta}_{it}$ corresponds to the variability of the threshold level of debt default; it is the empirical counterpart of the shock on Λ_t in the theoretical model, recalling that this latter variable has an impact on the default threshold as can be seen from proposition 3.8.

The following normal distribution is assumed for these shocks (which in addition are supposed to be independent and identically distributed over periods and countries):

$$\begin{pmatrix} \varepsilon^{d}_{it} \\ \varepsilon^{g}_{it} \\ \varepsilon^{\delta}_{it} \end{pmatrix} \rightsquigarrow \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_d & 0 & 0 \\ 0 & \sigma^2_g & 0 \\ 0 & 0 & 1 \end{pmatrix} \right)$$

Since equation (3.15) which defines the crisis dummy is essentially a probit, identifiability is guaranteed by setting the variance of $\varepsilon^{\delta}_{it}$ to unity.

3.5.2 Identification and multiple equilibria

Since there is a circular dependency between the three endogenous variables, the econometric model can not be identified at this stage. Indeed, for a given set of exogenous $X_{i,t-1}$ and for a given draw of the random variables ε^{d}_{it}, ε^{g}_{it} and $\varepsilon^{\delta}_{it}$, the model does not rule out the possibility of having two vectors $(d_{it}, g_{it}, \delta_{it})$ satisfying equations (3.13), (3.14) and (3.15): of these two vectors, one would be a no-crisis scenario ($\delta_{it} = 0$), and the other a crisis scenario ($\delta_{it} = 1$). This feature is precisely the possibility of multiple equilibria that I am trying to modelize.

In order to address this identification issue, two extensions are made to the model: first, restrictions stemming from economic theory are imposed on the parameters, which eliminate multiple equilibria for some values of the exogenous variables; and for the remaining cases where multiple equilibria are possible, a stochastic variable (with only two possible values) is introduced, which determines which equilibrium to choose: it is a *sunspot* variable, as it is sometimes called in the literature, *i.e.* a variable with no relation to economic fundamentals but which makes agents coordinate on one equilibrium when several are possible.

Let g^0_{it} and d^0_{it} be the growth and the debt-to-GDP ratio conditional to no crisis occurring ($\delta_{it} = 0$). Conversely, let g^1_{it} and d^1_{it} be the growth and the debt-to-GDP ratio conditional to a crisis occurring ($\delta_{it} = 1$).

One can easily see that:

$$g^0_{it} = X^g_{i,t-1}\eta^g + \varepsilon^g_{it}$$

$$g^1_{it} = X^g_{i,t-1}\eta^g + X^{g,\delta}_{i,t-1}\eta^{g,\delta} + \varepsilon^g_{it} = g^0_{it} + X^{g,\delta}_{i,t-1}\eta^{g,\delta}$$

$$d^0_{it} = X^d_{i,t-1}\eta^d + g^0_{it}X^{d,g}_{i,t-1}\eta^{d,g} + \varepsilon^d_{it}$$

$$d^1_{it} = X^d_{i,t-1}\eta^d + g^1_{it}X^{d,g}_{i,t-1}\eta^{d,g} + \varepsilon^d_{it} = d^0_{it} + X^{g,\delta}_{i,t-1}\eta^{g,\delta}X^{d,g}_{i,t-1}\eta^{d,g} \quad (3.16)$$

With these notations, a solution to equations (3.13), (3.14) and (3.15) is necessarily $(d^0_{it}, g^0_{it}, 0)$ or $(d^1_{it}, g^1_{it}, 1)$.

In relation to economic theory, the following assumptions over the parameters of the model are made:

$$\forall i,t:\ X^{d,g}_{i,t-1}\eta^{d,g} < 0 \quad (3.17)$$

$$\forall i,t:\ X^{g,\delta}_{i,t-1}\eta^{g,\delta} < 0 \quad (3.18)$$

$$\forall i,t:\ X^{\delta,d}_{i,t-1}\eta^{\delta,d} > 0 \quad (3.19)$$

Constraint (3.18) implies that $g^1_{it} < g^0_{it}$: growth is always lower in a crisis scenario than in a no-crisis scenario, *ceteris paribus*.

Constraint (3.17) means that the debt-to-GDP ratio is a decreasing function of growth. Combined with (3.18), it implies that $d^0_{it} < d^1_{it}$: the debt-to-GDP ratio is always worse in a crisis scenario than in a no-crisis scenario, *ceteris paribus*.

Constraint (3.19) simply states that the probability of a debt crisis—as given by equation (3.15)—is an increasing function of the debt-to-GDP ratio.

Finally, I introduce a fourth random variable ζ_{it} following a Bernoulli distribution of parameter p (that is: $\mathbb{P}(\zeta_{it} = 1) = p$ and $\mathbb{P}(\zeta_{it} = 0) = 1 - p$). The variable ζ_{it} is a sunspot: its role is to discriminate between the two equilibria when both are possible.

Given these extensions, it is now possible to describe how the model behaves. For a given set of exogenous $X_{i,t-1}$, and for a given draw of random variables ε^d_{it}, ε^g_{it}, ε^δ_{it} and ζ_{it}, three cases are possible:

- The *crisis equilibrium*, inexorably driven by economic fundamentals, when $X^\delta_{i,t-1}\eta^\delta + d^0_{it}X^{\delta,d}_{i,t-1}\eta^{\delta,d} + \varepsilon^\delta_{it} > 0$. In that case, a no-crisis equilibrium is impossible, and because of equations (3.17), (3.18) and (3.19), one has $X^\delta_{i,t-1}\eta^\delta + d^1_{it}X^{\delta,d}_{i,t-1}\eta^{\delta,d} + \varepsilon^\delta_{it} > 0$, *i.e.* a crisis is triggered.

– The *no-crisis equilibrium*, when $X^{\delta}_{i,t-1}\eta^{\delta} + d^1_{it}\, X^{\delta,d}_{i,t-1}\eta^{\delta,d} + \varepsilon^{\delta}_{it} < 0$. A crisis equilibrium is impossible, and because of equations (3.17), (3.18) and (3.19), one has $X^{\delta}_{i,t-1}\eta^{\delta} + d^0_{it}\, X^{\delta,d}_{i,t-1}\eta^{\delta,d} + \varepsilon^{\delta}_{it} < 0$, *i.e.* no crisis occurs.
– The *multiple equilibria case*, when $X^{\delta}_{i,t-1}\eta^{\delta} + d^1_{it}\, X^{\delta,d}_{i,t-1}\eta^{\delta,d} + \varepsilon^{\delta}_{it} > 0 > X^{\delta}_{i,t-1}\eta^{\delta} + d^0_{it}\, X^{\delta,d}_{i,t-1}\eta^{\delta,d} + \varepsilon^{\delta}_{it}$. Both equilibria are possible. The outcome is given by the sunspot: $\delta_{it} = \zeta_{it}$ (and g_{it} and d_{it} are set accordingly, *i.e.* $g_{it} = g^{\delta_{it}}_{it}$ and $d_{it} = d^{\delta_{it}}_{it}$). A *self-fulfilling crisis* occurs if $\zeta_{it} = 1$: it could have been avoided (if the sunspot had been different), since the fundamentals are compatible with a no-crisis equilibrium.

The derivation of the likelihood function of the model can be found in appendix 3.7.3.

3.5.3 Estimating the self-fulfilling effect

The econometric model presented in sections 3.5.1 and 3.5.2 is estimated with full information maximum likelihood (FIML) on the dataset of debt crisis episodes presented in section 3.4. Details about the estimation procedure can be found in appendix 3.7.4.

Table 3.1 reports the results for various specifications. The present section discusses only columns (1) and (2); the remaining ones will be discussed in the following section.

All exogenous variables are taken in $t-2$ (*i.e.* two years before the beginning of the episode). The parameter p is calibrated: its estimation has not been possible with a reasonable accuracy. Two different values have been used for its calibration: $p = 1$, which reflects the assumption that, when two equilibria are possible, the market always chooses the worst of the two; and $p = 0.5$, which means that, when there is a possibility of a self-fulfilling crisis, a coin is flipped and the crisis takes place half of the time.

The upper part of the table reports the debt-to-GDP ratio dynamics (equation (3.13)), the middle part reports the growth dynamics (equation (3.14)), and the lower part reports the crisis probability (equation (3.15)). In particular, recall that the coefficient lines beginning with $\eta^{d,g}$ (resp. $\eta^{g,\delta}$, $\eta^{\delta,d}$) present regressors that are interacted with growth (resp. the crisis dummy and the debt-to-GDP ratio).

The current debt-to-GDP ratio is explained by past debt-to-GDP ratio, and in addition by the interaction of *current growth* with past debt-to-GDP ratio (under the category $\eta^{d,g}$). This second term is meant to capture the accounting effect of growth in the denominator of the debt-to-GDP ratio.

Current growth is explained by three factors: past growth, the occurrence of a crisis—which lowers the level of growth by a constant amount, and the level of real GDP per capita—in order to capture the international convergence effect.

The occurrence—or not—of a debt crisis is explained by the *current debt-to-GDP ratio*, the level of real GDP per capita, and the overvaluation of the exchange rate (measured as the ratio of GDP expressed in current US$ to GDP expressed in international PPP US$). The level of real GDP per capita is included because richer countries seem to exhibit less crisis in the data; the overvaluation of the exchange rate is meant to capture the fact that currency misalignment increases the risk of currency crisis which in turn increases the risk of debt crisis, since debt

Table 3.1: Estimation results

	(1)	(2)	(3)	(4)	(5)
Debt/GDP ratio dynamics					
η^d: Debt/GDP $(t-2)$	1.204***	1.205***	1.104***	1.197***	1.104***
	(0.023)	(0.023)	(0.075)	(0.066)	(0.073)
η^d: Crisis prob $\times$ Growth gap $\hat{g}$ $(t/t-2)$			0.821**	1.313*	0.825**
			(0.262)	(0.559)	(0.266)
η^d: Crisis prob $\times$ Debt/GDP $(t/t-2)$				-0.321	
				(0.244)	
η^d: Growth $(t-2)$ $-$ Mean Growth $(t-2/t-4)$					-0.017
					(0.212)
$\eta^{d,g}$: Debt/GDP $(t-2)$ $\times$ Growth (t)	-1.722***	-1.719***	-1.651***	-1.897***	-1.669***
	(0.214)	(0.210)	(0.320)	(0.318)	(0.317)
σ_d	0.124***	0.125***	0.120***	0.116***	0.121***
	(0.006)	(0.006)	(0.008)	(0.011)	(0.008)
Growth dynamics					
η^g: Log per capita PPP real GDP $(t-2)$	-0.023**	-0.025**	-0.023**	-0.022**	-0.023**
	(0.008)	(0.008)	(0.007)	(0.007)	(0.007)
η^g: Growth $(t-2)$	0.281**	0.277**	0.281**	0.284***	0.278**
	(0.101)	(0.101)	(0.086)	(0.077)	(0.087)
η^g: Constant	0.268***	0.290***	0.271***	0.263***	0.270***
	(0.064)	(0.064)	(0.059)	(0.059)	(0.060)
$\eta^{g,\delta}$: Debt crisis dummy (t)	-0.059***	-0.077***	-0.062***	-0.059***	-0.061***
	(0.015)	(0.014)	(0.015)	(0.014)	(0.015)
σ_g	0.094***	0.093***	0.094***	0.094***	0.094***
	(0.004)	(0.004)	(0.004)	(0.005)	(0.004)
Debt crisis determinants					
η^δ: Log per capita PPP real GDP (t-2)	-0.365**	-0.426**	-0.356**	-0.363*	-0.363**
	(0.132)	(0.133)	(0.135)	(0.141)	(0.135)
η^δ: US$ GDP / PPP GDP (t-2)	1.477**	1.582**	1.454**	1.387*	1.475**
	(0.535)	(0.530)	(0.525)	(0.542)	(0.525)
η^δ: Constant	0.237	0.705	0.202	0.313	0.261
	(1.071)	(1.070)	(1.085)	(1.108)	(1.084)
$\eta^{\delta,d}$: Debt/GDP (t)	2.883***	2.971***	2.815***	2.748***	2.801***
	(0.456)	(0.465)	(0.429)	(0.454)	(0.430)
Calibrated parameter					
p: Sunspot Bernoulli parameter	1.000	0.500	1.000	1.000	1.000
Self-fulfilling probability	0.111	0.077	0.111	0.119	0.110
Self-enforcing probability			0.124	0.192	0.124
Number of observations	253	253	251	251	248
Log-likelihood	301.683	300.872	306.744	313.844	300.338
AIC	-579.366	-577.744	-587.489	-599.688	-572.675

is generally denominated in foreign currency.

In Table 3.1, one can see in columns (1) and (2) that most of the parameters of interest are estimated with the expected sign, and with a good accuracy.

As expected, the debt dynamics exhibits high inertia (the coefficient on past debt-to-GDP is close to unity), and the interaction of *current growth* with past debt-to-GDP ratio has a strong effect (with a coefficient close to -2, which is logical given the fact that lagged variables are taken two periods in the past).

The growth dynamics has some serial auto-correlation, though not very high. The convergence effect of poor countries appears clearly. And, as expected, a debt crisis lowers the level of growth by more than 5% on average.

The estimators for the determinants of debt crises are also consistent: debt crises are made more likely by a high *current* debt-to-GDP ratio, low real income level and overvaluation of the local currency.

In addition to the results of parameter estimations, the tables also report information about the percentage of crises that were of a self-fulfilling nature. Indeed, with this econometric model, it is possible for a given crisis, to compute the *a posteriori* probability that it was of a self-fulfilling nature, by opposition to being solely driven by fundamentals and exogenous shocks (see below section 3.5.5). The line entitled "Self-fulfilling probability" in the tables reports the mean of that probability over all the crises in the dataset. In column (1) where p is calibrated to 1—that is, when the markets are considered as "panic prone"—about 11% of debt crises are reported as being self-fulfilling. In column (2), where p is calibrated to 0.5, the proportion of self-fulfilling crises is consistently almost halved, being around 7%.

3.5.4 Estimating the Panglossian effect

Since the theoretical model of section 3.2 predicts that some countries will adopt a prudent behavior while others will accumulate debt, ignoring the risk of a crisis in the Panglossian mode, this hypothesis is tested in the data. More precisely, using equation (3.11), I construct a proxy variable for the Panglossian effect which I define as $\pi_{it}(g_{it}^{+} - g_{it}^{-})$; this expression appears in the debt dynamics equation (3.12).

The first step for constructing this variable consists in estimating π_{it}, the probability of a debt crisis for country i at date t, given variables in $t-2$. For that purpose, a simple probit is estimated on the dataset of episodes, where the probability of a debt crisis is a function of several exogenous variables. [4] This makes possible it to compute at every date the probability of a debt crisis two periods ahead, as predicted by the probit model, independently of the actual realization or not of a crisis. [5]

4. Those variables are: the debt-to-GDP ratio, the log of per capita real PPP GDP, the total debt service-to-exports ratio and the overvaluation of exchange rate (measured by US$ GDP to PPP GDP ratio). All exogenous are taken two years before the beginning of the episode. The methodology is exactly that of Kraay and Nehru (2006), using a slightly different set of exogenous variables.

5. One possible criticism against this methodology is that the crisis probability as defined by a probit is not consistent with the crisis probability as defined by the larger system of simultaneous equations. The main reason for adopting this methodology is that estimating a model-consistent probability is a very difficult problem from a

The second step consists in estimating the *growth gap* $\hat{g}_{it} = g^+_{it} - g^-_{it}$, *i.e.* the expected growth conditionally on the absence of a crisis occurring minus the expected growth conditionally on the occurrence of a crisis. For a given π_{it}, the corresponding growth gap $g^+_{it} - g^-_{it}$ is computed by taking the mean growth rate (across the whole data sample) above and below the quantile π_{it}. This method is rigorously true when a common factor drives (up to uncorrelated disturbances) the determinant of growth and that of the probability of default.

The Panglossian variable thus constructed is used in the estimations of columns (3), (4) and (5) of Table 3.1. Note that since the Panglossian effect is a generated regressor, the standard errors of the parameter estimates—as generated by the FIML estimator—need to be corrected to take into account the sampling error of the first step probit. For that purpose, I implemented the generic method proposed by Murphy and Topel (1985) for two stage maximum likelihood estimation.

The estimation reported in column (3) shows that the Panglossian effect enters in the debt dynamics equation, consistently with the theoretical model. Its coefficient has the expected sign and is significant at the 0.2% level.

The table also reports information about the percentage of crises that are of a self-enforcing nature, *i.e.* that are the direct consequence of the Panglossian effect. More precisely, after having canceled the self-fulfilling effect, one can compute the probability that a crisis would not have occurred if the Panglossian effect had not been operative between $t-2$ and t. Note that the self-fulfilling and the self-enforcing probabilities thus computed are additive by construction. This leads, on average, to a self-enforcing probability of about 12%.

Note that the self-enforcing probabilities reported here only take into account the impact of the Panglossian effect between dates $t-2$ and t. In section 3.5.6 below I present another quantitative measure of the importance of the Panglossian effect, taking into account its cumulative effect a longer period.

Robustness checks are also performed in order to show that what is measured with the Panglossian variable is indeed the effect exhibited in the theoretical model and not a proxy for another economic mechanism.

First, one may argue that what is captured in the Panglossian effect is simply the mechanical effect of the risk premium asked by investors when the level of risk is higher. In column (4), the Panglossian variable is tested against the variable $\pi_{it}\frac{D_{i,t-2}}{Q_{i,t-2}}$, which is a proxy for the risk premium effect (since the risk premium is supposed to be highly correlated with the crisis probability). The results show that the Panglossian variable remains significant—though at a lower level—while the risk premium variable is not significant and has the wrong sign.

Secondly, one may argue that the Panglossian variable is simply a proxy for "bad news,"

computational point of view: it involves the computation of a fixed point in the maximum likelihood estimation, and there is no well-known methodology for computing the standard errors of the coefficients thus estimated. From an economic point of view, the methodology that I adopt is equivalent to the hypothesis that agents in the economy only know the probit model, but not the simultaneous equations model, and use the probit model to form their expectations about the future. This hypothesis is not fully satisfactory, but can nevertheless be justified by the fact that crisis forecasting is generally done with very simple models, as the probit one, both in policy institutions and in credit rating agencies.

and that the increase in debt that is measured when such a bad news occurs would also be predicted by a standard inter-temporal consumption smoothing. To test that hypothesis, I construct a measure of the business cycle, equal to growth in $t-2$ minus mean growth over $t-4$ to $t-2$. If the inter-temporal consumption smoothing hypothesis was true, this variable should enter in the debt dynamics, since it captures temporary shocks. On the contrary, the results in column (5) show that this variable is not significant, and does not diminish the explanatory power of the Panglossian variable.

3.5.5 *A posteriori* self-fulfilling probabilities

For each crisis in the sample, it is possible to compute the *a posteriori* probability that it was self-fulfilling. The probability is computed as the measure of the set of events where the (unobservable) trigger of default $\varepsilon_{it}^{\delta}$ is such that $X_{i,t-1}^{\delta}\eta^{\delta} + d_{it}^{1} X_{i,t-1}^{\delta,d}\eta^{\delta,d} + \varepsilon_{it}^{\delta} > 0 > X_{i,t-1}^{\delta}\eta^{\delta} + d_{it}^{0} X_{i,t-1}^{\delta,d}\eta^{\delta,d} + \varepsilon_{it}^{\delta}$. Since only crisis episodes are considered, the value of d_{it}^{1} is directly observable, and that of d_{it}^{0} can be easily found using equation (3.16). It is then straightforward to compute the probability given that $\varepsilon_{it}^{\delta}$ is assumed to be normally distributed.[6]

The results of this computation are given in Table 3.2. The crises are ordered by their likelihood of being self-fulfilling episodes. The probabilities are computed on the basis of the specification (3) of Table 3.1. Note that in this specification, the self-fulfilling parameter p is calibrated to 1; a lower value would give correspondingly lower self-fulfilling probabilities. The values reported can therefore be considered as upper bounds of the real probabilities.

Table 3.2: Individual crises self-fulfilling probabilities

Country	Year	Crisis length	Self-fulfill prob. (in %)
Jordan	1989	16	0.2
Somalia	1981	24	1.4
Rwanda	1994	11	1.4
Congo, Rep.	1985	20	1.6
Nigeria	1986	19	1.9
Cote d'Ivoire	1981	16	3.1
Guinea-Bissau	1981	23	3.7
Madagascar	1980	25	4.5
Congo, Dem. Rep.	1976	29	4.6
Turkey	1978	7	4.6
Uruguay	1983	4	5.0
Ethiopia	1991	14	5.1
Benin	1983	16	5.4
Benin	1970	9	5.9

Continued on next page

6. See equation (3.20) in appendix 3.7.3 for the derivation of the algebraic formula.

Continued from previous page

Country	Year	Crisis length	Self-fulfill prob. (in %)
Chile	1983	7	6.5
India	1981	3	6.6
Egypt, Arab Rep.	1977	4	7.8
Uruguay	2002	3	7.9
Mexico	1983	10	8.0
Sudan	1977	28	9.0
Gabon	1986	19	9.1
Peru	1977	4	9.9
Ghana	1970	7	10.0
Solomon Islands	2002	3	10.3
Brazil	1998	7	10.3
Kenya	1975	3	10.6
Pakistan	1972	5	10.8
Senegal	1980	23	10.8
Philippines	1976	3	10.9
Paraguay	1986	9	11.5
Brazil	1983	3	11.6
Niger	1983	22	11.9
Ecuador	2000	5	12.4
Kenya	1992	5	12.6
Bangladesh	1979	3	12.9
Honduras	1979	23	13.0
Egypt, Arab Rep.	1984	12	13.0
Colombia	1999	3	13.2
Dominican Republic	1983	17	14.2
Turkey	1999	6	14.5
Kenya	2000	3	14.7
Indonesia	1970	3	14.7
Ecuador	1983	14	14.8
Jamaica	1977	24	14.9
Comoros	1987	18	15.2
Tunisia	1986	6	15.3
Ghana	1996	3	15.5
Algeria	1994	4	15.7
Chile	1972	5	15.8
Morocco	1980	15	15.9

Continued on next page

Continued from previous page

Country	Year	Crisis length	Self-fulfill prob. (in %)
Trinidad and Tobago	1988	5	16.0
Thailand	1997	3	16.3
Costa Rica	1980	16	16.3
Cameroon	1987	18	16.6
Pakistan	1980	4	17.0
Kyrgyz Republic	2002	3	18.1
Pakistan	1994	10	18.7
Venezuela, RB	1989	4	19.3
Indonesia	1997	8	19.6
El Salvador	1990	3	19.9
Argentina	1983	13	20.3

In words, the Jordan crisis of 1989 or the Rwandan crisis of 1994 were almost surely not created by a self-fulfilling process. They could not have been avoided by simply restoring confidence.

In contrast, the crises of Argentina in 1983, El Salvador in 1990 or Indonesia in 1997 may have been self-fulfilling. There is about one chance in five that they could have been avoided if confidence had been maintained and panic avoided.

3.5.6 Simulating the model

I now turn to the simulation of the estimated model. The strategy is to simulate the dynamic model described by equations (3.13), (3.14) and (3.15) over several periods, for a given trajectory of the random draws $(\varepsilon_{it}^{d}, \varepsilon_{it}^{g}, \varepsilon_{it}^{\delta})$ and of the exogenous values X_{it}, given the estimated parameters η.

More precisely, I simulate the specification reported in column (3) of Table 3.1, for given values of both the set of exogenous and of parameters (as obtained by maximum-likelihood estimation). The log of per capital PPP real GDP and of the US$ GDP to PPP GDP ratio are set constant across time and equal to the sample mean. The starting point of the simulations is a debt-to-GDP ratio of 60%. The probability π_{it} used for the Panglossian effect is recomputed at each period, using the simple probit described in section 3.5.4. I simulate 2500 series of 5 periods (*i.e.* of 10 years, since lagged variables are taken 2 years earlier).

The dynamics of the model are affected by four shocks that may be switched off for comparison purposes: shocks to the law of motion of debt (ε_{it}^{d}), to growth (ε_{it}^{g}), to the crisis equation ($\varepsilon_{it}^{\delta}$), plus the sunspot ($\zeta_{it}$). I also consider simulations where the Panglossian effect is switched off (just by removing the corresponding term in the debt equation). Thus, there is a total of $2^5 = 32$ possible combinations according to whether some of these five effects are activated or not.

When the five effects are activated, 89.4% of the simulations exhibit a crisis episode in at

Table 3.3: Simulated contributions of shocks and Panglossian effect to crises

Effect	Contribution
Crisis shock ($\varepsilon_{it}^{\delta}$)	55.8%
Debt shock (ε_{it}^{d})	15.2%
Panglossian effect	12.0%
Growth shock (ε_{it}^{g})	11.0%
Self-fulfilling effect (ζ_{it})	6.1%
Total	100.0%

least one of the 5 simulation periods. This high occurrence rate of crisis is the consequence of the relatively high level of the debt-to-GDP ratio that has been chosen as the starting point for simulations.

In order to compute the contribution of each of these five effects to these crises, each of them is shut off one by one, and I observe by how much the number of crises diminishes, which gives the contribution of each one.[7]

Table 3.3 reports the contribution of each effect so computed: it shows the percentage of crisis episodes that can be considered a direct consequence of each effect.

One can see that the largest contributor is by far the crisis shock $\varepsilon_{it}^{\delta}$ which explains more than 55% of crises: this means that most crises are triggered by events not related to the level of the debt-to-GDP ratio. For the remaining crises, the Panglossian effect comes third, explaining about 12% of the crises, while the self-fulfilling effect accounts for about 6%.

3.6 Conclusion

This chapter has tried to distinguish two attitudes towards debt: the attitude of prudent borrowers, who attempt to stabilize their debt at low levels even in the event of an adverse shock, and Panglossian borrowers, who only take into account the best scenarios possible, rationally anticipating to default on their debt if hit by an unfavorable shock (or by a sequence of them). It has been shown empirically that this distinction is consistent with the data.

Two types of debt crises have also been distinguished: those that are the effect of an exogenous shock, and those that are created in a self-fulfilling manner by the financial markets themselves. I have shown that the large majority of crises are of the first kind, although the probability of self-fulfilling cases is not negligible.

These results have a few policy implications that are left to future work. For one thing, if the "earthquake model" is correct, then there is room for improving the stability of financial markets by the use of more conditional sovereign lending, contingent on other lenders following suit. It indeed remains a question to understand why sovereign debt arrangements

7. An issue is that the results depend on the order in which the effects are shut down: this problem is solved by making these computations for the 120 possible orders, and by computing the average contributions.

contain so few contingency clauses.

Regarding the self-fulfilling case, if the above results can be trusted, while the now old debate on sovereign debt restructuring remains important, it may be relatively less so than finding more innovative source of finance.

3.7 Appendix

3.7.1 Debt crises

Tables 3.4 and 3.5 present the complete list of the crisis episodes identified according to the methodology of section 3.4.

In Table 3.4, for each crisis episode, the first three columns give the country, the year of the crisis outbreak, and the number of years it lasted. The columns labeled "Type of crisis" tell whether the crisis was characterized by a Paris Club relief, accumulated arrears or IMF intervention (or several of these options). The last column is the debt-to-GDP ratio at three points in time (3 years before the outbreak, in the year of the outbreak and three years later)

Table 3.5 gives other macroeconomic indicators about the country: the debt-to-PPP-GDP ratio (at the same three points in time), the debt service-to-exports ratio, the mean annual growth before the crisis and the mean effective interest rate charged on the debt before the crisis.

Table 3.4: List of crisis episodes (with debt indicators)

Country	Year	Length	Type of crisis			D/GDP		
			Paris Club	*Arrears*	*SBA/EFF*	$t-3$	t	$t+3$
Indonesia	1970	3	Y	N	N	46.9	46.9	42.3
Benin	1970	9	N	Y	N	12.5	12.5	11.7
Ghana	1970	7	N	Y	N	25.8	25.8	30.6
Guinea	1970	35	Y	Y	Y	NA	NA	NA
Chile	1972	5	Y	Y	Y	33.1	30.7	76.4
Pakistan	1972	5	Y	N	N	34.0	43.7	50.7
Tanzania	1972	33	Y	Y	Y	NA	NA	NA
Kenya	1975	3	N	N	Y	27.6	39.6	41.0
Congo, Dem. Rep.	1976	29	Y	Y	Y	13.2	30.2	30.0
Philippines	1976	3	N	N	Y	27.4	35.3	48.3
Jamaica	1977	24	Y	Y	Y	60.4	51.7	71.4
Egypt, Arab Rep.	1977	4	N	N	Y	24.5	80.2	83.5
Peru	1977	4	Y	N	Y	38.8	64.4	45.4
Sudan	1977	28	Y	Y	Y	28.9	35.1	68.0

Continued on next page

Continued from previous page

Country	Year	Length	Type of crisis			D/GDP		
			Paris Club	*Arrears*	*SBA/EFF*	$t-3$	t	$t+3$
Panama	1978	3	N	N	Y	50.4	93.8	77.7
Turkey	1978	7	Y	N	Y	10.9	22.3	28.9
Honduras	1979	23	Y	Y	Y	27.1	52.6	63.5
Bangladesh	1979	3	N	N	Y	19.8	19.5	28.0
Mauritius	1979	3	N	N	Y	NA	NA	53.3
Costa Rica	1980	16	Y	Y	Y	42.9	56.8	133.1
Madagascar	1980	25	Y	Y	Y	33.1	30.6	57.9
Senegal	1980	23	Y	Y	Y	31.7	49.3	83.8
Morocco	1980	15	Y	Y	Y	50.8	51.7	93.5
Pakistan	1980	4	Y	N	Y	50.0	41.9	41.9
India	1981	3	N	N	Y	12.4	12.1	16.5
Romania	1981	5	Y	N	Y	NA	NA	NA
Cote d'Ivoire	1981	16	Y	Y	Y	48.6	96.5	124.9
Guinea-Bissau	1981	23	Y	Y	N	43.9	97.8	183.1
Somalia	1981	24	Y	Y	Y	91.8	151.0	190.0
Argentina	1983	13	Y	Y	Y	35.3	44.2	47.3
Niger	1983	22	Y	Y	Y	34.4	52.7	74.3
Benin	1983	16	Y	Y	N	30.2	68.1	74.0
Brazil	1983	3	Y	N	Y	30.4	48.5	40.7
Chile	1983	7	Y	N	Y	43.8	90.7	119.3
Dominican Republic	1983	17	Y	Y	Y	30.2	34.0	60.2
Ecuador	1983	14	Y	Y	Y	50.4	67.9	90.5
Mexico	1983	10	Y	N	Y	29.5	62.5	77.9
Uruguay	1983	4	N	N	Y	16.4	64.8	66.7
Egypt, Arab Rep.	1984	12	Y	Y	Y	94.3	105.1	109.0
Congo, Rep.	1985	20	Y	Y	Y	91.8	141.2	184.9
Lebanon	1986	6	N	Y	N	NA	NA	37.7
Sao Tome and Principe	1986	19	Y	Y	N	83.9	122.9	291.9
Gabon	1986	19	Y	Y	Y	27.0	57.1	80.0
Nigeria	1986	19	Y	Y	Y	50.2	109.9	126.3
Paraguay	1986	9	N	Y	N	25.2	58.9	54.6
Tunisia	1986	6	N	N	Y	48.6	65.9	69.0
Cameroon	1987	18	Y	Y	Y	37.2	37.9	59.7
Comoros	1987	18	N	Y	N	97.5	103.5	71.8
Trinidad and Tobago	1988	5	Y	Y	Y	19.6	46.7	46.7
Vietnam	1988	17	Y	Y	Y	0.4	2.4	243.4

Continued on next page

Continued from previous page

Country	Year	Length	Type of crisis			D/GDP		
			Paris Club	*Arrears*	*SBA/EFF*	$t-3$	t	$t+3$
Jordan	1989	16	Y	Y	Y	78.2	177.2	150.0
Venezuela, RB	1989	4	N	N	Y	58.3	76.8	64.7
El Salvador	1990	3	Y	N	N	50.2	44.8	29.3
Seychelles	1990	15	N	Y	N	69.4	49.7	38.7
Ethiopia	1991	14	Y	Y	N	99.9	95.8	181.7
Kenya	1992	5	Y	Y	N	71.2	83.9	80.8
Algeria	1994	4	Y	N	Y	62.3	71.1	64.5
Rwanda	1994	11	Y	Y	N	42.4	126.6	60.1
Pakistan	1994	10	Y	N	Y	51.4	52.8	48.2
Ghana	1996	3	Y	N	N	76.7	83.6	83.3
Indonesia	1997	8	Y	Y	Y	61.0	63.1	87.5
Thailand	1997	3	N	N	Y	45.3	72.7	64.9
Brazil	1998	7	N	N	Y	22.8	30.7	45.5
Colombia	1999	3	N	N	Y	29.7	39.9	40.7
Turkey	1999	6	N	N	Y	44.1	55.6	71.3
Kenya	2000	3	Y	N	N	49.3	48.4	45.6
Ecuador	2000	5	Y	N	Y	65.2	86.0	62.0
Kyrgyz Republic	2002	3	Y	N	N	139.0	115.3	NA
Solomon Islands	2002	3	N	Y	N	49.7	79.5	NA
Uruguay	2002	3	N	N	Y	35.3	86.4	NA

Table 3.5: List of crisis episodes (with other macro indicators)

Country	Year	D/PPP-GDP			TDS/X	Growth	Interest rate
		$t-3$	t	$t+3$		$t-3\ldots t-1$	$t-3\ldots t-1$
Indonesia	1970	16.0	16.0	16.0	13.0	6.9	1.0
Benin	1970	5.7	5.7	6.0	3.4	2.6	1.0
Ghana	1970	12.6	12.6	13.4	10.8	3.2	2.1
Guinea	1970	14.6	14.6	22.8	NA	NA	1.3
Chile	1972	17.2	17.5	27.6	27.3	4.9	3.5
Pakistan	1972	14.5	14.8	14.3	33.2	5.8	2.0
Tanzania	1972	8.6	49.4	63.7	NA	NA	0.9
Kenya	1975	12.0	21.3	23.6	8.6	9.0	3.5
Congo, Dem. Rep.	1976	9.8	28.9	26.0	9.2	2.1	3.4
Philippines	1976	7.3	11.2	16.9	22.5	6.0	3.3

Continued on next page

Continued from previous page

Country	Year	D/PPP-GDP			TDS/X	Growth	Interest rate
		$t-3$	t	$t+3$		$t-3\ldots t-1$	$t-3\ldots t-1$
Jamaica	1977	43.8	41.8	43.2	37.2	-3.7	6.9
Egypt, Arab Rep.	1977	7.3	26.0	25.9	16.3	8.7	1.7
Peru	1977	19.7	27.0	18.6	42.5	4.9	5.0
Sudan	1977	18.7	29.2	46.1	12.7	14.6	1.8
Panama	1978	30.2	53.5	47.8	NA	1.5	4.4
Turkey	1978	9.2	19.3	18.2	19.1	7.0	5.6
Honduras	1979	13.2	28.1	33.4	29.1	10.3	5.1
Bangladesh	1979	5.0	5.4	6.7	28.8	5.1	1.9
Mauritius	1979	2.7	12.0	14.9	NA	NA	3.3
Costa Rica	1980	20.6	31.5	43.8	22.0	6.7	5.0
Madagascar	1980	18.4	24.3	32.2	37.7	3.2	2.5
Senegal	1980	15.8	29.6	31.2	7.1	0.1	3.8
Morocco	1980	24.6	29.6	30.6	19.1	4.4	4.8
Pakistan	1980	14.8	12.7	10.6	31.9	5.3	2.5
India	1981	4.2	4.0	4.5	16.1	2.4	2.7
Romania	1981	1.9	13.5	7.8	NA	NA	4.3
Cote d'Ivoire	1981	37.6	62.3	47.5	16.4	0.8	6.5
Guinea-Bissau	1981	22.9	44.9	67.5	10.5	-0.3	1.0
Somalia	1981	16.1	27.3	36.2	3.0	-1.0	0.3
Argentina	1983	15.9	23.7	24.6	107.4	-2.2	8.8
Niger	1983	23.4	19.0	26.9	22.9	-0.0	9.2
Benin	1983	21.5	30.8	29.3	9.1	6.3	3.0
Brazil	1983	15.2	18.9	15.1	69.4	1.8	12.1
Chile	1983	29.1	42.1	41.4	43.0	0.9	11.7
Dominican Republic	1983	15.6	15.9	17.6	29.8	3.9	9.1
Ecuador	1983	22.0	24.5	28.0	34.0	2.4	9.4
Mexico	1983	20.2	25.8	26.8	52.7	5.8	12.0
Uruguay	1983	12.3	24.4	23.7	19.6	-0.8	9.2
Egypt, Arab Rep.	1984	26.1	29.5	37.6	19.9	7.0	4.3
Congo, Rep.	1985	56.3	75.8	123.5	20.2	12.1	5.7
Lebanon	1986	NA	NA	NA	NA	NA	8.0
Sao Tome and Principe	1986	46.5	76.5	117.5	24.2	NA	1.7
Gabon	1986	19.1	35.7	51.0	11.7	3.6	7.9
Nigeria	1986	37.5	42.2	36.9	53.8	-0.1	9.4
Paraguay	1986	12.8	16.3	14.1	13.7	1.3	4.0
Tunisia	1986	17.1	22.0	21.5	22.2	5.4	5.9

Continued on next page

Continued from previous page

Country	Year	D/PPP-GDP			TDS/X	Growth	Interest rate
		$t-3$	t	$t+3$		$t-3\ldots t-1$	$t-3\ldots t-1$
Cameroon	1987	15.4	21.0	28.1	15.9	7.4	6.2
Comoros	1987	20.4	34.0	26.9	28.9	2.8	1.2
Trinidad and Tobago	1988	13.8	23.9	24.0	11.0	-4.0	7.6
Vietnam	1988	NA	NA	28.2	NA	3.4	0.5
Jordan	1989	44.9	70.3	63.2	32.8	2.7	6.0
Venezuela, RB	1989	38.9	29.7	27.7	43.7	5.3	8.6
El Salvador	1990	14.7	14.2	10.5	34.6	1.8	3.9
Seychelles	1990	43.8	31.8	24.0	9.1	7.0	5.2
Ethiopia	1991	46.7	47.2	37.2	48.6	0.9	1.0
Kenya	1992	25.8	25.6	22.3	37.2	3.4	4.6
Algeria	1994	24.7	26.8	22.4	68.9	-0.5	7.1
Rwanda	1994	10.3	25.0	16.9	16.5	-1.6	1.3
Pakistan	1994	10.3	10.1	9.5	25.4	4.8	3.6
Ghana	1996	23.6	25.7	23.9	24.4	4.1	2.1
Indonesia	1997	16.9	16.5	17.1	30.4	7.9	5.0
Thailand	1997	18.4	26.3	19.7	14.0	8.0	4.3
Brazil	1998	15.1	20.7	18.0	39.7	3.4	6.2
Colombia	1999	13.5	15.2	13.0	36.6	2.0	6.4
Turkey	1999	25.0	28.1	33.6	28.0	5.9	5.7
Kenya	2000	17.7	16.0	16.7	22.1	2.0	3.0
Ecuador	2000	29.3	24.6	26.7	31.2	-0.0	6.0
Kyrgyz Republic	2002	12.3	10.7	NA	20.9	4.8	3.3
Solomon Islands	2002	15.5	20.5	NA	5.0	-7.9	1.9
Uruguay	2002	20.7	34.3	NA	27.6	-2.6	6.8

3.7.2 Proofs

Proof of corollary 3.6. Suppose the economy is in a state (D, Q, Λ) such that:

$$D > \frac{Q}{1 - \frac{\mathbb{E}(g)}{1+r}}$$

If the country decides to repay, lemma 3.5 shows that it can borrow at most $L = \frac{\mathbb{E}(g)Q}{(1+r)\left(1-\frac{\mathbb{E}(g)}{1+r}\right)}$.

It is easy to see that country consumption $Q - D + L$ cannot be positive in that case. So default is the only option. □

Proof of lemma 3.7. Let $L \in \mathscr{L}(Q)$ such that $L - D \geq -\Lambda(1-\lambda)Q - \lambda Q$. By definition, one has:

$$J^r(D,Q) \geq u(Q-D+L) + \beta \int_{\mathscr{D}(\tilde{D}'(L,Q),Q)} J^d(g'Q,\Lambda')\mathrm{d}\mathcal{F}(g')\mathrm{d}\mathcal{G}(\Lambda') + \beta \int_{\mathscr{R}(\tilde{D}'(L,Q),Q)} J^r(\tilde{D}'(L,Q),g'Q)\mathrm{d}\mathcal{F}(g')\mathrm{d}\mathcal{G}(\Lambda')$$

Then:

$$J^r(D,Q) - J^d(Q,\Lambda) \geq u(Q-D+L) - u((1-\Lambda)(1-\lambda)Q) + \beta \int_{\mathscr{R}(\tilde{D}'(L,Q),Q)} (J^r(\tilde{D}'(L,Q),g'Q) - J^d(g'Q,\Lambda))\mathrm{d}\mathcal{F}(g')\mathrm{d}\mathcal{G}(\Lambda')$$

Since, by definition, J^r is greater than J^d over the repayment set $\mathscr{R}$, one has:

$$J^r(D,Q) - J^d(Q,\Lambda) \geq u(Q-D+L) - u((1-\Lambda)(1-\lambda)Q)$$

So $J^r(D,Q) - J^d(Q,\Lambda) \geq 0$ by definition of L, and since u is increasing. □

Proof of proposition 3.8. Suppose the economy is in a state (D,Q,Λ) such that $D \leq V(Q,\Lambda) + \lambda Q$. Then let:

$$L = \frac{1}{1+r} \int V(g'Q,\Lambda')\mathrm{d}\mathcal{F}(g')\mathrm{d}\mathcal{G}(\Lambda')$$

It is easy to see that $L \in \mathscr{L}(Q)$. Indeed, if the country asks for that level today, the investors can ask for $\tilde{D}'(L,Q) = \frac{g^{\max} Q}{1-\frac{\mathbb{E}(g)}{1+r}}$, which verifies the zero-profit condition (because at such a level of indebtment, the country will default tomorrow with probability one).

Given that value for L, one has $L - D \geq -\Lambda(1-\lambda)Q - \lambda Q$ (because $D \leq V(Q,\Lambda) + \lambda Q$, and using the definition of $V(Q,\Lambda)$).

By lemma 3.7, default is therefore not optimal. The country decides to repay in that situation.

The result follows using the homogeneity properties. □

Proof of proposition 3.10. Formally, for a given default set $\mathscr{D}$ (such that $(g',\Lambda') \in \mathscr{D}(D',Q) \Leftrightarrow \frac{D'}{g'Q} > d^*(\Lambda')$), there exists a unique continuous function $\tilde{D}'(L,Q)$ satisfying the zero-profit condition (3.6) in the smooth default case. The function $\tilde{D}'(L,Q)$ is determined by the implicit equation (3.6). This equation can be rewritten as:

$$f(L,Q,\tilde{D}'(L,Q)) = 0$$

where:

$$f(L,Q,D') = D'\,\mathbb{P}[\mathscr{R}(D',Q)] + \int_{\mathscr{D}(D',Q)} V(g'Q,\Lambda')\mathrm{d}\mathcal{F}(g')\mathrm{d}\mathcal{G}(\Lambda') - L(1+r)$$

The implicit function theorem states that there is a unique solution to this implicit equation if the derivative of f with respect to D' is non negative.

Using the specific structure of $\mathscr{D}$ and $\mathscr{R}$, this can be rewritten as:

$$f(L,Q,D') = \int \left(\int_{\frac{D'}{d^*(\Lambda')Q}}^{g^{\max}} D' \mathrm{d}\mathcal{F}(g') + \int_0^{\frac{D'}{d^*(\Lambda')Q}} V(g'Q,\Lambda')\mathrm{d}\mathcal{F}(g') \right) \mathrm{d}\mathcal{G}(\Lambda') - L(1+r)$$

Taking the derivative with respect to D', one gets:

$$\frac{\partial f}{\partial D'}(L,Q,D') = \mathbb{P}[\mathscr{R}(D',Q)] - \frac{D'}{Q} \int \frac{d^*(\Lambda') - V(1,\Lambda')}{d^*(\Lambda')^2} \mathrm{d}\mathcal{G}(\Lambda')$$

In the general case the sign of this derivative is not constant, since both terms in the expression are positive (the second term is positive because of proposition 3.8). But since in this particular case I assumed that $d^*(\Lambda') = V(1,\Lambda')$ (smooth default), one has:

$$\frac{\partial f}{\partial D'}(L,Q,D') = \mathbb{P}[\mathscr{R}(D',Q)] \geq 0$$

So the derivative is non null, except for the points where $\mathbb{P}[\mathscr{R}(D',Q)] = 0$. But in this latter case, the zero profit condition implies that:

$$L = \frac{1}{1+r} \int V(g'Q,\Lambda')\mathrm{d}\mathcal{F}(g')\mathrm{d}\mathcal{G}(\Lambda').$$

Hence the derivative is non null everywhere except on a set of points of empty interior. Using the implicit function theorem, this implies that there is a unique continuous function verifying the zero-profit condition. □

3.7.3 Likelihood derivation

In this section I derive the likelihood function of the econometric model described in section 3.5. The likelihood of a single observation $(d_{it}, g_{it}, \delta_{it})$ is $\mathcal{L}_\Theta(d_{it}, g_{it}, \delta_{it}|X_{i,t-1})$ given the exogenous values $X_{i,t-1}$ and the vector of parameters $\Theta = (\eta^d, \eta^{d,g}, \eta^g, \eta^{g,\delta}, \eta^\delta, \eta^{\delta,d}, \sigma_d, \sigma_g, p)$.

For the remaining of this subsection, the i and t subscripts are dropped for the sake of simplicity.

Let's note φ the probability density function of the standard normal distribution (zero mean and unit variance), and Φ its cumulative density function.

Given (d, g, δ), ε^d and ε^g can be immediately inferred. The likelihood function is therefore, by independence of the four shocks $(\varepsilon^d, \varepsilon^g, \varepsilon^\delta, \zeta)$:

$$\mathcal{L}_\Theta(d,g,\delta|X) = \mathbb{P}_\Theta(\varepsilon^d = d - X^d\eta^d - gX^{d,g}\eta^{d,g})\, \mathbb{P}_\Theta(\varepsilon^g = g - X^g\eta^g - \delta X^{g,\delta}\eta^{g,\delta})\, \mathbb{P}_\Theta(\delta|d,X)$$

The first two factors are:

$$\mathbb{P}_\Theta(\varepsilon^d = d - X^d\eta^d - gX^{d,g}\eta^{d,g}) = \frac{1}{\sigma_d}\varphi\left(\frac{d - X^d\eta^d - gX^{d,g}\eta^{d,g}}{\sigma_d}\right)$$

$$\mathbb{P}_\Theta(\varepsilon^g = g - X^g\eta^g - \delta X^{g,\delta}\eta^{g,\delta}) = \frac{1}{\sigma_g}\varphi\left(\frac{g - X^g\eta^g - \delta X^{g,\delta}\eta^{g,\delta}}{\sigma_g}\right)$$

The third factor is discussed below.

Crisis case

If $\delta = 1$, one knows that $d = d^1$ and $g = g^1$. Then:

$$\begin{aligned}\mathbb{P}(\delta = 1|d^1, X) &= \mathbb{P}(X^\delta\eta^\delta + d^0X^{\delta,d}\eta^{\delta,d} + \varepsilon^\delta > 0) + \\ &\quad p\,\mathbb{P}(X^\delta\eta^\delta + d^1X^{\delta,d}\eta^{\delta,d} + \varepsilon^\delta > 0 > X^\delta\eta^\delta + d^0X^{\delta,d}\eta^{\delta,d} + \varepsilon^\delta)\end{aligned}$$

In this equation, the first term corresponds to a crisis driven solely by fundamentals and exogenous shocks, and the second term to the self-fulfilling case.

Using (3.16), it can be rewritten as:

$$\begin{aligned}\mathbb{P}(\delta = 1|d^1, X) &= \Phi[X^\delta\eta^\delta + (d^1 - X^{g,\delta}\eta^{g,\delta}X^{d,g}\eta^{d,g})X^{\delta,d}\eta^{\delta,d}] + \\ &\quad p\left\{\Phi(X^\delta\eta^\delta + d^1X^{\delta,d}\eta^{\delta,d}) - \Phi[X^\delta\eta^\delta + (d^1 - X^{g,\delta}\eta^{g,\delta}X^{d,g}\eta^{d,g})X^{\delta,d}\eta^{\delta,d}]\right\}\end{aligned}$$

For a given crisis observation, it is therefore possible to compute the *a posteriori* probability that the crisis is of a self-fulfilling nature (by opposition to a crisis solely driven by fundamentals and exogenous shocks). This probability is given by:

$$\mathcal{S}_\Theta(d^1, X) = \frac{p\left\{\Phi(X^\delta\eta^\delta + d^1X^{\delta,d}\eta^{\delta,d}) - \Phi[X^\delta\eta^\delta + (d^1 - X^{g,\delta}\eta^{g,\delta}X^{d,g}\eta^{d,g})X^{\delta,d}\eta^{\delta,d}]\right\}}{\mathbb{P}_\Theta(\delta = 1|d^1, X)} \tag{3.20}$$

No-crisis case

If $\delta = 0$, one knows that $d = d^0$. Then:

$$\begin{aligned}\mathbb{P}(\delta = 0|d^0, X) &= \mathbb{P}(X^\delta\eta^\delta + d^1X^{\delta,d}\eta^{\delta,d} + \varepsilon^\delta < 0) + \\ &\quad (1-p)\,\mathbb{P}(X^\delta\eta^\delta + d^1X^{\delta,d}\eta^{\delta,d} + \varepsilon^\delta > 0 > X^\delta\eta^\delta + d^0X^{\delta,d}\eta^{\delta,d} + \varepsilon^\delta)\end{aligned}$$

In this equation, the first term corresponds to the no-crisis equilibrium driven by strong fundamentals, and the second term to the self-fulfilling case in which the country escapes the crisis.

Using (3.16), it can be rewritten as:

$$\begin{aligned}\mathbb{P}(\delta = 0|d^0, X) &= 1 - \Phi[X^\delta \eta^\delta + (d^0 + X^{g,\delta}\eta^{g,\delta}X^{d,g}\eta^{d,g})X^{\delta,d}\eta^{\delta,d}] + \\ &(1-p)\left\{\Phi[X^\delta \eta^\delta + (d^0 + X^{g,\delta}\eta^{g,\delta}X^{d,g}\eta^{d,g})X^{\delta,d}\eta^{\delta,d}] - \Phi(X^\delta \eta^\delta + d^0 X^{\delta,d}\eta^{\delta,d})\right\}\end{aligned}$$

3.7.4 Estimation methodology

The model is estimated with full information maximum (log-)likelihood, *i.e.* by computing the following:

$$\underset{\Theta \in \mathcal{B}}{\operatorname{argmax}} \sum_{(i,t)} \log \mathcal{L}_\Theta(d_{it}, g_{it}, \delta_{it}|X_{i,t-1})$$

where $\mathcal{B}$ is a set of constraints over parameters to ensure that constraints (3.17), (3.18) and (3.19) are satisfied and that $\sigma_d > 0$, $\sigma_g > 0$. The parameter p is calibrated.

The programs performing the estimations are written using the R environment for statistical computing. [8]

Constraints

The constrained-optimization algorithm that I use is the L-BFGS-B method (see Byrd et al., 1994), which allows box constraints (*i.e.* each variable can be given a lower and/or upper bound).

The constraints over σ_d and σ_g already fit into this category.

Constraints (3.17), (3.18), (3.19) (respectively over $\eta^{d,g}$, $\eta^{g,\delta}$, $\eta^{\delta,d}$) are enforced by replacing them with tighter constraints, in the following way:

- First, I only choose constant sign regressors in $X^{d,g}$, $X^{g,\delta}$, $X^{\delta,d}$ (that is, all elements of a given column in these matrices have a constant sign).
- Second, every component of $\eta^{d,g}$, $\eta^{g,\delta}$, $\eta^{\delta,d}$ is constrained to have the sign that will enforce the constraint.

Therefore, constraints (3.17), (3.18), (3.19) are clearly satisfied, and the constraints over $\eta^{d,g}$, $\eta^{g,\delta}$, $\eta^{\delta,d}$ can be dealt with by the L-BFGS-B algorithm.

Non-concavity

The second issue is the fact that the log-likelihood function is not globally concave, which implies that different initial values in the optimization algorithm can lead to different local maxima.

This problem is dealt with using a simple randomization algorithm. The following procedure is repeated 50,000 times:

- Generate a random initial value for the maximization algorithm. I alternate between two algorithms for generating this point (each algorithm is used half of the time):

8. See http://www.r-project.org and R Development Core Team (2011).

    - Draw a totally random point. For unconstrained parameters $(\eta^d, \eta^g, \eta^\delta)$, a standard normal distribution is used. For sign-constrained parameters $(\eta^{d,g}, \eta^{g,\delta}, \eta^{\delta,d}, \sigma_d, \sigma_g)$, a χ_1^2 distribution is used (multiplied by -1 for the relevant components of $\eta^{d,g}$, $\eta^{g,\delta}$, $\eta^{\delta,d}$).
    - Draw a point in the neighborhood of the point which has the highest likelihood so far. For all parameters, a normal distribution centered around that point is used, using the same standard error than the maximum likelihood estimator.
- Run the L-BFGS-B algorithm using the initial value thus generated.
- If the result has a greater log-likelihood than the previous best point, keep it, otherwise discard it.

The results obtained in this way exhibit good numerical stability.

Chapter 4

Sovereign defaults in RBC models

4.1 Introduction

So far I have studied quantitative sovereign debt models that are based on the framework initiated by Eaton and Gersovitz (1981). On the front of replicating realistic debt levels and default probabilities, the success of these quantitative models has been relatively mixed so far, as discussed in section 2.2 (and a solution to this issue is precisely the purpose of the whole chapter 2). But the main objective of the initiators of the new quantitative sovereign debt literature—such as Arellano (2008) and Aguiar and Gopinath (2006)—was more to replicate key business cycle properties of emerging countries rather than replicating sovereign risk facts themselves. In this respect, these models have been quite successful, as recalled in section 1.3.

There is another parallel strand of the literature which has aimed at replicating business cycle properties of emerging countries, using real business cycle (RBC) models and more recently dynamic stochastic general equilibrium (DSGE) models. This literature has been initiated by Mendoza (1991) who examined a developed small open economy (SOE) model using the RBC paradigm. More recently, Uribe and Yue (2006) and Neumeyer and Perri (2005) have analyzed RBC models calibrated for emerging small open economy, and have obtained quite convincing results regarding the interaction of sovereign spreads and the business cycle of these countries.

With the notable exception of Mendoza and Yue (2012), these two trends of the literature (SOE-RBC on one side and endogenous default à la Eaton and Gersovitz (1981) on the other side) have been pursued independently and largely ignored each other. Each paradigm has its strengths and weaknesses: the endogenous default models assume a purely exogenous process for output, while the SOE-RBC models assume a more realistic process with capital accumulation and labor supply; but SOE-RBC models are unable to endogeneize the default decision and are therefore forced to rely on relatively *ad hoc* formulations for incorporating sovereign interest rate spreads.

It should be noted that SOE-RBC models are not self-consistent, at least on the surface: on the one hand they do not allow for defaults since, by very construction, they assume that the country always pays back its debts; on the other hand, they incorporate positive spreads in some form (possibly as a function of macroeconomic fundamentals). This is a contradiction: since the model assumes that no default ever takes place, model consistent spreads are zero!

This chapter tries to see if the gap between SOE-RBC and endogenous defaults models could be filled in the easy way. The idea is to introduce the possibility of a default in SOE-RBC models, without however breaking the simplicity of the RBC framework. The idea is the following. I compute an out-of-model value function corresponding to what the country would get by defaulting (using the typical modeling tools of the Eaton and Gersovitz (1981) framework), and I compare this default value function (J^d) with the value function of the SOE-RBC (J^r) which by construction assumes that there is no default ever. In this way, I am able to compute a default probability quasi-consistent with the model. Of course this probability is not fully consistent, since the country still does not internalize the fact that it may default in the future. But this is the best that can done within the RBC framework.

Once this extension to the SOE-RBC model has been made, I then explore whether this extended model is able to deliver default probabilities and debt levels which are close to those observed in the data. I can also answer another question, which is whether the SOE-RBC models are internally consistent when they assume no default. Of course, it is one or the other: either the model has realistic default probabilities and is not consistent (since it assumes no default), or it is the opposite.

The results that I arrive actually show that, even if for some parametrizations the extended model is self-consistent (*i.e.* it exhibits no default), it can hardly ever deliver data-consistent results. Depending on the parametrization, the probability of default implied by the extended RBC model is either far too low or far too high. The model behaves in a very dichotomic way, being always on one side or on the other. It is clear that the model does not naturally deliver implied default probabilities lying in a realistic range, at least for the benchmark calibrations. In particular, this raises concerns about the relevance of RBC models for studying business cycle fluctuations in emerging countries where the role of interest rate spreads has been shown to be quite important.

The rest of this chapter is organized as follow. In section 4.2, I present the SOE-RBC model that I use for the purpose of this exercise, and I outline the methodology used to quantify the risk of default in such a model. In section 4.3, I present a benchmark calibration of this model and discuss its properties regarding default probabilities. In section 4.4, I study the sensitivity of these results to some parameter values and some modelling choices. Finally, in section 4.5, I draw some implications of this exercise.

4.2 Quantifying default in a small open economy RBC model

4.2.1 The model core

The model that I outline below is taken from Aguiar and Gopinath (2007). It is in the tradition of SOE-RBC models as in Mendoza (1991) and Schmitt-Grohé and Uribe (2003).

The economy consists of a country with a representative agent which produces (using capital and labor), trades and consumes. The difference between consumption and output is financed by international capital markets and results in the accumulation (or decumulation) of external debt. The interest rate charged is the sum of a constant exogenous riskless rate and of a risk premium which depends on the external indebtment level.

Output is given by a Cobb-Douglas production function and is affected by both transitory and permanent productivity shocks. The permanent component is embedded into a stochastic productivity trend. It is precisely the introduction of a shock to the productivity trend that is the contribution of Aguiar and Gopinath (2007) and that, according to the authors, is the main driver of the business cycle of emerging countries. We therefore have:

$$Q_t = a_t K_t^{1-\alpha} (\Gamma_t \ell_t)^{\alpha} \tag{4.1}$$

where a_t is the transitory productivity shock, K_t the is capital stock at the beginning of the period, Γ_t is the stochastic productivity trend, ℓ_t is the labor supply and $\alpha \in (0,1)$ is the share of labor in production.

The transitory productivity shock evolves accordingly to an AR(1) process:

$$\log(a_t) = \rho_a \log(a_{t-1}) + \varepsilon_t^a \tag{4.2}$$

where $\rho_a \in (0,1)$ and $\varepsilon_t^a \rightsquigarrow \mathcal{N}(0, \sigma_a^2)$.

The stochastic productivity trend Γ_t is such that:

$$\Gamma_t = g_{t-1}\Gamma_{t-1} = \prod_{T=0}^{t-1} g_T \tag{4.3}$$

$$g_t = e^{y_t} \tag{4.4}$$

$$y_t - \mu_y = \rho_y(y_{t-1} - \mu_y) + \varepsilon_t^y \tag{4.5}$$

where $\rho_y \in [0,1)$, $\varepsilon_t^y \rightsquigarrow \mathcal{N}(0, \sigma_y^2)$. Note that there is a slight abuse of notation because strictly speaking g_t is not growth but only its permanent component. I denote $\mu_g = e^{\mu_y}$ the steady state level of g_t.

The preferences of the representative agent in the country are modeled using GHH preferences (Greenwood et al., 1988):

$$u_t = \frac{(C_t - \tau \Gamma_t \ell_t^{\nu})^{1-\gamma}}{1-\gamma}$$

where $\nu > 1$ and $\tau > 0$. The risk aversion is $\gamma > 0$ and the elasticity of labor supply is given by $\frac{1}{\nu-1}$. Note that it is necessary to introduce the productivity trend inside the instantaneous utility function in order to maintain a bounded utility along the balanced growth path. Another necessary condition for utility boundedness is $\beta \mu_g^{1-\gamma} < 1$. In the following, detrended variables are denoted with a hat. For example the detrended debt is $\hat{D}_t = \frac{D_t}{\Gamma_t}$.[1]

1. The fact that current permanent shock y_t does not enter Γ_t guarantees that if X_t is in the information set at

The resource constraint of the economy is:

$$C_t + K_{t+1} = Q_t + (1-\kappa)K_t - \frac{\phi}{2}\left(\frac{K_{t+1}}{K_t} - \mu_g\right)^2 K_t - D_t + L_t \tag{4.6}$$

where $\kappa \in [0,1]$ is the depreciation rate of capital and $\phi > 0$ governs the quadratic adjustment cost of capital stock. Net foreign borrowing is equal to $L_t - D_t$.

In the canonical default model of section 1.3.1, the risk premium asked by the international investors is fully endogenous and depends on the risk of default implied by the model, as shown in (1.9). In the RBC model, since the default decision is not made endogenously, such a specification is no longer possible. The modeler has to explicitly specify a reduced form version of the risk premium mechanism. A first interesting case would be the zero premium (*i.e.* interest rate always equal to the riskless rate r); this is however not an easy option because, as is well known in the literature, the debt level would not be well defined in steady state and would exhibit a non stationary dynamics (Schmitt-Grohé and Uribe, 2003). The risk premium function in RBC models therefore plays two roles: it is an approximation of the reaction of the markets to perceived default risk, but it is also a technical hypothesis necessary to make the model well-specified.

In the following, let's denote Δ_t the risk premium. It is related to the borrowing supply by international investors as follows:

$$L_t = \frac{D_{t+1}}{1 + r + \Delta_t} \tag{4.7}$$

The specification for the risk premium is taken from Schmitt-Grohé and Uribe (2003):

$$\Delta_t = \psi\left(e^{\frac{D_{t+1}}{\Gamma_{t+1}} - \bar{d}} - 1\right) = \psi\left(e^{\hat{D}_{t+1} - \bar{d}} - 1\right) \tag{4.8}$$

where $\psi > 0$. This functional form implies a risk premium equal to zero for a debt-to-GDP ratio equal to $\bar{d}$ and which grows exponentially for higher levels of debt.

The problem of the representative agent can be stated, in detrended form, as follows:

$$J^r(\hat{K}_t, \hat{D}_t, a_t, g_t) = \max_{\hat{C}_t, \ell_t, \hat{K}_{t+1}, \hat{D}_{t+1}} \left\{ u(\hat{C}_t, \ell_t) + \beta\, g_t^{1-\gamma}\, \mathbb{E}_t J^r(\hat{K}_{t+1}, \hat{D}_{t+1}, a_{t+1}, g_{t+1}) \right\} \tag{4.9}$$

where $u(\hat{C}_t, \ell_t) = \frac{\left(\hat{C}_t - \tau\, \ell_t^{\nu}\right)^{1-\gamma}}{1-\gamma}$, subject to (4.6) and (4.7) combined together and detrended:

$$\hat{C}_t + g_t \hat{K}_{t+1} = \hat{Q}_t + (1-\kappa)\hat{K}_t - \frac{\phi}{2}\left(g_t \frac{\hat{K}_{t+1}}{\hat{K}_t} - \mu_g\right)^2 \hat{K}_t - \hat{D}_t + \frac{g_t \hat{D}_{t+1}}{1 + r + \Delta_t} \tag{4.10}$$

This leads to the following three first order conditions:

date $t-1$, then so is $\hat{X}_t$.

– Euler equation with respect to capital:

$$(\hat{C}_t - \tau\,\ell_t^{\nu})^{-\gamma}\left(1+\phi\left(g_t\frac{\hat{K}_{t+1}}{\hat{K}_t}-\mu_g\right)\right) =$$
$$\beta\, g_t^{-\gamma}\,\mathbb{E}_t\left\{(\hat{C}_{t+1}-\tau\,\ell_{t+1}^{\nu})^{-\gamma}\left[1-\kappa+(1-\alpha)\frac{\hat{Q}_{t+1}}{\hat{K}_{t+1}}+\frac{\phi}{2}\left(\left(g_{t+1}\frac{\hat{K}_{t+2}}{\hat{K}_{t+1}}\right)^2-\mu_g^2\right)\right]\right\} \tag{4.11}$$

– Euler equation with respect to debt:

$$(\hat{C}_t - \tau\,\ell_t^{\nu})^{-\gamma} = \beta\, g_t^{-\gamma}\,(1+r+\Delta_t)\,\mathbb{E}_t(\hat{C}_{t+1}-\tau\,\ell_{t+1}^{\nu})^{-\gamma} \tag{4.12}$$

– Arbitrage between consumption and leisure:

$$\tau\,\nu\,\ell_t^{\nu-1} = \alpha\frac{\hat{Q}_t}{\ell_t} \tag{4.13}$$

The core of the RBC model is therefore constituted of equations (4.1), (4.2), (4.4), (4.5), (4.8), (4.10), (4.11), (4.12), (4.13). In the remaining of this chapter, I refer to this part of the model as the *core model*.

4.2.2 Modelling the implied default risk

In parallel with the core model exposed above, I consider a *satellite model* whose purpose is to quantify the risk of default in the core model. This satellite model is described below.

As in the canonical endogenous default model I assume that, after a default, a penalty is imposed on the country in the form of a proportional cost to GDP and that the country remains in financial autarky for some time (with a probability x of reentering international financial markets at every period). As a consequence of the first cost, the post-default GDP is (in detrended form):

$$\hat{Q}_t = (1-\lambda)a_t\hat{K}_t^{1-\alpha}\ell_t^{\alpha} \tag{4.14}$$

where λ governs the magnitude of the default cost. The two productivity shocks a_t and Γ_t evolve according to the same law as in the RBC model. Moreover, the resource constraint therefore gets simplified to the following expression:

$$\hat{C}_t + g_t\hat{K}_{t+1} = \hat{Q}_t + (1-\kappa)\hat{K}_t - \frac{\phi}{2}\left(g_t\frac{\hat{K}_{t+1}}{\hat{K}_t}-\mu_g\right)^2\hat{K}_t \tag{4.15}$$

The value function of the country after a default is given by:

$$J^d(\hat{K}_t,a_t,g_t) = \max_{\hat{C}_t,\ell_t,\hat{K}_{t+1}}\left\{u(\hat{C}_t,\ell_t)+\beta\, g_t^{1-\gamma}\,\mathbb{E}_t\big[(1-x)\,J^d(\hat{K}_{t+1},a_{t+1},g_{t+1}) + x\,J^r(\hat{K}_{t+1},0,a_{t+1},g_{t+1})\big]\right\} \tag{4.16}$$

where x is the probability of reentering the international capital markets. Note that, for the sake of simplicity, I assume that the recovery value of debt is zero ($V = 0$).

The first order conditions are similar to those of the core model, except that of course there is no Euler equation with respect to debt:

- Arbitrage between consumption and leisure: same equation than (4.13).
- The Euler equation with respect to capital is essentially the same equation than (4.11). However, it should be noted that the expectancy term on the right hand side incorporates the probability of reentering the financial markets; this means that the control variables belonging to the information set of date $t+1$ (*i.e.* $\hat{C}_{t+1}$, ℓ_{t+1}, $\hat{Q}_{t+1}$, $\hat{K}_{t+2}$) are computed with probability $1-x$ using the policy functions of the satellite model, and with probability x using the policy functions of the core model. A more precise (but notationally heavier) way of writing this Euler equation is therefore:

$$
\begin{aligned}
&(\hat{C}_t - \tau\,\ell_t^{\nu})^{-\gamma}\left(1+\phi\left(g_t\frac{\hat{K}_{t+1}}{\hat{K}_t}-\mu_g\right)\right) = \\
&\beta\, g_t^{-\gamma}\,\mathbb{E}_t\Bigg\{(1-x)\,(\hat{C}_{t+1}-\tau\,\ell_{t+1}^{\nu})^{-\gamma}\left[1-\kappa+(1-\alpha)\frac{\hat{Q}_{t+1}}{\hat{K}_{t+1}}+\frac{\phi}{2}\left(\left(g_{t+1}\frac{\hat{K}_{t+2}}{\hat{K}_{t+1}}\right)^2-\mu_g^2\right)\right] \\
&+x\,(\tilde{C}(s_{t+1})-\tau\,\tilde{\ell}(s_{t+1})^{\nu})^{-\gamma}\left[1-\kappa+(1-\alpha)\frac{\tilde{Q}(s_{t+1})}{\hat{K}_{t+1}}+\frac{\phi}{2}\left(\left(g_{t+1}\frac{\tilde{K}'(s_{t+1})}{\hat{K}_{t+1}}\right)^2-\mu_g^2\right)\right]\Bigg\}
\end{aligned}
\tag{4.17}
$$

  where $\tilde{C}$, $\tilde{\ell}$, $\tilde{Q}$ and $\tilde{K}'$ are the policy functions of the core model and $s_{t+1} = (K_{t+1}, 0, a_{t+1}, g_{t+1})$ is the state of the economy tomorrow in case of redemption (note that debt is reset to zero).

The satellite model of default is therefore constituted of equations (4.14), (4.2), (4.4), (4.5), (4.15), (4.17), (4.13).

Note that the core model is self-contained and does not depend on the satellite model, because default is not endogenous. But the satellite model depends on the core one, because after a default there is a possibility for the country to reenter the market (*i.e.* redemption is endogenous to the second model). In technical terms, this dependency translates into the appearance of the policy functions of the core model in the satellite model.

The comparison of the value function of the core model J^r with that of the satellite model J^d delivers the implicit default probability of the SOE-RBC model, as I explain in more detail in the following section.

4.3 Calibration and benchmark results

I calibrate the model using the parameter values that Aguiar and Gopinath (2007) estimated for Mexico. For the additional parameters that come from the satellite model and which are related to default, I use the values from the canonical endogenous default model of section 1.3.1. Table 4.1 summarizes the calibration.

Table 4.1: Calibration of the SOE-RBC model

Parameter	Symbol	Value
Steady state growth rate of the stochastic trend	μ_g	1.006
Auto-correlation of the permanent productivity shock	ρ_y	0.72
Innovation variance of the permanent productivity shock	σ_y	1.09%
Auto-correlation of the transitory productivity shock	ρ_a	0.94
Innovation variance of the transitory productivity shock	σ_a	0.41%
Discount factor	β	0.98
Risk aversion	γ	2
Weight of disutility of labor	τ	1.4
Parameter governing elasticity of labor supply	ν	1.6
Share of labor in production	α	0.68
Depreciation rate of capital	κ	3%
Riskless interest rate	r	$\simeq$ 3.3%
Sensitivity of risk premium to debt level	ψ	0.001
Equilibrium debt level (% of quarterly GDP)	$\bar{d}$	10%
Loss of output in autarky (% of GDP)	λ	2%
Probability of settlement after default	x	10%

Quarterly frequency.

Note that r, β, γ, μ_g and the steady state value of the risk premium $\bar{\Delta}$ must satisfy the following relationship which is implied by (4.12):

$$\beta \mu_g^{-\gamma}(1 + r + \bar{\Delta}) = 1 \tag{4.18}$$

In the benchmark calibration I set $r = \mu_g^{-\gamma}/\beta - 1$, which implies $\bar{\Delta} = 0$ and therefore $\bar{\tilde{D}} = \bar{d}$ (according to (4.8)).

The model is solved as follows. First, I solve the core model, including the repayment value function J^r given by (4.9). This computation delivers a mean debt-to-GDP ratio and various business cycle statistics, but it does not give a default probability since there is no endogenous default in the core model. I also compute a simulation path of 10,000 periods for all the model variables. Then, as a second stage, I compute the default value function J^d from the satellite model as given by (4.16) (using the policy functions computed for the core model as an input). As a third stage, I compare the value of J^r and J^d on the 10,000 simulation points, and I compute the implied default probability (equal to the percentage of periods in which $J^d > J^r$); the result shows how often the country would default in this model if it were allowed to (since by very construction RBC models do not allow for default). From a computational point of view, rational expectation solutions of the core and satellite models are computed with a second order perturbation method using Dynare[2] on top of GNU Octave.[3]

The business cycle statistics of the model are well known and are documented by Aguiar and Gopinath (2007) among others: in particular the model is able to replicate a countercyclical current account and a consumption more volatile than output, two well known stylized facts of emerging countries. Table 4.2 reports the statistics that are of interest for the present exercise, *i.e.* those relating to debt and default.

Table 4.2: Debt statistics of the benchmark SOE-RBC model

Mean debt-to-GDP ratio (annualized)	−65%
Standard deviation of debt-to-GDP ratio (annualized)	47%
Default threshold as debt-to-GDP ratio (annualized)	4.5%
Implicit rate of default (per year)	0.13%

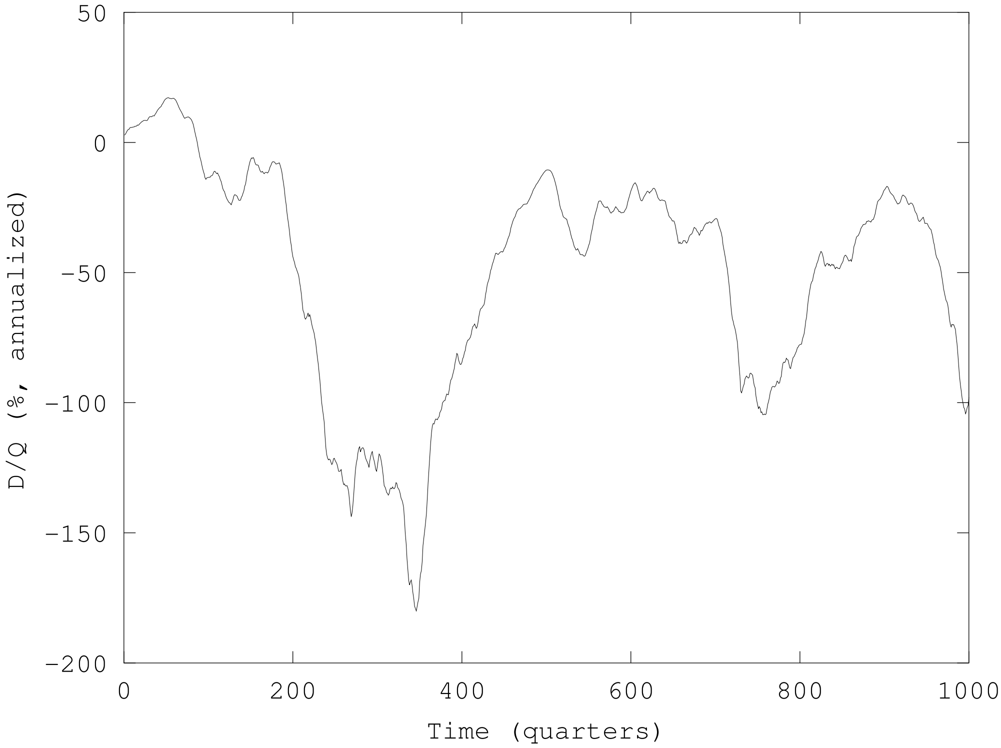
The model is solved using a second order approximation. The simulations results are theoretical moments for the debt-to-GDP ratio and an empirical moment over a simulation of 10,000 points for the rate of default (the 500 first observations are dropped to mitigate the effect of initial conditions). The default debt-to-GDP threshold is computed at mean productivity and capital levels.

A striking fact emerges from this computation and is not mentioned in the original paper by Aguiar and Gopinath (2007): the average debt-to-GDP ratio is actually highly negative. In other words, the representative agent builds up a high level of foreign assets (65% of its annual GDP) and as a logical consequence the implied probability of default is almost zero. Also note that the debt-to-GDP ratio is highly volatile: the one-standard-deviation interval goes from −112% to −18%. A sample simulation path of the debt-to-GDP ratio is reported in Figure 4.1; the ratio behaves pretty much like a random walk process, with ample and persistent movements.[4] The very rare default events occur when the debt-to-GDP goes sufficiently positive, above the default threshold (which is around 4.5% of annualized GDP): this is rare because the debt-to-GDP is highly negative on average, but this is still possible because the debt ratio is highly volatile.

This result is the consequence of two factors: the precautionary demand for savings and the low elasticity of interest rates to debt. Since the country is risk averse, it decides to consume less than it would in a world without uncertainty and therefore accumulates foreign assets.[5] And since the sensitivity of interest rates to the level of assets (embodied in the parameter ψ) is very low, it is not before the country has accumulated a high level of assets that its desire for safety is satisfied. The high volatility of foreign assets is also a direct

2. See http://www.dynare.org and Adjemian et al. (2011).
3. See http://www.octave.org and Eaton et al. (2008).
4. Actually it would strictly be a random walk if the parameter ψ was 0. Since $\psi = 0.001$, we are close to a random walk.
5. This is analog to the buffer-stock savings behavior exhibited by Carroll (1997). Indeed, the sovereign country fulfills the two reasons exhibited in that paper for engaging into a buffer-stock savings behavior: it is both "prudent" (it has a precautionary saving motive due to the risk aversion parameter) and "impatient" (in the sense that it would consume more than it produces—by taking debt at the $\bar{d}$ level—if there was no uncertainty in the model).

Figure 4.1: Sample simulation path of the debt-to-GDP ratio in the SOE-RBC model



The model is solved using a second order approximation. The simulation starts at the steady state level of debt-of-GDP, *i.e.* $\bar{d} = 2.5\%$ of annual GDP.

consequence of the low sensitivity of interest rates to assets.

In technical terms, the equality (4.18) that holds in a static equilibrium becomes an inequality in a dynamic context (because of the risk aversion and the Jensen inequality) so that one must have $\beta g_t^{-\gamma}(1 + r + \Delta_t) < 1$ on average; and because of the choice that was made for r, this implies that $\Delta_t < 0$ on average. The low elasticity of Δ_t with respect to D_t then implies that D_t will be highly negative and volatile.

It is interesting to note that if the model was solved using a first order approximation rather than a second order approximation, the results would radically different. The reason is well-known: in a first order approximation, the certainty equivalence property holds. The precautionary motive disappears, and the mean debt-to-GDP ratio is therefore equal to $\bar{d} =$ 2.5% of annual GDP, still with a high volatility (standard deviation of 47%). As a consequence, the probability of default is very high, around 84% in annual terms. This again shows that the method of resolution should be carefully chosen and that an imprecise method can lead to very different—and possibly blatantly wrong—results. [6]

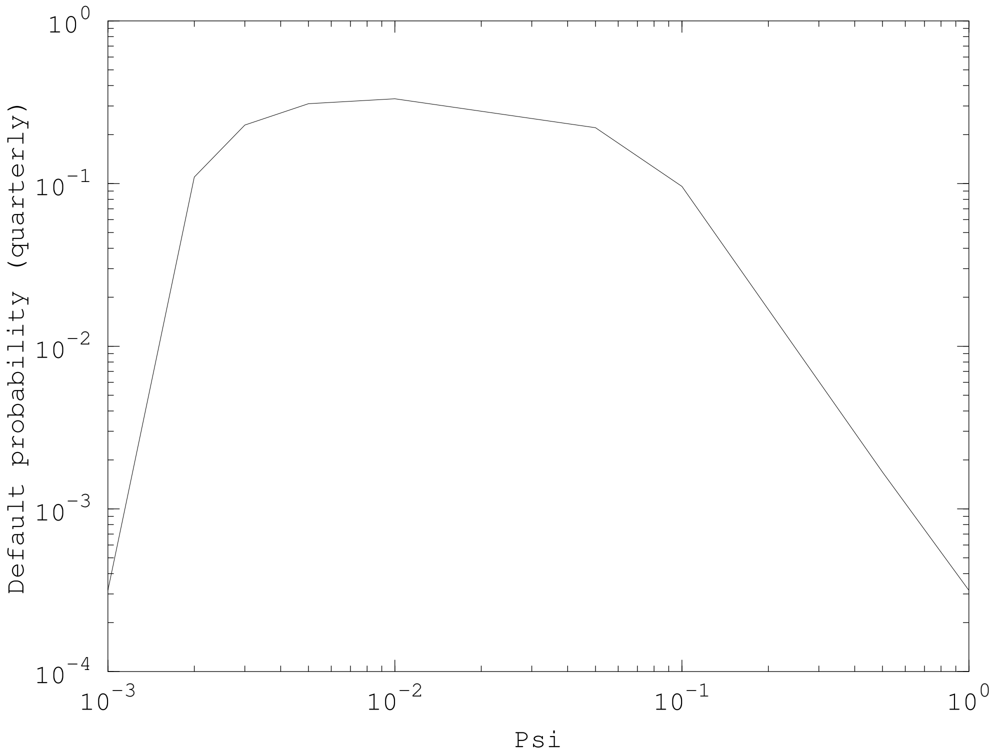
6. There is another way of looking at this issue, if the parameter $\bar{d}$ is reinterpreted differently. This parameter is usually interpreted as the steady state level of debt, but it can also be considered to be the mean debt level. When the model is solved at first order, the steady state and mean levels are the same, so both interpretations are equivalent. But at second order the two differ. Therefore, under the mean level interpretation, one could argue that the formula of the risk premium should be changed at the second order so that $\bar{d}$ remains the mean debt level. By construction, the first and second order solutions would then be much more similar than in the steady state interpretation used in the presented simulations.

In our case, a third order approximation gives almost the same results than the second order approximation, so it is clear that a second order approximation is enough to get good results. With confidence one can say that the true solution of the model exhibits a highly negative and volatile debt level and, as a consequence, no defaults.

4.4 Sensitivity analysis

As discussed in the previous section, the results obtained on the benchmark calibration of the model are driven by the low elasticity of the risk premium to the level of indebtment. One is therefore naturally inclined to analyze the sensitivity of these results to the parameter ψ governing this elasticity. Figure 4.2 summarizes the results of such a sensitivity exercise when ψ ranges from 0.001 (the benchmark value) to 1, while keeping other parameters constant.

Figure 4.2: Default probability as a function of the risk premium sensitivity ψ



The scale is logarithmic on both axes. All parameters except ψ are set to their benchmark values given in Table 4.1. The model is solved using a second order approximation. The simulations results are empirical moments over a simulation of 10,000 points (the 500 first observations are dropped to mitigate the effect of initial conditions).

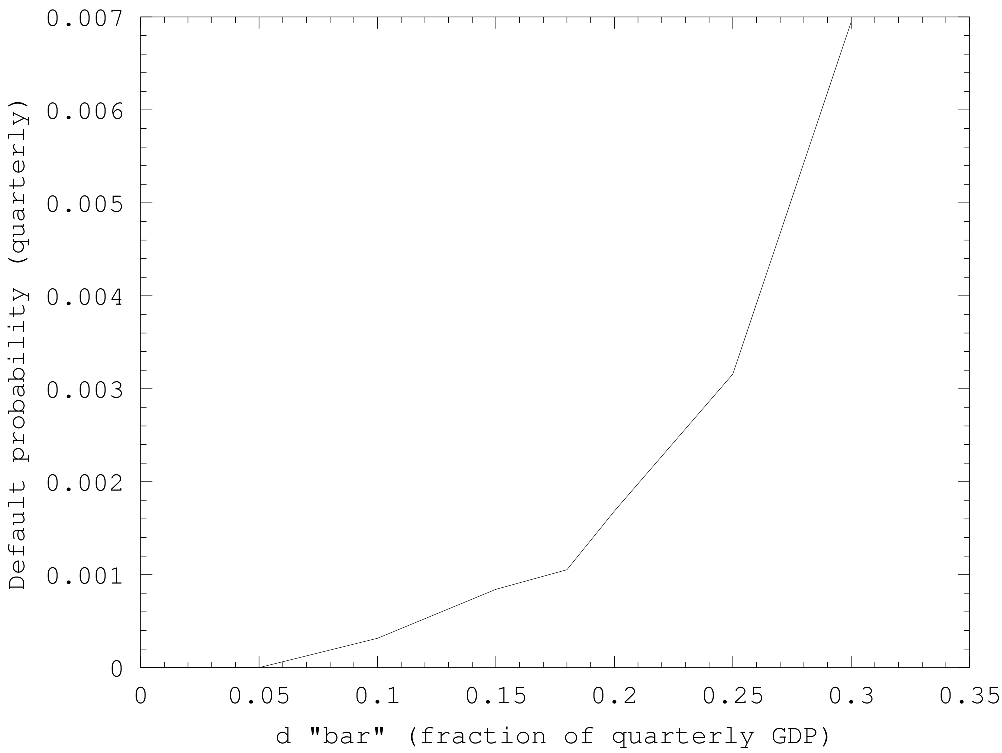
The first striking fact is that the default probability is extremely sensitive to this parameter, and approximately ranges from 0.0003 to 0.3 (in quarterly terms) over the range chosen for ψ: this means that by changing only this parameter, one can obtain an economy with virtually no default as well as an economy where default is extremely (and excessively) frequent. The other striking fact is the inverted U-shape of the graph.

The explanation of these two results is the following. In the left part of the graph, as ψ

increases, the mean debt-to-GDP ratio increases (because the higher risk premium diminishes the demand for precautionary savings), but at the same time the volatility of the debt-to-GDP ratio remains high (because the elasticity of the risk premium to debt level is still low). As a consequence, the frequency with which the debt-to-GDP ratio crosses the default threshold is greatly increased, and one therefore observes a dramatic increase of the default probability. In the right part of the graph, the mean debt-to-GDP ratio continues to increase (and converges towards $\bar{d}$) but at the same time the volatility of the debt-to-GDP ratio declines substantially. For very high values of ψ, the debt-to-GDP ratio becomes quasi-constant around the equilibrium value $\bar{d}$, which is smaller than the default threshold d^* given in Table 4.2 (and which is independent of the value of ψ); as a consequence, when ψ is very high, the default probability converges to zero since the country is always below the default threshold. It should be noted that if the parameters were instead such that $\bar{d} > d^*$, then the default probability would have converged to one as ψ increases, and the default probability as a function of ψ would have been a monotonically increasing function.

This observation naturally leads to the sensitivity analysis of the results to the equilibrium debt level $\bar{d}$. The results are shown in Figure 4.3.

Figure 4.3: Default probability as a function of the equilibrium debt level $\bar{d}$ with a low ψ

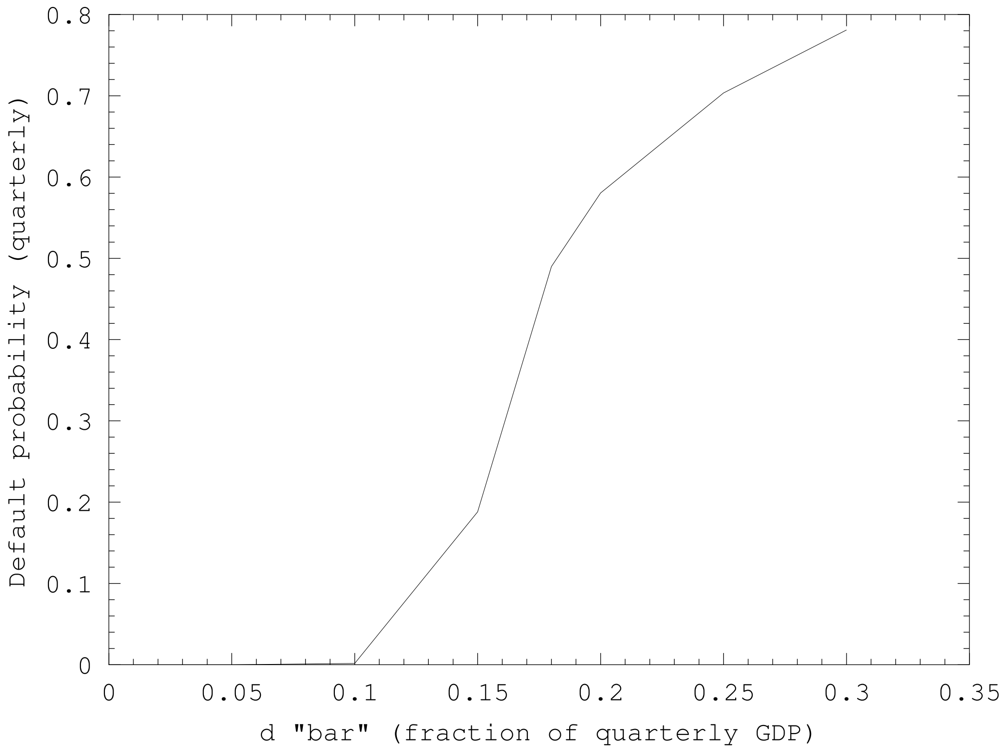


All parameters except $\bar{d}$ are set to their benchmark values given in Table 4.1. The simulation technique is the same as in Figure 4.2.

As expected, defaults are more frequent for higher values of the equilibrium debt-to-GDP level. But the probabilities of default remain very low, because the average simulated debt-to-GDP ratio (not shown) is still highly negative, and is only marginally affected by $\bar{d}$.

In order to get much action with respect to $\bar{d}$, one needs to use a higher value for the parameter governing the sensitivity of risk premium to debt level: as a consequence, the simulated mean debt-to-GDP ratio will remain close to the value given for the parameter $\bar{d}$, hence making this latter parameter much more influential over the result. The results are shown in Figure 4.4 where the value $\psi = 0.5$ is used.

Figure 4.4: Default probability as a function of the equilibrium debt level $\bar{d}$ with a high ψ



All parameters except $\bar{d}$ and $\psi = 0.5$ are set to their benchmark values given in Table 4.1. The simulation technique is the same as in Figure 4.2.

As expected, the results are very different from Figure 4.3, and everything depends on whether $\bar{d}$ is below or above the default threshold. With a value as low as $\bar{d} = 5\%$ of annual GDP (20% in quarterly terms), the quarterly default probability is as high as 60%, while it is almost 0 for $\bar{d} = 2.5\%$ of annual GDP (the benchmark value). This is as expected: with a high ψ—*i.e.* with a lowly volatile debt-to-GDP ratio—the model becomes completely dichotomic depending on whether the equilibrium debt-to-GDP exogenously imposed is above or below the default threshold. Note that in such a setup it is very difficult to get a realistic default probability (around 3% in annual terms as argued in section 2.2) because the model jumps very quickly from zero default to a highly unrealistic default probability.

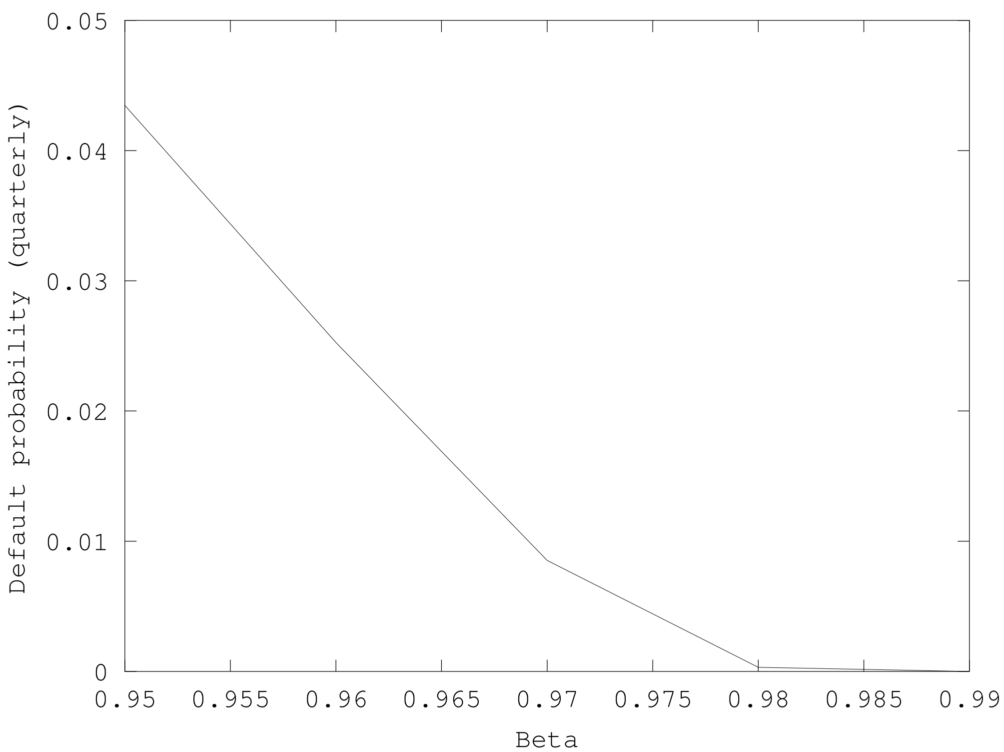
The two sensitivity analysis exercises performed on ψ and $\bar{d}$—which are the two parameters governing the risk premium function Δ_t—show how difficult it is to reach realistic levels of debt-to-GDP ratio and default probabilities within the class of SOE-RBC models. Either ψ is very low, so that the risk premium function plays only a marginal role on the dynamics of the model, and in that case the average debt-to-GDP ratio is highly negative and the default rate is consequently ridiculously low. Or ψ is set to a higher value, being a real force preventing the debt-to-GDP ratio from going far from its equilibrium level, but then the results are entirely driven by the parameter $\bar{d}$, and tend to exhibit polar cases (either almost zero or on the contrary excessively frequent defaults). In either case, the value chosen for these parameters are largely *ad hoc*, and the model is unable to endogenously deliver realistic business statistics concerning debt and default.

Note that changing the value of $\bar{d}$ is broadly equivalent—in terms of default probabilities—to changing the value of the default threshold d^*, which is itself governed by the parameters λ and x. It is therefore expected that the default probability is a decreasing function of λ and an increasing function of x.

As a last exercise, I present sensitivity results to the time discount factor β (in Figure 4.5) and for the risk aversion γ (in Figure 4.6). It should be noted that, when changing any of these two parameters, the riskless interest rate must consequently be adjusted in order to maintain the long term equilibrium relationship (4.18).

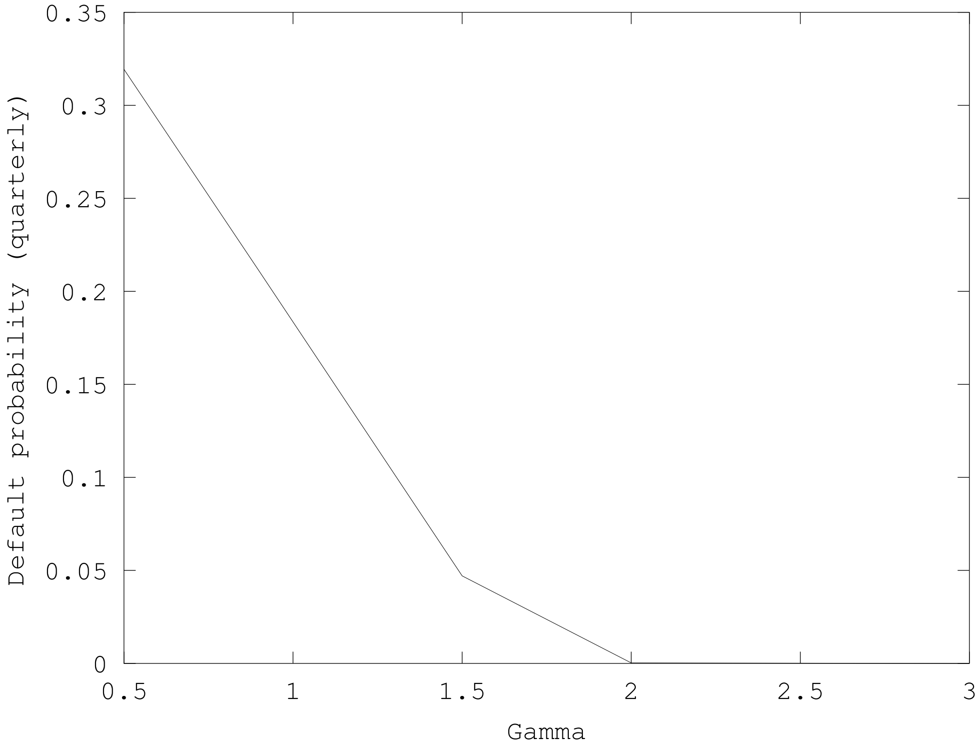
The results are as expected: the country defaults more as it is more impatient and less risk averse. Note that the risk aversion parameter has the potential of moving the default rate to very high values, by significantly diminishing the average level of foreign assets that the country accumulates.

Figure 4.5: Default probability as a function of the the discount factor β



All parameters except β are set to their benchmark values given in Table 4.1. The simulation technique is the same as in Figure 4.2.

Figure 4.6: Default probability as a function of risk aversion γ



All parameters except γ are set to their benchmark values given in Table 4.1. The simulation technique is the same as in Figure 4.2.

4.5 Conclusion

In this chapter, I have tried to analyze how small open economy real business cycle (SOE-RBC) models can be used to analyze the phenomenon of sovereign default. Since these models do not endogeneize default, they rely on some form of *ad hoc* risk premium on their external debt to sustain their equilibrium. Starting from an example of such a standard SOE-RBC model, I have augmented it with a satellite model aimed at measuring the risk of default in the core model. The result is a model where default can happen but whose possible occurrence is not internalized *ex ante* by the sovereign, contrarily to the canonical sovereign default model presented in section 1.3.1.

Two main conclusions can be drawn from this exercise. The first is that in a typical SOE-RBC with a standard calibration, the sovereign country is a *net creditor* to the rest of the world most of the time. This is the consequence of the precautionary motive and of the low sensitivity of interest rate to indebtment levels typically assumed by these models. This fact is not mentioned in the original paper by Aguiar and Gopinath (2007) (of which the model presented in this chapter is a variation). This fact casts some doubts on the authors' claims regarding the performance of their model, in particular when they argue that their model can correctly replicate the business cycle statistics of emerging countries with respect to interest rate spreads. Even if the model is technically able to produce counter-cyclical spreads for example, one can hardly argue that the result is economically meaningful since the country is actually a net creditor most of the time and should rather face a constant riskless rate.

The second result is that the model does not deliver—at least in simple variations of a benchmark calibration—realistic debt levels and default probabilities (as those presented in section 2.2). For the benchmark calibration of Aguiar and Gopinath (2007), the default probability is virtually zero, since the country is a net creditor most of the time. It is possible to invert this result by increasing the elasticity of the risk premium to the debt level, in which case the model tends to oscillate between two polar cases (either almost zero or on the contrary excessively frequent defaults) depending on the exogenously imposed equilibrium debt level. It is possible to reach realistic default probabilities by making the country more impatient, but this does not provides a realistic indebtment level since the country remains a net creditor on average.

At the end of this exercise it therefore seems clear that SOE-RBC models—though very useful for delivering key insights on the business cycle of emerging countries—do not seem immediately amendable in order to deliver a realistic behavior regarding sovereign debt statistics. Endogeneizing the default decision as in models à la Eaton and Gersovitz (1981) is not a characteristics implemented just for the sake of beauty or of model self-consistency: it critically affects the behavior of the model. A direction for further research on SOE-RBC models could therefore be to improve the risk premium function over the one that is adopted in this chapter (as well as in many papers in the literature), since it is clear that this risk premium function is the part of the model which drives the results regarding debt statistics.

Chapter 5

Accelerating the resolution of sovereign debt models using an endogenous grid method

5.1 Introduction

The choice of a numerical solution method is an important decision when studying a sovereign debt model, as it actually is for any other class of quantitative model. As shown by Hatchondo et al. (2010), an imprecise method can lead to significant numerical errors over the solution of sovereign debt models, up to the point where some of the main conclusions of innovative papers turn out to be just wrong. Having a precise enough method at one's disposal is therefore critical.

But the economist faces a trade-off: a more precise method usually requires a higher computing time, and is often (though not always) more difficult to implement. In the field of sovereign debt models, the speed-accuracy frontier of solution methods is particularly unfavorable compared to other classes of models (such as the family of RBC and DSGE models).[1] Indeed, RBC/DSGE models benefit from fast and advanced techniques based on first order conditions, while the sovereign debt models have been so far limited to the slower value function iteration procedure (hereafter referred to as VFI). The main reason for this situation is that sovereign debt models cannot be entirely specified in terms of first order conditions since the default decision involves a comparison between two value functions; therefore standard DSGE techniques do not apply and alternative techniques have to be designed.

In this chapter I present a new method for solving sovereign debt models which significantly improves the existing speed-accuracy frontier. This method is an adaptation to sovereign debt models of the *endogenous grid method* (hereafter referred to as EGM) introduced by Carroll (2006) and extended by Barillas and Fernández-Villaverde (2007). I call the

1. See for example Aruoba et al. (2006) and Kollmann et al. (2011) for overviews of recent solution techniques for RBC/DSGE models and for characterizations of the current speed-accuracy frontier.

new method the "doubly endogenous grid method" (hereafter referred to as 2EGM). As a second contribution, I explore the accuracy of solution methods (whether VFI or 2EGM) in a more systematic way than previously done in the literature on sovereign debt models, by applying tests based on the Euler errors. The main result of the present chapter is that both VFI and 2EGM are capable of delivering accurate solutions for the canonical sovereign debt model, but that 2EGM is much faster (by a factor of 5 to 10) than VFI for a comparable level of accuracy.

This chapter is organized as follows. Section 5.2 reviews the existing solution methods for sovereign debt models and briefly discusses their respective advantages. Section 5.3 presents the doubly endogenous grid method. Section 5.4 assesses the accuracy of both VFI and 2EGM on the canonical model presented in section 1.3.1. Section 5.5 presents a similar exercise on the "trembling times" model of section 2.4. Section 5.6 concludes. Appendix 5.7 gives some additional implementation details.

5.2 Solution methods: the state of the art

Solving a sovereign debt model, such as the one presented in section 1.3.1, consists in computing the value functions J^r and J^d and the credit supply function $\tilde{L}$. The value function J^* and the policy functions $\tilde{D}'$ and $\tilde{\delta}'$ are then trivial to deduce given the others.

Since in the general case these function do not have a closed form solution, the economist is only able to compute approximations of these functions, which I will denote by $\check{J}^r$, $\check{J}^d$ and $\check{L}$ (more generally, in the following, $\check{X}$ designates a numerical approximation of X).

I briefly describe the value function iteration (VFI) technique below:[2]

1. Define a finite grid of points $(D_i, Q_j)_{(i,j)\in\mathcal{I}\times\mathcal{J}}$ (where $\mathcal{I}$ and $\mathcal{J}$ are finite indexing sets), which will be used for interpolating $\check{J}^r$ and $\check{J}^d$.
2. Let n be the iterations counter and start with $n = 0$. Choose initial values $\check{J}^{r,(0)}$ and $\check{J}^{d,(0)}$ for the value functions (see section 5.7.2 for a discussion on the choice of the initial value). Let $\check{J}^{*,(0)} = \max\{\check{J}^{r,(0)}, \check{J}^{d,(0)}\}$.
3. At each point of the grid, compute the value functions $\check{J}^{r,(n+1)}$ and $\check{J}^{d,(n+1)}$ for the next iteration by solving equations (1.7) and (1.8) recursively:

$$\check{J}^{r,(n+1)}(D_i, Q_j) = \max_{D'} \left\{ u(Q_j - D_i + \check{L}^{(n)}(Q_j, D')) + \beta \int \check{J}^{*,(n)}(D', Q') \mathrm{d}\mathcal{F}(Q'|Q_j) \right\} \tag{5.1}$$

$$\check{J}^{d,(n+1)}(Q_j) = u((1-\lambda)Q_j) + \beta \int \left[(1-x)\check{J}^{d,(n)}(Q') + x\,\check{J}^{*,(n)}(0, Q') \right] \mathrm{d}\mathcal{F}(Q'|Q_j)$$

where $\mathcal{F}$ is the cumulative distribution function of tomorrow's output conditional to today's output.

2. A similar description can be found in the appendix of Hatchondo et al. (2010).

Note that this step involves the computation of an integral[3] and a function maximization. The credit supply function $\check{L}^{(n)}$ simultaneously needs to be computed, and this can be done using one of the two alternative ways that are described further below.

4. Let $\check{J}^{*,(n+1)} = \max\{\check{J}^{r,(n+1)}, \check{J}^{d,(n+1)}\}$. If $\check{J}^{*,(n+1)}$ is close enough to $\check{J}^{*,(n)}$ (up to some target accuracy level), then stop. Otherwise, set $n = n+1$ and go to step 3.

This algorithm has a nice intuitive interpretation: it is equivalent to solving the finite horizon problem whose number of periods is equal to the number of iterations in the algorithm. The solution for the infinite horizon problem is therefore approximated by the solution to a finite horizon problem with a large enough number of periods.

Moreover, this algorithm is known to converge because it closely replicates the constructive proof of the existence of an equilibrium (which itself relies on the well-known contraction fixed point theorem, see proof of proposition 1.2).

The most costly step in the whole solution procedure is the maximization involved at step 3; it is precisely this maximization that is no longer needed in the 2EGM procedure, as I will show below.

Existing solution techniques are all based on VFI, and differ mainly in two dimensions: whether they treat the state space as a discrete or a continuous set; and how they interpolate the value function outside of the grid that is used for approximation.

The most popular solution technique is the *discrete state space* (DSS) method, which consists in a complete discretization of the problem: the state space (D, Q) and the choice space D' are discretized, the law of motion of GDP is approximated by a discrete Markov chain; as a consequence the maximization problem (5.1) is a easy to solve since it only involves a finite number of choices. In step 3, the credit supply function $\check{L}$ is simply computed using the following approximation of equation (1.10):

$$\check{L}^{(n+1)}(Q_j, D'_i) = \frac{D'_i}{1+r} \sum_k \mathbb{1}_{\check{J}^{r,(n)}(D'_i, Q'_k) \geq \check{J}^{d,(n)}(Q'_k)} \mathbb{P}(Q'_k | Q_j)$$

where $\mathbb{P}(Q'_k | Q_j)$ is the transition probability of the discretized Markov chain. Note that DSS does not require any interpolation of the value functions outside the grid.

The DSS method is fast and easy to implement. But, as shown by Hatchondo et al. (2010), it is very imprecise unless a very fine grid (with thousands of points) is used.

The other solution technique based on VFI relies on *interpolation*, and treats the state space and the choice space as continuous. Between the points of the grid, the value functions are interpolated using well-know techniques such as Chebychev polynomials or cubic splines. The maximization in step 3 involves a costly nonlinear optimization. The credit supply function is approximated by:

$$\check{L}^{(n)}(Q, D') = \frac{D'}{1+r}(1 - \mathcal{F}(Q'^{*}|Q)) \tag{5.2}$$

3. This can be done using the quadrature techniques described for example in Judd (1998, chapter 7). See appendix 5.7.1 for details on the quadrature technique chosen in the implementation.

where $\mathcal{F}$ is the conditional cumulative distribution function of Q' given Q, and Q'^* is such that $\check{J}^{r,(n)}(D', Q'^*) = \check{J}^{d,(n)}(Q'^*)$ (note that a nonlinear solver is involved here).

The interpolation method is slower and more complicated to implement than DSS. But, as shown by Hatchondo et al. (2010), it is very precise, even when the grid involves a few dozens of points.

One of the main lessons from the comparison exercise between DSS and the interpolation methods performed by Hatchondo et al. (2010) is that the interpolation methods have a much better speed/accuracy ratio. In order to achieve the same accuracy than interpolation, DSS needs to be given so many discretization points that it becomes painfully slow and basically pointless. No one should therefore continue to use DSS, except for didactic or comparison exercises.

5.3 The "doubly endogenous grid method" (2EGM)

In this section I present a new solution method for the family of sovereign debt models in the tradition of Eaton and Gersovitz (1981) and Cohen and Sachs (1986).

This technique builds on the *endogenous grid method* (EGM), initially introduced by Carroll (2006) and extended by Barillas and Fernández-Villaverde (2007).

The basic idea of this method is the following: it consists in using a fixed grid for the control variable (here D'), instead of a fixed grid for the state variable (here D) as in VFI. For a given value of the control variable D', the value of the state variable D for which D' is the optimal choice is derived using the first-order condition of the maximization problem. The grid over D becomes endogenous, hence the name of the method.

This method is much faster because it does not rely on a maximization at every point of the grid for every iteration; first order conditions are used instead of the maximization problem. The dramatic gain of speed obtained with the EGM is illustrated by Barillas and Fernández-Villaverde (2007): on a standard neoclassical growth model without labor, EGM is faster than VFI by more than a factor of 11 (for a comparable accuracy level), and on the same model augmented with labor, EGM is faster by a factor of 56.

In the remainder of this section I describe how the EGM can be applied to the canonical model of sovereign debt presented in section 1.3.1. I choose to call "doubly endogenous grid method" (2EGM) the resulting algorithm, for reasons that will become clear below. Extensions of this method to other sovereign debt models is straightforward, and such an extension to the "trembling times" model of section 2.4 is studied in section 5.5.

The first step is to derive the Euler equation, *i.e.* the first order condition associated with the maximization problem (1.7):

$$u'(C_t)\frac{\partial \tilde{L}}{\partial D_{t+1}}(Q_t, D_{t+1}) = -\beta\,\mathbb{E}_t\frac{\partial J^*}{\partial D_{t+1}}(D_{t+1}, Q_{t+1}) \tag{5.3}$$

where $C_t = Q_t - D_t + \tilde{L}(Q_t, D_{t+1})$.

As is customary for an Euler equation, this equation says that if debt is increased by one unit, the marginal gain of utility today must equal the corresponding marginal loss in utility tomorrow.

I now turn to a description of the EGM applied to the canonical model of sovereign debt. Then I will explain why this does not work out of the box, and how to modify the algorithm in order to yield the functional 2EGM.

Let's first introduce the following function, which represents tomorrow's discounted expected utility:

$$\mathbb{J}(D', Q) = \beta \int J^*(D', Q') \mathrm{d}\mathcal{F}(Q'|Q) \tag{5.4}$$

Note that the right hand side of the Euler equation (5.3) is equal (up to the sign) to the derivative of $\mathbb{J}$ with respect to D', so that (5.3) can be rewritten in the following form:

$$C_t = u'^{-1}\left(-\frac{\frac{\partial \mathbb{J}}{\partial D_{t+1}}(D_{t+1}, Q_t)}{\frac{\partial \check{L}}{\partial D_{t+1}}(Q_t, D_{t+1})}\right) \tag{5.5}$$

Then the EGM is as follows:

1. Define a finite grid of points for *tomorrow*'s debt $(D'_i)_{i\in\mathcal{I}}$, and another one for today's output $(Q_j)_{j\in\mathcal{J}}$ (where $\mathcal{I}$ and $\mathcal{J}$ are finite indexing sets). These grids will remain constant during the iterations of the algorithm.
2. Set $n = 0$. Choose initial interpolation grids and initial values $\check{J}^{r,(0)}$ and $\check{J}^{d,(0)}$. The interpolation grid for $\check{J}^r$ will vary over the iterations, hence the name of the *endogenous grid* method. The initial grid is $\left(D^{(0)}_{ij}, Q_j\right)_{(i,j)\in\mathcal{I}\times\mathcal{J}}$ where $D^{(0)}_{ij} = D'_i$. Let $\check{J}^{*,(0)} = \max\{\check{J}^{r,(0)}, \check{J}^{d,(0)}\}$.
3. Construct an approximation $\check{\mathbb{J}}^{(n)}$ of the function $\mathbb{J}$. This is done by applying formula (5.4) at every point of the fixed grid (D'_i, Q_j), using a quadrature formula and the approximated function $\check{J}^{*,(n)} = \max\{\check{J}^{r,(n)}, \check{J}^{d,(n)}\}$. Interpolation is used outside the fixed grid.
4. Compute $\check{J}^{d,(n+1)}$ as you would in VFI:

$$\check{J}^{d,(n+1)}(Q_j) = u((1-\lambda)Q_j) + \beta \int \left[(1-x)\check{J}^{d,(n)}(Q') + x\,\check{J}^{*,(n)}(0, Q')\right] \mathrm{d}\mathcal{F}(Q'|Q_j) \tag{5.6}$$

5. The step for computing $\check{J}^{r,(n+1)}$ is as follows. For every point of the fixed grid (D'_i, Q_j), use the rewritten Euler equation (5.5) to find the level of today's consumption consistent with that choice for *tomorrow's* debt:

$$C^{(n+1)}_{ij} = u'^{-1}\left(-\frac{\frac{\partial \check{\mathbb{J}}^{(n)}}{\partial D'}(D'_i, Q_j)}{\frac{\partial \check{L}^{(n)}}{\partial D'}(Q_j, D'_i)}\right)$$

This step involves a nonlinear solver (for the computation of $\check{L}^{(n)}$ as in VFI) and two numerical differentiations (which are cheap), but *no maximization*.

The level of today's debt consistent with this level of consumption is computed immediately using the resource constraint (1.5):

$$D_{ij}^{(n+1)} = Q_j - C_{ij}^{(n+1)} - \check{L}^{(n)}(Q_j, D_i')$$

The function $\check{J}^{r,(n+1)}$ will be interpolated over the grid $\left(D_{ij}^{(n+1)}, Q_j\right)$, hence the name of the method: this grid is determined endogenously during the resolution of the model.

The value of the function at these points is simply:

$$\check{J}^{r,(n+1)}\left(D_{ij}^{(n+1)}, Q_j\right) = u\left(C_{ij}^{(n+1)}\right) + \check{\mathbb{J}}^{(n)}(D_i', Q_j) \tag{5.7}$$

6. Let $\check{J}^{*,(n+1)} = \max\{\check{J}^{r,(n+1)}, \check{J}^{d,(n+1)}\}$. If $\check{J}^{*,(n+1)}$ is close enough to $\check{J}^{*,(n)}$ (up to some target accuracy level), then stop. Otherwise, set $n = n+1$ and go to step 3.

This method is very appealing and intuitive despite its apparent complexity. It shares the core of the VFI method: choose an approximation grid and iterate backwards in time as in a finite-horizon model. The crucial difference is how the optimal decision rule for the level of debt is computed: in VFI, the algorithm deduces the choice for tomorrow's debt given today's debt; in EGM, it is the reverse: the algorithm takes as given a level for tomorrow's debt, and deduces the level of today's debt which is consistent with this choice. The analytical tools are therefore different between the two methods: VFI uses the maximization given by the Bellman equation (1.7) while EGM uses the first-order condition (5.3). And the latter happens to be much less computationally expensive to solve than the former, hence the dramatic gain in performance of EGM over VFI.

There is however a characteristic of the canonical sovereign debt model that makes the EGM fail if applied blindly. The problem comes from the fact that the choice function $\tilde{D}'(D, Q)$ for tomorrow's level of debt is very "flat," *i.e.* it takes a narrow range of values. I illustrate this in Figure 5.1: over the range of values for which the model is solved ($D \in [0, 0.3]$), the choice function D' takes its values in the interval $[0.14, 0.20]$, which is much narrower.

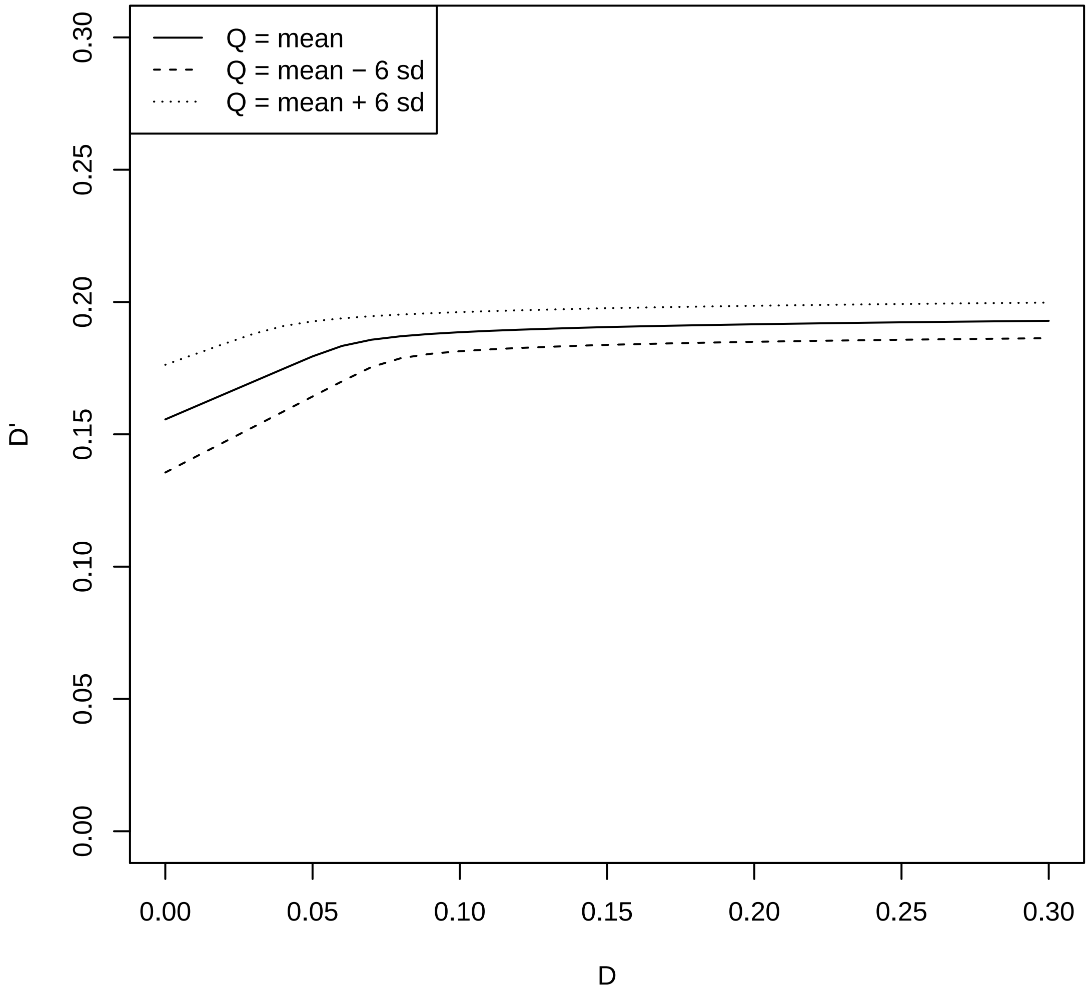
As a consequence, if the full range $[0, 0.3]$ is used for the fixed grid for D' (in step 1 of the EGM procedure), then the values deduced for D (in step 5) will be either very large or, worse, invalid (*e.g.* corresponding to a negative consumption). The algorithm will therefore fail.

A solution would be to use the interval $[0.14, 0.20]$ for the fixed grid over D', but this is not a practical solution since this range is precisely an output of the computation and cannot be guessed *ex ante* in the general case.

The solution that I suggest is to adapt the algorithm so that the grid over D' becomes also endogenous and converges towards the ergodic set as iterations run.[4] Hence the suggested

4. The *ergodic set* is the set of points of the state space that are reached in equilibrium. This is a probabilistic

Figure 5.1: Choice function for tomorrow's level of debt, given today's level



The choice function is plot for 3 values of Q which cover most of the ergodic distribution. The scales on horizontal and vertical axes are identical. *sd* stands for standard deviation.

name of "doubly endogenous grid method" (2EGM).

Here is the modified algorithm:

1. Define a finite grid of points for output $(Q_j)_{j\in\mathcal{J}}$ ($\mathcal{J}$ is a finite indexing set). This grid will remain constant during the iterations of the algorithm. Also define a finite indexing set $\mathcal{I}$ for debt values.
2. Set $n = 0$. Define a minimum debt value $\underline{D} = 0$ and a maximum debt value $\bar{D}^{(0)}$. The maximum debt value will be updated during iterations.
3. Choose initial interpolation grids and initial values for $\check{J}^{r,(0)}$ and $\check{J}^{d,(0)}$. The interpolation grid for $\check{J}^r$ will vary over the iterations. The initial grid is $\left(D_{ij}^{(0)}, Q_j\right)_{(i,j)\in\mathcal{I}\times\mathcal{J}}$ such that for a given j, the points $D_{ij}^{(0)}$ are evenly distributed in the range $[\underline{D}, \bar{D}^{(0)}]$. The functional of the initial values for the value functions are given in section 5.7.3. Let $\check{J}^{*,(0)} = \max\{\check{J}^{r,(0)}, \check{J}^{d,(0)}\}$.
4. Construct an approximation $\check{\mathbb{J}}^{(n)}$ of the function $\mathbb{J}$. First, generate a grid (D'_i, Q_j) where the D'_i are evenly distributed in the range $[\underline{D}, \bar{D}^{(n)}]$. Then apply formula (5.4) at every point of the grid , using a quadrature formula and the approximated function $\check{J}^{*,(n)} = \max\{\check{J}^{r,(n)}, \check{J}^{d,(n)}\}$. Interpolation is used outside the grid.
5. Compute $\check{J}^{d,(n+1)}$ as in (5.6).
6. The step for computing $\check{J}^{r,(n+1)}$ is as follows.
   - Consider the function f that maps tomorrow's debt to today's debt (using the Euler equation (5.3) and the resource constraint (1.5)):

   $$f^{(n)}(D', Q) = Q - u'^{-1}\left(-\frac{\frac{\partial \check{\mathbb{J}}^{(n)}}{\partial D'}(D', Q)}{\frac{\partial \check{L}^{(n)}}{\partial D'}(Q, D')}\right) - \check{L}^{(n)}(Q, D') \tag{5.8}$$

   Then for every $j \in \mathcal{J}$, use a dichotomy-based algorithm to compute:

   $$\bar{D}_j'^{(n)} = \max\left\{D' \mid f^{(n)}(D', Q_j) \in [\underline{D}, \bar{D}^{(n)}]\right\}$$
   $$\underline{D}_j'^{(n)} = \min\left\{D' \mid f^{(n)}(D', Q_j) \in [\underline{D}, \bar{D}^{(n)}]\right\}$$

   The grid for tomorrow's debt $D_{ij}'^{(n)}$ is computed by drawing points in the range $\left[\underline{D}_j'^{(n)}, \bar{D}_j'^{(n)}\right]$ (see section 5.7.3 for a discussion on the distribution of these points within the range).

   The idea here is to limit the grid for tomorrow's debt to points that are compatible with a level of today's debt lying in the range of interest.
   - The interpolation grid for $\check{J}^{r,(n+1)}$ is $\left(D_{ij}^{(n+1)}, Q_j\right)$, defined by $D_{ij}^{(n+1)} = f^{(n)}\left(D_{ij}'^{(n)}, Q_j\right)$
   - Compute the value of $\check{J}^{r,(n+1)}$ at interpolation points as in (5.7)

concept because potentially any point in the state space can be reached after a big enough shock. One should rather talk about the ergodic set *at a given probability level—e.g.* 99%: it is the subset of the state space where the model evolves at least 99% of the time in equilibrium.

7. Update the maximum debt value:

$$\bar{D}^{(n+1)} = \max_{j \in \mathcal{J}} \bar{D}_j'^{(n)} + o$$

where o is an offset (typically 1%).

The idea here is to limit the debt range to levels that are effectively chosen by the sovereign in equilibrium.

8. Let $\check{J}^{*,(n+1)} = \max\{\check{J}^{r,(n+1)}, \check{J}^{d,(n+1)}\}$. If $\check{J}^{*,(n+1)}$ is close enough to $\check{J}^{*,(n)}$ (up to some target accuracy level), then stop. Otherwise, set $n = n + 1$ and go to step 3.

The 2EGM algorithm is very flexible and robust because all the grids that it uses are endogenous and updated at every iteration. As a by-product, these grids will converge towards the ergodic set. This increases the efficiency of the algorithm: approximations of the decision and value functions are only computed on areas of interest in the state space. No resource is lost in computing functions in areas of the state space that are never reached in equilibrium.

The 2EGM algorithm is slightly more computationally expensive than the original EGM, because there is a need to compute minimum and maximum bounds for tomorrow's debt using a dichotomy-based algorithm. But this extra cost does not seem substantial, since it does not prevent 2EGM from being far more efficient than VFI, as I show in the next section.

5.4 Assessing solution methods with the canonical sovereign debt model

In this section I apply both VFI and 2EGM to the canonical model of section 1.3.1, and I assess their respective performance in the terms of speed, accuracy and complexity of implementation.

The two implementations have been written using the C++ programming language, and have been tested on a 8-cores computing workstation.[5] The implementations have been parallelized in order to exploit as much as possible of the power of all the CPU cores. Then the programs have been run both in single-threaded mode and in multi-threaded mode, in order to quantify the benefits of parallelization for both algorithms. More details on the implementations can be found in appendix 5.7.

In order to compare the accuracy of the two methods, I use two devices. The first one is simply the moments of the model: these are the most interesting objects from the point of view of the economist, and they were used in the comparison study of Hatchondo et al. (2010). But using only the moments to compare the accuracy of the two methods is hardly satisfactory. First, it is in theory possible that a highly inaccurate solution yields the right moments under consideration, while being wrong in other dimensions. Second, and more

5. The hardware characteristics of the workstation are the following: two Intel Xeon X5460 quad-core processors clocked at 3.16Ghz with 256kb of L1 cache and 12Mb of L2 cache, 8Gb of DDR2 RAM clocked at 667Mhz. The workstation is running Debian GNU/Linux.

fundamentally, the moments do not give an absolute measure of accuracy, since one does not know the true moments in the absence of a closed-form solution: the moments give an idea of the distance between two given solutions, but not their distance to the true solution.

I therefore choose to use an accuracy check based on Euler equation errors. This type of accuracy check has been first introduced in Judd (1992) and has since become the standard way to assess the accuracy of solutions to rational expectation models (Jin and Judd, 2002; Barillas and Fernández-Villaverde, 2007; Juillard and Villemot, 2011). The idea is simply to check to which extent the solution satisfies the first-order conditions of the model. On a simple RBC model, this amounts to verifying that the model satisfies the Euler equation, hence the name of the method.

The Euler equation of our canonical model can be expressed as follows (after substituting out the value function from (5.3)):

$$u'(C_t)\frac{\partial \tilde{L}}{\partial D_{t+1}}(Q_t, D_{t+1}) = \beta \int_{\delta_{t+1}=0} u'(C_{t+1})\mathrm{d}\mathcal{F}(Q_{t+1}|Q_t) \tag{5.9}$$

where δ_{t+1} is a dummy equal to 1 if the country defaults tomorrow, 0 if it repays.

Note that the expectation of tomorrow's utility is only computed over states of nature for which there is no default tomorrow. The technical reason is simply that $\frac{\partial J^d}{\partial D} = 0$. This is the Panglossian effect that I already discussed in section 3.3.3: the country rationally ignores *ex ante* the future states of the nature in which he will not repay.

The unit-free Euler error of a solution is then defined as the relative difference between the left and right hand-sides of (5.9). The policy functions of the solution are used for computing today's and tomorrow's control variables. The Euler error can therefore be expressed as:

$$\mathcal{R}(D,Q) = 1 - \frac{\beta \int_{\check{\delta}'(\check{D}'(D,Q),Q')=0} u'[\check{C}(\check{D}'(D,Q),Q')]\mathrm{d}\mathcal{F}(Q'|Q)}{u'[\check{C}(D,Q)]\frac{\partial \check{L}}{\partial D'}(Q,\check{D}'(D,Q))} \tag{5.10}$$

where $\check{D}'(D,Q)$ denotes the policy function for tomorrow's level of debt (conditionally to repayment), $\check{\delta}'(D,Q)$ the decision function for default, $\check{C}(D,Q)$ the consumption function in case of repayment and $\check{L}(Q,D')$ the credit supply function.

The main property of the Euler error is that it is equal to zero for the true solution of the model. As a consequence, the Euler error can be used as a loose metrics for measuring the distance of an approximated solution to the true solution. [6]

The Euler error defined in (5.10) is constructed for a given point in the state space, *i.e.* for a given pair (D,Q), and conditionally to repayment. In order to create an overall measure of the "distance" of an approximated solution to the true solution, one needs to compute Euler errors at several representative points of the state space, and then report an aggregate measure. For this purpose, I simulate a series of 10,000 points in the state space, starting from $D = 0$ and $y = 0$ (where y is defined in (1.3)), and at each period I recursively apply

6. Note however that the Euler error cannot be used to construct a distance (in the topological sense) to the true solution. Such a construction is not possible in the generic case.

the policy functions. Then at each point of the simulation for which the country decides to repay, I compute the Euler error, and I report the mean and maximum of the absolute values of these errors across the simulation; this gives an idea of both the average and worst case performance of the solution. Note that no error is computed for the periods at which the country is in financial autarky (since there is no Euler equation in that case), but this is not an issue since no real computational challenge is involved there: no optimization is performed, the country simply behaves hand-to-mouth.

Table 5.1 shows the results of the comparison of VFI and 2EGM over the canonical model. Column (1) corresponds to VFI and column (2) to 2EGM using the same number of grid points than for VFI. Column (3) reports calculations done by Hatchondo et al. (2010) on the same model: their results should be comparable to column (1) since they use the same method and the same grid.

I first discuss the relative accuracy of the two algorithms. When used on the same grid and with the same convergence criterion, VFI and 2EGM are of comparable accuracy, as can be seen from both the Euler errors and the simulated moments. The Euler errors are of comparable magnitude (2EGM being slightly less precise, but by a tiny margin). The simulated moments are the same across the two methods up to the second decimal.

Concerning speed, I report two computing times: the first one when only a single thread (*i.e.* a single processor) is used, the second when 8 threads (or processor cores) are used simultaneously. Comparing single- and multi-thread computing times give an idea of how parallelizable both algorithms are: this information is critical since it is to be expected that future technology improvements in computers will be in terms of number of cores rather than in speed of individual cores (as was the rule in the past).

The results show that 2EGM is much faster than VFI. With a single thread, 2EGM is almost 10 times faster. With 8 threads, 2EGM is still much faster, but the ratio is reduced to 5. This suggests that VFI benefits more from parallelization than 2EGM, at least in the way I implemented both algorithms.

It should also be noted that the computing time that I report for VFI is 21 times smaller than what Hatchondo et al. (2010) achieved. This is to some extent the consequence of better hardware, but it is probably also the consequence of a better implementation. Given that I have made a very efficient implementation of VFI, my results showing the superiority of 2EGM can therefore not be considered as biased against VFI because of a poor implementation of the latter. If there is any such bias, it more likely plays in the other direction, since in implementing 2EGM I could not take advantage of the experience gained from pre-existing implementations (which were nonexistent).

The last thing to note is that VFI and 2EGM have more or less the same degree of complexity when it comes to programming the algorithms: the two algorithms are implemented using about one thousand of single lines of code.[7]

7. As reported by the SLOCCount program by David A. Wheeler, see `http://www.dwheeler.com/sloccount/sloccount.html`.

Table 5.1: Comparison of VFI and 2EGM on canonical model

	(1)	(2)	(3)
Solution characteristics			
Method	VFI	2EGM	VFI
Grid points for Q	15	15	15
Grid points for D	30	30	30
Convergence criterion (in $\log_{10}$ units)	-6	-6	
Lines of C++ code	1,000	1,080	
Solution time			
Single thread	54.4s	5.8s	1182s
8 threads	15.9s	3.1s	
Moments			
Rate of default (%, per year)	0.86	0.86	0.88
Mean debt output ratio (%, annualized)	4.68	4.68	4.75
$\sigma(Q)$ (%)	4.40	4.40	4.43
$\sigma(C)$ (%)	4.64	4.64	4.68
$\sigma(TB/Q)$ (%)	0.92	0.92	0.94
$\sigma(\Delta)$ (%)	0.06	0.06	0.07
$\rho(C, Q)$	0.98	0.98	0.98
$\rho(TB/Q, Q)$	-0.18	-0.18	-0.18
$\rho(\Delta, Q)$	0.05	0.05	0.09
$\rho(\Delta, TB/Q)$	0.53	0.53	0.52
Euler errors (in $\log_{10}$ *units)*			
Mean	-4.38	-4.20	
Max	-3.47	-3.39	

The model and calibration are those of section 1.3.1. Columns (1) and (2) report calculations by the author, while column (3) report calculations by Hatchondo et al. (2010). The convergence criterion is the maximum difference tolerated between value functions of two consecutive iterations when convergence is achieved. Moments are obtained by averaging over 500 simulated series of 1,500 points each, of which the first 1,000 are discarded. Q is GDP, C is consumption, TB/Q is trade balance over GDP, Δ is spread. GDP, consumption, trade balance and spread are detrended with an HP filter of parameter 1600. Euler errors are computed according to (5.10), and I report the mean and maximum errors over a path of 10,000 points simulated using the computed solution.

5.5 Assessing solution methods with the "trembling times" model

In this section I apply the 2EGM algorithm to the model of sovereign default with "trembling times" (developed in section 2.4), and I compare the accuracy and speed of this algorithm with traditional VFI.

This model is computationally more challenging than the canonical model because it has a state space of dimension three (instead of two): in addition to the level of debt D and to the Brownian component of the growth process y, this model adds a Poisson component to the growth process z.[8]

Similarly to the previous section, I report in Table 5.2 a comparison across several dimensions of VFI and 2EGM on the "trembling times" model.

Compared to the solution of the canonical model presented in the previous section, I chose a coarser grid along the dimensions for D and y (only 10×10 points here, while I used 30×15 in the previous model). Since I used also 10 points for z, the total number of points is 1,000 here (against 450 in the previous model).

Also, I had to use a relatively big convergence criterion for both VFI and 2EGM, but for different reasons. In the VFI case, the algorithm does not stop when the criterion is set to a tighter value (*i.e.* in that case the criterion set in step 4 of section 5.2 is never met). In the 2EGM case, the algorithm converges for lower values of the criterion (*e.g.* 10^{-5}), but the extra iterations do not improve the quality of the solution (at least from the angle of the Euler error), and are therefore a loss of computing time. The conclusion that can be drawn from these considerations is that, for the chosen grid, the results shown here correspond to the best outcome that both algorithms can deliver.

The resulting solutions appear to be much less accurate than the one obtained for the canonical model in the previous section. Where the average relative Euler error was about 10^{-4}, it is now 100 times bigger, around 1%. Also note that the average error of the 2EGM algorithm is smaller than that of VFI, by almost 20%. Concerning the maximum error, both algorithms perform very poorly: over a 10,000 periods simulation, the maximum error is about 10% for VFI and 34% for 2EGM. While these figures are important, they concern only a very small number of simulation points: for example, along the simulation path for 2EGM, only 0.05% of the points exhibit an Euler error greater than 10% (which is the maximum error for VFI). These points can therefore be considered as outliers and, given that the on average 2EGM performs better than VFI by a significant margin, one can reasonably conclude that the two solutions shown here are of comparable accuracy.

In terms of simulated moments, the two solution techniques deliver results which are qualitatively similar, and quantitatively close albeit slightly different. The largest discrepancy comes from the probability of default: VFI gives a probability of 1.23%, when 2EGM gives a result of 2.54% which is about two times bigger. This discrepancy likely comes from the fact the probability of default is highly sensitive to the value of the parameter q around the

8. There is also a fourth state variable tracking whether we are in normal or trembling times, but since it can take only two values it does not increase the dimensionality of the problem.

Table 5.2: Comparison of VFI and 2EGM on "trembling times" model

	(1)	(2)
Solution characteristics		
Method	VFI	2EGM
Grid points for y	10	10
Grid points for z	10	10
Grid points for D	10	10
Convergence criterion (in $\log_{10}$ units)	-1.7	-3.0
Lines of C++ code	1,423	1,525
Solution time		
Single thread	3,588s	413s
8 threads	1,396s	195s
Moments		
Rate of default (%, per year)	1.24	2.50
Mean debt output ratio (%, annualized)	38.58	38.17
$\sigma(Q)$ (%)	4.45	4.45
$\sigma(C)$ (%)	6.47	6.04
$\sigma(TB/Q)$ (%)	3.11	2.63
$\sigma(\Delta)$ (%)	0.40	0.57
$\rho(C, Q)$	0.90	0.92
$\rho(TB/Q, Q)$	-0.44	-0.41
$\rho(\Delta, Q)$	-0.47	-0.60
$\rho(\Delta, TB/Q)$	0.79	0.64
Euler errors (in $\log_{10}$ *units)*		
Mean	-1.99	-2.08
Max	-0.98	-0.46

The model and calibration are those of section 2.4.4. The convergence criterion is the maximum difference tolerated between value functions of two consecutive iterations when convergence is achieved. Moments are obtained by averaging over 500 simulated series of 1,500 points each, of which the first 1,000 are discarded. Q is GDP, C is consumption, TB/Q is trade balance over GDP, Δ is spread. GDP, consumption, trade balance and spread are detrended with an HP filter of parameter 1600. Euler errors are computed over a path of 10,000 points simulated using the computed solution.

value that was chosen for it (5%), as shown in Figure 2.2. For lower or higher values of q, the two solution methods give similar results (*i.e.* almost no default if q is large or, at the other extreme, a default frequency close to p if q is small).

Since the hardware and the programming techniques are the same between the implementations of both the canonical and the "trembling times" models, the computing times are directly comparable. The ratio is between 62:1 and 88:1 depending on the algorithm and the number of threads chosen. This is a striking illustration of the curse of dimensionality: the choice of a coarser grid and of a looser convergence criterion does not prevent the computation time from exploding when only a single dimension is added, and in addition the result is a much less precise solution.

In terms of computing time, the comparison between VFI and 2EGM delivers a similar picture than the one obtained in the previous section: 2EGM is faster than VFI by a factor of 8.5 when there is a single thread, and by a factor of almost 7 when there are 8 threads.

Again, the programming complexity is roughly the same for both algorithms: about 1,500 lines of code. Note also that this is a 50% increase compared to the canonical model.

5.6 Conclusion

Building on earlier work by Carroll (2006) and Barillas and Fernández-Villaverde (2007) on the endogenous grid method, I have presented in this chapter a new solution method for sovereign debt models à la Eaton and Gersovitz (1981). This technique is easy to implement and significantly improves the speed-accuracy frontier compared to pre-existing techniques based on value function iteration (VFI). For a similar accuracy, this *doubly endogenous grid method* (2EGM) is faster than VFI by a factor lying between 5 and 10. These properties have been verified on a simple sovereign debt model such as the one presented in section 1.3.1, and on the more complex model with 3 state variables and 2 stochastic shocks presented in section 2.4.

Having a fast and accurate algorithm such as the 2EGM opens several interesting possibilities. One is to make easier the study of bigger sovereign debt models than the ones currently found in the literature; I already did so in this thesis by analyzing the "trembling times" model which has more state variables and stochastic shocks than any model of the related literature to date. Future models at the juncture between the RBC/DSGE and the endogenous default traditions, in the spirit of Mendoza and Yue (2012), will likely feature a state space of even higher dimension and could also benefit from the 2EGM technique. Another possibility worth exploring is the estimation of sovereign debt models with bayesian techniques: since such an estimation necessitates to solve the model a great number of times at different points in the parameter space, the 2EGM could be of great help in such an endeavour.

5.7 Appendix: Implementation details

In this section, I document several details of the implementation of the two algorithms compared in this chapter.

5.7.1 Implementation details common to VFI and 2EGM

The programs used in this chapter are written in the C++ language and compiled using the GNU C++ compiler.[9] Numerical computations are done using double-precision floating points numbers (IEEE 754), and the implementation makes a heavy use of various routines provided by the GNU Scientific Library.[10] Parallelization of algorithms is achieved using OpenMP directives.[11]

Value functions and decision rules are interpolated outside the grid using cubic splines (interpolation is done both *between* grid points and *outside* the convex envelope of the grid). Interpolations occurs sequentially along each dimension in the following order: first along the debt dimension D (except for value functions in case of default), then along the Brownian component of growth y, and finally along the Poisson component of growth z (only for the "trembling times" model). The interpolation engine is designed to be able to cope with values of $-\infty$ in some parts of the state space (in that case, interpolation is done using only the finite values): this feature is particularly helpful when exploring points of the state space for which, in case of repayment, the markets provide no level of lending compatible with a positive consumption (*i.e.* points for which $J^r = -\infty$).

Numerical integration over the Gaussian distribution is achieved using a Gauss-Legendre quadrature (the distribution is truncated below and above 4 standard deviations) using 16 points for the canonical model and 10 points for the "trembling times" model (see Abramowitz and Stegun, 1964, p. 887, eq. 25.4.29).

The credit supply function given in (5.2) is computed using "Brent's method": it is a well-known nonlinear solver combining an interpolation strategy and the bisection algorithm.

5.7.2 Implementation details specific to VFI

The maximization of equation (5.1) is done in two steps:

- first, a global search is performed by computing the objective over a pre-defined grid of points over tomorrow's debt D';
- the result of the global search is used as a starting point for the "Brent minimization algorithm," which combines a parabolic interpolation with the golden section algorithm.

9. See http://www.gnu.org/software/gcc.
10. See http://www.gnu.org/software/gsl/ or Galassi et al. (2003).
11. See http://www.openmp.org.

Concerning the initial values, a natural candidate is the continuation value at the last period of the finite horizon version of the model:

$$\begin{aligned} \check{J}^{r,(0)}(D_i, Q_j) &= u(Q_j - D_i) \\ \check{J}^{d,(0)}(Q_j) &= u((1-\lambda)Q_j) \end{aligned} \tag{5.11}$$

However, if the domain over which the problem is solved includes points for which $Q - D < 0$, then this particular initial value for J^r cannot be used (because u is only defined for positive values). In a quarterly model, this problem appears as soon as the (annualized) debt-to-GDP ratio is of 25%: since many countries (both emerging and developed) have bigger ratios, this is certainly a problem which one wants to circumvent.

The solution that I adopt is to use the following alternative initial value:

$$\check{J}^{r,(0)}(D_i, Q_j) = u(Q_j - r\, D_i) \tag{5.12}$$

Since r is typically about 1%, this functional form is compatible with realistic debt-to-GDP ratios. It also has an economic interpretation: it corresponds to the utility that the country gets if it keeps its indebtment at a constant level.

5.7.3 Implementation details specific to 2EGM

Initial values

The initial values for J^r and J^d that I presented in the previous section for VFI do not work for 2EGM. The fundamental problem with (5.12) is that its derivative with respect to D is far from the true derivative of J^r (which is close to one), and the 2EGM relies on this derivative in order to converge. The form in (5.11) has a better derivative, but is still not applicable for the same reasons than those exposed in the previous section.

In the end, I used the following form which solves both problems:

$$\check{J}^{r,(0)}\left(D_{ij}^{(0)}, Q_j\right) = a \cdot u\left(Q_j - \frac{D_{ij}^{(0)}}{a}\right)$$

where a is a constant set to 3 for the canonical model and 10 for the "trembling times" model.

Also, I have observed that the algorithm only converges if the initial values are such that they imply no default ever, so I chose a value for $\check{J}^{d,(0)}$ which is always smaller than $\check{J}^{r,(0)}$:

$$\check{J}^{d,(0)}(Q_j) = a \cdot u\left(Q_j - \frac{\bar{D}^{(0)}}{a}\right)$$

Generation of the endogenous grid

In step 6 (p. 118) of the 2EGM algorithm, the grid for tomorrow's debt level is endogenously generated (this is precisely the step that I added relatively to the original EGM).

The first step is to compute the boundaries $\bar{D}_j'^{(n)}$ and $\underline{D}_j'^{(n)}$ of the range of interest. This is done with a dichotomy-based algorithm: for computing the lower bound $\underline{D}_j'^{(n)}$, I start with the hypothesis that this bound is located in a wide range, then at each iteration I cut the interval in two by keeping the right half (guessing which half to keep is done by applying the function (5.8) to the point at the middle of the interval); the same algorithm is used to find the upper bound $\bar{D}_j'^{(n)}$.

Once the bounds have been computed, the second step is to draw the points $D_{ij}'^{(n)}$ in this interval. The natural way of doing this would be to draw evenly distributed points in the interval, but this is highly inefficient and leads to poor accuracy results, because the ergodic set is located in the neighborhood of the upper bound $\bar{D}_j'^{(n)}$ (as shown on figure 5.1). The optimal solution is to put more points where the country spends the more time, using a cubic formula:

$$D_{ij}'^{(n)} = \bar{D}_j'^{(n)} - \left(\frac{i}{|\mathcal{I}|-1}\right)^3 \left(\bar{D}_j'^{(n)} - \underline{D}_j'^{(n)}\right)$$

(assuming that the indexing set $\mathcal{I}$ consists of integers from 0 to $|\mathcal{I}|-1$).

Refinement iterations

Since iterations in 2EGM are based on the Euler equation (5.3), one would expect 2EGM to deliver very small Euler errors *by construction*. But things are not that simple, because Euler errors are computed using the expectancy of tomorrow's marginal utility (see (5.9)), while the formula used in 2EGM iterations is based on the derivative of the value function with respect to D (see (5.5)). In the true solution, these two are equal. But unfortunately, in the solution delivered by 2EGM, the discrepancy between the marginal utility of consumption and the derivative of the value function is not negligible and can lead to substantial Euler errors.

Hopefully there is an easy way to improve that situation. After the convergence of the 2EGM algorithm, I run a few extra iterations where I use a modified version of equation (5.8): I naturally replace the derivative of the value function by the marginal utility of consumption (based on the policy function computed at the previous iteration). On the canonical model, this has a very small computational cost (only one extra iteration is necessary), but it dramatically improves the accuracy of the solution. Without this extra refinement iteration, 2EGM would have performed much worse than VFI. Unfortunately, I was not able to run this refinement step on the "trembling times" model: instead of improving the convergence, it leads to a divergence. A better understanding of this issue could be the subject of further research.

Chapter 6

Conclusion

In this thesis I have tried to contribute to a better understanding of the mechanisms at work behind sovereign default. The results that I have presented rely on the theoretical foundations laid by Eaton and Gersovitz (1981) and Cohen and Sachs (1986) who studied the strategic behavior of a sovereign country facing the option to default on its external debt. I also built on the quantitative sovereign debt literature initiated by Aguiar and Gopinath (2006) and Arellano (2008) who have shown that such models are well equipped to match several empirical regularities specific to small open emerging countries.

My first contribution is to suggest a solution to what can be called the "sovereign default puzzle:" the tendency of most quantitative sovereign default models to predict default at very low debt-to-GDP thresholds, in clear contradiction with what is observed in the data (many countries in the emerging and advanced world live with ratios of 50%—and of more than 100% for some of them—without having any difficulty to fulfill their obligations). The solution that I have presented to this puzzle relies on two main ideas: first, the observation that countries generally do not want to default but are rather forced into it by negative market anticipations which have real consequences; second, the understanding that large and discontinuous shocks are necessary to generate defaults, and that on the contrary smooth and continuous shocks can always be mitigated by the country. The resulting model is able to sustain both a high level of debt and a realistic default probability, while maintaining the good business cycle properties of previous quantitative models. During the course of this study, it also appeared that a critical parameter is the speed at which the country exits out of a market-generated confidence crisis: the faster the country reacts, the lower the default probability will be from an *ex ante* perspective. This result has clear policy implications: it underlines the need for fast and coordinated decision processes and also for "financial emergency plans" prepared in such a way to quickly extinguish bursting crises.

As another contribution, I have established a typology of the debt crises that can arise in the above model. Crises can be broadly classified in three categories: those crises that are the consequence of an exogenous shock to the country's fundamentals; those that are of the nature of a *self-fulfilling* prophecy; and those *self-enforcing* crises that are the consequence of a rational tendency to over-borrow when the risk of an exogenous negative shock becomes high.

After having theoretically characterized these three types of crises, I have tried to quantify their relative importance in the data. It appears that the proportion of self-fulfilling and self-enforcing crises is about 10% in each case. This is clearly not negligible and these kinds of crises therefore deserve to be taken seriously. Yet, this means that the vast majority of crises are of the exogenously-driven type; this validates the hypotheses made when constructing the quantitative model mentioned above.

I have also studied how sovereign default could be understood and then incorporated into real business cycle models of small open economies. In these models, default is typically never explicitly modelled, and some *ad hoc* interest rate risk premium function has to be postulated. I have therefore extended the benchmark SOE-RBC model into a model where default can happen but whose possible occurrence is not internalized *ex ante* by the sovereign. The main conclusion of this exercise is that default so defined is either inexistent or very frequent, depending on the choice of the parameter values. At any rate, it seems impossible to analyze default in the context of pure RBC models, and the approach undertaken for example by Mendoza and Yue (2012) who merge elements of a RBC model into an endogenous default model à la Eaton and Gersovitz (1981) definitely seems the right way to go.

Finally, I have made a methodological contribution by presenting a new computational method for solving endogenous default models, using a variation on the endogenous grid method introduced by Carroll (2006). This method turns out to be clearly superior to the traditional value function methods in the space of speed-accuracy trade-offs. It is also quite simple to implement. This solution method therefore has the potential to make possible new empirical investigations such as the estimation of endogenous debt models via Bayesian methods or the resolution of models with a higher number of state variables.

Even if I hope these contributions help at arriving at a better understanding of the determinants of sovereign debt crises, many questions still need to be answered before economists can claim to have arrived at a full understanding of the issues at hand. A first open question, which is actually a real puzzle, is why *contingent* debt instruments are not more widely used. It is clear that debt instruments whose repayments are indexed to some fundamental variable affecting the ability to repay would diminish the risk of default. In the extreme case, if markets were complete, defaults would never happen. Of course, complete markets are only a theoretical object and cannot be achieved in the real world. But, still, partially contingent instruments are more efficient—they help prevent socially costly disorderly defaults—and it is not yet adequately understood why their use is so rare.

In order to arrive at a better understanding of the ongoing sovereign debt crisis which is currently unfolding in Europe, modelling challenges are also numerous.

One interesting research direction would be to study the impact of currency denomination within the context of endogenous default models. As far as I know, the literature has so far focused exclusively on models where debt is expressed in real terms and where therefore monetary and exchange rate policies play no role. Models of nominal sovereign debt could be useful for studying at least two situations. The first one is the case of emerging or low-income countries who borrow abroad but in local currency unit (LCU), instead of borrowing in foreign currency as is usually the case. Such loans, while still being the exception, have been recently on the rise. An integrated framework incorporating the monetary policy reaction function of the sovereign country would be helpful to better understand the issues at hand. The second case study is given by a country member of a currency union (like Greece in the eurozone), who may be tempted to default on its debt while at the same time being forced to leave the currency union. This problem has been studied from the angle of fiscal limits (Daniel and Shiamptanis, 2008) but not from the angle of the willingness to pay which is the core of endogenous default models.

Another interesting research direction would be to incorporate fiscal policy rules in sovereign debt models. This would in particular imply to distinguish between publicly- and privately-issued sovereign debt, since both contribute to the current account. One could even allow for the government to take responsibility for privately-issued debt in the wake of crisis, as as been observed during the recent global financial crisis. Such a framework could provide a useful tool for analyzing long term fiscal sustainability while maintaining the endogeneity of sovereign default. Some elementary fiscal rules have already been studied in the form of debt ceilings (see section 2.5.2 or Martinez et al., 2012), but much more work is needed in order to incorporate more realistic fiscal setups including taxes and government expenditure.

Since the addition of these elements implies the modelling of the production sector, this effort would be part of the broader effort—initiated by Mendoza and Yue (2012)—of merging elements of the DSGE paradigm into endogenous default models. As already argued in section 1.3.4, the construction of endogenous default models which incorporate features borrowed from DSGE models seems the natural way forward for the quantitative sovereign debt literature. The range of questions that could be addressed within such a class of models would be very large, and this would be particularly relevant in the context of the sovereign debt crisis that we are now facing. One of the main obstacles to the development of such models is computational: being more complex, they would inevitable have more state variables, and since existing solution methods (including the 2EGM that I presented in chapter 5) are vulnerable to the so-called *curse of dimensionality*, these models could quickly become intractable. A solution to this problem could reside in an improvement of the existing algorithms (both VFI and 2EGM) using sparse grid methods Malin et al. (2011), which are much less vulnerable to the curse of dimensionality.

Résumé substantiel en Français

Introduction

Cette thèse s'intéresse à la problématique de la dette souveraine, définie ici comme la dette émise par un État souverain et détenue par des créanciers étrangers (publics et/ou privés). Plus précisément, ce travail porte sur les crises de dette souveraine, c'est-à-dire sur les situations où le débiteur souverain ne respecte pas l'intégralité de ses engagements initiaux vis-à-vis de ses créanciers, que ce soit du point de vue des montants dûs ou des délais de paiements. Cette définition des crises inclut bien entendu les décisions unilatérales de refus de paiement, mais également les accords négociés de rééchelonnement ou de réduction de dette (par exemple dans le cadre du Club de Paris ou de Club de Londres).

Bien que la question de la dette souveraine soit particulièrement d'actualité au vu de l'endettement public important de la zone Euro et des États-Unis d'Amérique, il ne s'agit nullement d'une question nouvelle. Le premier défaut souverain identifié dans l'histoire remonte à la période de la Grèce antique, et l'époque moderne a été ponctuée de nombreux épisodes de défaut ; la période récente a notamment été marquée par la vague de défauts en Amérique Latine pendant les années 1980, suivie des crises mexicaine et russe durant les années 1990 puis de la crise argentine au début du 21$^{\text{ème}}$ siècle.

Parmi les évolutions récentes, on retiendra notamment que l'endettement externe moyen, calculé sur l'ensemble des pays du globe, a augmenté entre le début des années 1970 et le milieu des années 1990, pour ensuite redescendre sous l'effet notamment de l'initiative en faveur des pays pauvres très endettés (PPTE/HIPC) et du développement des marchés domestiques dans les pays émergents. Le profil des créditeurs a également évolué entre 1970 et 2009 : les pays les plus pauvres continuent d'être fortement dépendants de créanciers publics institutionnels, tandis que les pays émergents ont clairement réussi à séduire les créanciers privés. Le type d'instruments a également évolué, avec le développement des prêts syndiqués pendant les années 1970 au détriment des obligations d'État ; les années 2000 ont quant à elles vu le développement de clauses d'action collective pour diminuer les problèmes de coordination entre créditeurs.

Dans ce contexte, la littérature s'est attachée depuis le début des années 1980 à mieux comprendre la problématique de la dette souveraine. De nombreuses questions ont été explorées : quels sont les coûts d'un défaut souverain (pour le débiteur comme pour les créanciers) ? Quels sont les facteurs qui déclenchent les défauts ? Pourquoi les pays s'endettent-ils auprès de l'étranger ? Comment se fait-il qu'un équilibre avec une dette strictement positive soit soutenable, alors que les créanciers disposent de très peu de moyens coercitifs pour récupérer leur mise, et que les débiteurs semblent donc avoir peu d'incitations à rembourser ? Certaines crises sont-elles le résultat de prophéties auto-réalisatrices ou, autrement dit, existe-t-il des équilibres multiples ? Peut-on reproduire qualitativement et quantitativement les principaux faits stylisés relatifs à la dette souveraine en utilisant des modèles en équilibre général ? Quelles pistes pour diminuer la fréquence et le coût des crises ?

De nombreux éléments de réponse ont été apportés par la littérature. Le chapitre 1 de cette thèse propose ainsi un aperçu des contributions les plus importantes sur ces questions. Les chapitres suivants présentent ensuite des contributions originales qui participent à la résolution de certaines de ces questions. Dans le chapitre 2, je propose une solution au problème suivant : plupart des modèles de dette souveraine prédisent le défaut pour des valeurs très faibles du ratio dette sur PIB, en contradiction avec ce qui est observé dans les données ; en partant de l'observation que les pays ne souhaitent généralement pas faire défaut mais y sont forcés par les marchés, je présente un modèle qui peut reproduire quantitativement les principaux faits stylisés concernant le risque souverain. Dans le chapitre 3, je propose une typologie des crises de dette en trois catégories : les crises qui sont la conséquence d'un choc exogène, celles qui sont des prophéties *auto-réalisatrices*, et les crises *auto-imposées* qui sont la conséquence d'une tendance rationnelle au surendettement lorsque le risque d'un choc négatif est élevé ; la proportion de crises auto-réalisatrices et auto-imposées dans les données est estimée à environ 10% pour chacune de ces deux catégories. Dans le chapitre 4, j'étudie comment le défaut souverain peut se comprendre dans les modèles de cycles réels en petite économie ouverte. Il ressort que ces modèles oscillent entre deux cas polaires : le défaut y est soit inexistant soit trop fréquent ; ces modèles sont donc peu adaptés à l'étude du risque de défaut, risque qui doit être endogénéisé pour obtenir des résultats utiles. Enfin, dans le chapitre 5, je propose une contribution méthodologique en présentant une nouvelle méthode de résolution des modèles de défaut souverain endogène ; cette méthode améliore significativement la frontière vitesse/précision pré-existante.

Enjeux de la modélisation du défaut souverain et leçons pour l'Europe

L'Europe a récemment été frappée par une crise de la dette qui a conduit trois de ses membres à être expulsés des marchés financiers. Ces trois pays — la Grèce, l'Irlande et le Portugal — ont été contraints de demander l'aide des autres pays de la zone Euro pour refinancer leur dette. En outre, dans le cas de la Grèce, une décote nominale de plus de 50% a été entérinée. En réponse à cette crise inattendue, l'Europe a décidé de s'imposer une discipline budgétaire beaucoup très stricte, avec l'objectif d'atteindre des déficits publics quasiment nuls. Comment se fait-il que l'Europe soit devenue soudainement si vulnérable au risque souverain ? N'est-elle pas en train de surréagir en s'imposant des contraintes budgétaires trop restrictives ?

Les spécialistes des crises souveraines ont été interrogés. Comprendre pourquoi certains pays font défaut est en effet le thème d'une littérature abondante, comme mentionné ci-dessus. En particulier, Reinhart et al. (2003) ont introduit le concept de "défaillants en série", et la Grèce est certainement l'un d'entre eux, ayant fait défaut de nombreuses fois au cours des deux siècles précédents. Le paradoxe principal de la littérature sur le défaut souverain est néanmoins qu'il est très difficile de modéliser simultanément des probabilités de défaut et des niveaux d'endettement en accord avec les données. Par exemple, les travaux de Aguiar and Gopinath (2006) ou Arellano (2008) sont confrontés au fait qu'un niveau de dette sur PIB de seulement 5% suffit à déclencher un défaut pour des calibrations raisonnables des modèles. Ces articles obtiennent des résultats presque caricaturaux car ils prédisent un risque de défaut pour quasiment n'importe quel niveau d'endettement strictement positif.

Ces difficultés ont conduit Rogoff (2011) à considérer que l'approche descriptive des phénomènes de défaut conduisait à une meilleure compréhension du problème que les modèles calibrés et simulés (voir aussi Reinhart and Rogoff, 2009). Néanmoins, il s'agit clairement d'une position excessive. À part dans le cadre d'un modèle calibré, comment peut-on raisonner sur le niveau de dette approprié ? Plus globablement, comment peut-on alors comprendre la tentative des dirigeants européens d'imposer des plafonds de dette en vue d'éviter une autre crise ?

Dans la plupart des modèles existants, basés sur les travaux de Eaton and Gersovitz (1981), le défaut est une décision coûteuse que le pays compare avec l'alternative de rembourser sa dette. Du point de vue du modélisateur, l'arbitrage suivant apparaît : soit le coût du défaut est élevé, auquel cas un ratio dette sur PIB élevé peut être obtenu, mais au prix d'une fréquence de défaut faible, puisque les pays ne font pas défaut quand cela a un coût élevé ; soit inversement le coût du défaut est faible, auquel cas la fréquence de défaut correspond à celle observée dans les données, mais le niveau de dette soutenable devient anormalement bas. C'est dans cette dernière situation que se trouvent la plupart des modèles calibrés aujourd'hui.

Dans le chapitre 2, je reconsidère les modèles de défaut existants et je propose des modifications qui leur permettent de correspondre mieux aux données.

La première contribution tire son inspiration de la théorie des processus stochastiques de Lévy. Pour simplifier, ces processus sont la généralisation en temps continu des marches aléatoires du temps discret. Plus précisément, tout processus stochastique en temps continu avec des incréments stationnaires et indépendants est un processus de Lévy. La décomposition de Lévy-Itô établit que tout processus de Lévy est essentiellement la somme de deux composantes : un processus Brownien et un processus de Poisson composé. Je démontre ainsi que dans un modèle de défaut souverain où le PIB suit un processus de Lévy, seule la composante Poisson est susceptible d'engendrer des défauts si on se rapproche asymptotiquement du temps continu. À l'inverse, un processus Brownien n'a pas la capacité d'engendrer des défauts, car son fonctionnement est analogue à celui d'un modèle purement déterministe :

quel que soit le coût du défaut, la probabilité que le pays décide de ne pas rembourser est nulle, car il est toujours préférable de s'ajuster en continu face à des chocs infinitésimaux, plutôt que prendre une décision qui a un coût non-infinitésimal. Le défaut ne peut donc être déclenché que par des chocs exogènes qui engendrent des sauts discontinus dans la richesse du pays. De tels chocs sont bien représentés par les processus de Poisson.

La deuxième modification consiste à augmenter le coût d'un défaut par rapport à ce qui est habituellement supposé dans la littérature quantitative. La plupart des articles font l'hypothèse qu'à la suite d'un défaut, le pays est exclu des marchés financiers internationaux pour une durée moyenne de deux ans et demi. À cette pénalité, je rajoute une autre hypothèse de modélisation : à la suite du défaut, la dette du pays n'est pas annulée mais simplement ramenée à un niveau plus soutenable. En effet, comme l'ont documenté Sturzenegger and Zettelmeyer (2007) et Cruces and Trebesch (2011), les créditeurs subissent en moyenne une perte de 40% lors d'un défaut, ce qui est certes substantiel mais néanmoins éloigné d'une annulation totale. Le fait d'intégrer cet aspect à la modélisation permet d'augmenter significativement le niveau de dette soutenable. Cet ingrédient de modélisation est mentionné à plusieurs reprises dans la littérature, mais il a encore été peu exploité.

La troisième contribution provient de l'observation suivante, dont la situation grecque est une illustration : les pays ne veulent généralement pas faire défaut de façon unilatérale. En effet, comme le montrent l'Inter-American Development Bank (2007) et Levy-Yeyati and Panizza (2011), dans tous les cas de défaut (sauf un), la « décision » de faire défaut n'a pas été réellement prise par le pays : elle s'est imposée après que la crise s'est déclenchée. Le seul cas de « défaut stratégique » est celui de l'Équateur en 2009. Cette observation conduit naturellement à une nouvelle hypothèse de modélisation. Dans le modèle que je présente, la séquence des événements est inversée : la crise commence avant que la décision de défaut n'ait été prise. Il faut imaginer une panique bancaire ou un effondrement temporaire d'un secteur industriel clef. En ces « périodes de tremblement », le coût d'un défaut devient plus faible puisque la panique financière ou le choc économique ont déjà produit leurs effets. Le défaut ajoute des coûts supplémentaires, mais ceux-ci sont plus faibles que ceux qui auraient été encourus en « période normale ».

Une fois que les trois ingrédients mentionnés ci-dessus ont été intégrés dans un modèle de défaut souverain, il devient possible de reproduire à la fois des niveaux d'endettement élevés et des fréquences de défaut élevées. Avec la calibration de référence, le modèle prédit ainsi une fréquence de défaut annuelle de 2.5% et un ratio dette sur PIB (annuel) moyen de 38% : ces deux valeurs sont tout à fait réalistes au regard des moyennes historiques. Il est possible d'obtenir des ratios dette sur PIB encore plus élevés en jouant sur certains paramètres du modèle. À noter également que ces résultats sont obtenus sans pour autant perdre les bonnes propriétés des modèles de défaut souverain en matière de régularités du cycle économique ; le modèle prédit bien des taux d'intérêt et un compte courant contracycliques, en accord avec les données.

De cette analyse je tire plusieurs enseignements pour la zone Euro. Le premier est relatif aux plafonds d'endettement que les dirigeants de la zone cherchent à mettre en place. Si le modèle présenté ci-dessus est le bon, alors il apparaît clairement qu'il est contre-productif de mettre en place un plafond d'endettement unique et invariant dans le temps. Au contraire, le modèle fait apparaître clairement qu'il faut un plafond différent (plus élevé) pour les périodes de crises par opposition aux périodes normales. Un plafond unique serait économiquement inefficace. Un second aspect concerne le paramètre du modèle qui gouverne la vitesse à laquelle le pays sort de la « période de tremblement » : le risque de défaut apparaît comme inversement corrélé à ce paramètre. Dans le contexte européen, ce résultat peut se réinterpréter de la façon suivante : en étant incapable de se mettre rapidement d'accord sur une solution concertée à la situation grecque, les dirigeants européens ont probablement donné l'impression qu'ils avaient une faible capacité à sortir de cette « période de tremblement ». Ce faisant, ils ont augmenté le risque de défaut dans la zone. Il est donc peut-être plus important de rassurer les investisseurs sur la capacité de la zone euro à prendre les décisions qui s'imposent pour étouffer rapidement des crises naissantes, plutôt que de s'imposer des plafonds de dette trop contraignants.

Crises de dette endogènes

La motivation du chapitre 3 est la suivante : alors que les crises de dette souveraine sont si coûteuses pour le pays défaillant, pourquoi observe-t-on autant de pays qui tombent dans ce piège ? Ne devrait-on pas s'attendre à un comportement plus prudent de leur part ? La réponse théorique exposée dans ce chapitre est cependant plus nuancée.

Considérons la forme la plus simple d'une crise financière déclenchée par un choc totalement exogène. La prime de risque correspondante sur les titres de dette est élevée parce que le pays est considéré comme vulnérable à un événement hors de son contrôle, tel un tremblement de terre ou un choc persistent sur le prix des matières premières qu'il exporte. On pourrait s'attendre à ce que le pays se comporte alors avec plus de prudence ; en effet, plus la dette qu'il s'engage à repayer est élevée, plus le coût du tremblement de terre devient élevé relativement à l'état favorable de la nature. Mais d'un autre côté, si le tremblement de terre attendu est si violent que le pays anticipe qu'il fera défaut sur sa dette en ce cas, alors un « comportement panglossien » (ainsi que Krugman l'a nommé) peut devenir rationnel : la dette perdant toute valeur après le tremblement de terre, il devient alors absurde de ne pas emprunter plus avant coup. Le pays se comporte alors comme si le risque d'un choc défavorable pouvait être ignoré. Tout comme le Docteur Pangloss du *Candide* de Voltaire, le pays se comporte comme si « le meilleur des mondes possibles » devait se manifester. Dans ce cas, la dette engendre de la dette de façon endogène ; nous qualifions les crises qui en découlent d'*auto-imposées*.

Considérons maintenant le cas d'une crise causée par la défiance des marchés financiers à l'égard d'un pays, rendant ainsi le pays financièrement fragile par un effet auto-réalisateur. La littérature s'est attachée à étudier les crises de dette auto-réalisatrices sous différentes formes, décrites rapidement ci-dessous.

Dans le modèle de Cole and Kehoe (1996, 2000), les crises auto-réalisatrices sont une variante des crises de liquidité : un manque de coordination entre les créditeurs peut ainsi conduire un pays solvable vers le défaut. De telles crises peuvent être introduites dans un modèle canonique de dette souveraine en permettant un comportement stratégique des investisseurs internationaux, qui vont ainsi rendre leur décision de prêt conditionnelle à la décision des autres investisseurs. Cependant, comme l'a montré Chamon (2007), de telles crises de coordination peuvent être aisément évitées si les créditeurs peuvent proposer des prêts contingents à la manière de ceux organisés par les capital-risqueurs : si individuellement les créditeurs proposent des lignes de crédit conditionnellement au fait que les autres créditeurs font de même, alors les crises de liquidité ne peuvent plus se produire.

Les crises auto-réalisatrices ont également été analysées comme la conséquence d'un effet « boule de neige » : l'accumulation de dette jusqu'à un niveau insupportable est causée par la peur même que la dette devienne insoutenable (Calvo, 1988). En partant d'une intuition développée dans un modèle simple par Cohen and Portes (2006), je montre dans ce chapitre que les effets « boule de neige » ne peuvent apparaître que lorsque la crise de dette a le potentiel de nuire aux *fondamentaux* du pays endetté. Si une crise réduit le PIB d'un pays de 10%, il est clair que la défiance envers un pays peut dégénérer en une crise auto-réalisatrice. Si au contraire les fondamentaux sont inchangés par la crise, alors il est possible de montrer que les crises auto-réalisatrices à la Calvo sont (théoriquement) impossibles.

Cette observation conduit à la caractérisation suivante des crises auto-réalisatrices, sur laquelle je me concentre dans ce chapitre : ce sont les crises qui sont le produit d'une fragilisation *endogène* des fondamentaux du pays. Autrement dit, ce sont les crises qui entraînent une réduction du PIB du pays, par le biais des différentes perturbations que peut causer une défiance des marchés (fuite des capitaux, crise de taux de change. . .). Dans le cas d'une crise exogène (type « tremblement de terre »), c'est la séquence inverse qui prévaut : les fondamentaux sont d'abord détruits, puis la crise financière se produit.

Ces différentes idées sont étudiées dans un modèle théorique, qui permet de mettre en évidence et de comprendre l'effet Panglossien. Je montre dans le cas général que les crises auto-réalisatrices de type « boule de neige » ne peuvent se produire que lorsque la crise a un potentiel de destruction endogène des fondamentaux. Je développe également une typologie des situations au regard du risque de crise. En dessous d'un certain niveau d'endettement, le pays a tendance à se comporter prudemment et à réduire sa dette lorsqu'un choc négatif se produit. Au-delà d'un certain seuil d'endettement, typiquement suite à une série de chocs négatifs répétés, le pays va commencer à se comporter de façon panglossienne, en ignorant (rationnellement) les mauvaises nouvelles et en augmentant ainsi son endettement autant que les marchés l'y autorisent. Une crise peut alors se produire soit à cause d'un choc exogène, soit de façon auto-réalisatrice si cette crise a un potentiel de destruction endogène de la base de remboursement.

Les données historiques sur les crises souveraines sont ensuite analysées à l'aune de cet éclairage théorique. J'exploite une version légèrement modifiée de l'échantillon de données compilé par Kraay and Nehru (2006), que j'ai enrichi pour y inclure toutes les crises antérieures à 2004. De façon cohérente avec les travaux de ces deux auteurs, il apparaît que la probabilité d'une crise est bien expliquée par trois facteurs : le ratio dette sur PIB, le niveau de richesse par tête, ainsi qu'une mesure de la sur-évaluation du taux de change.

Afin d'estimer la prévalence des crises auto-réalisatrices, le modèle économétrique incorpore une loi d'évolution du ratio dette sur PIB en temps normal différente de celle qui s'applique en temps de crise. Une crise auto-réalisatrice se définit alors comme une crise qui n'aurait pas eu lieu si le ratio dette sur PIB avait suivi la trajectoire d'avant crise. Les crises auto-réalisatrices ainsi définies s'avèrent représenter une minorité des cas historiques. En moyenne, entre 6% et 12% des crises (selon la méthodologie) apparaissent de nature auto-réalisatrice. Cette proportion n'est néanmoins pas négligeable et le phénomène mérite donc d'être pris au sérieux.

L'intensité de l'effet panglossien est également calibrée. L'influence de ce mécanisme sur l'accumulation de dette est testée par le biais de simulations Monte-Carlo. L'effet panglossien s'avère substantiel et concerne environ 12% des cas de crises.

Il apparaît donc que la majorité des crises (plus de 75%) est donc la conséquence d'un choc exogène. Cela montre qu'il existe une possibilité d'améliorer la stabilité des marchés financiers en utilisant plus d'instruments contingents à l'état de la nature (tels que des contrats de dette indexés sur le prix des matières premières exportées par le pays). Comprendre pourquoi si peu de contrats d'endettement comportent des clauses contingentes reste une question ouverte et un sujet pour des recherches futures.

Le défaut souverain dans les modèles de cycles réels

Les analyses décrites ci-dessus sont basées sur des modèles de défaut endogène dans la filiation du travail de Eaton and Gersovitz (1981). Comme expliqué en introduction, ces modèles ont eu un succès mitigé lorsqu'il s'est agi de répliquer des niveaux de dette et des probabilités de défaut réalistes (et le chapitre 2 a justement pour objectif de proposer une solution à ce problème). Il est cependant nécessaire de préciser que l'objectif initial de la littérature quantitative sur la dette souveraine — initiée par Arellano (2008) et Aguiar and Gopinath (2006) — n'était pas de reproduire quantitativement les faits sur la dette et le défaut, mais plutôt de répliquer les faits stylisés du cycle des affaires dans les pays émergents. De ce point de vue, ces modèles ont largement atteint leur objectif, comme expliqué plus haut.

Il existe un autre courant de la littérature qui a également essayé de répliquer les faits stylisés du cycle des affaires dans les pays émergents, en utilisant des modèles de cycle réel (RBC) et plus récemment des modèles dynamiques et stochastiques d'équilibre général (DSGE). Cette littérature, a été initiée par Mendoza (1991) qui a examiné un modèle de petite économie ouverte (SOE) basé sur le paradigme RBC. Plus récemment, Uribe and Yue (2006) et Neumeyer and Perri (2005) ont analysé des modèles RBC calibrés pour des petites économies ouvertes, et ont obtenu des résultats assez convaincants au sujet de l'interaction entre les *spreads* souverains et le cycle des affaires de ces pays.

Hormis l'exception notable fournie par Mendoza and Yue (2012), ces deux courants de la littérature (SOE-RBC d'une part et modèles endogènes à la Eaton and Gersovitz (1981) d'autre part) ont évolué de façon indépendante en s'ignorant l'un l'autre. Chaque paradigme a ses propres forces et faiblesses : les modèles de défaut endogène font l'hypothèse d'un processus de PIB purement exogène, tandis que les modèles SOE-RBC font l'hypothèse d'un processus plus réaliste basé sur l'accumulation de capital et des décisions d'offre de travail ; mais les modèles SOE-RBC sont incapables d'endogénéiser la décision de défaut et sont donc contraints de s'appuyer sur des formulations relativement *ad hoc* pour incorporer le *spread* sur les taux d'intérêt des titres souverains.

Il faut ici remarquer que les modèles SOE-RBC ne sont pas auto-cohérents, du moins en apparence : d'un côté ils n'autorisent pas le défaut puisque, par construction même, ils font l'hypothèse que le pays rembourse toujours ses dettes ; de l'autre côté, ils incorporent une prime de risque strictement positive sous une forme ou une autre (généralement exprimée en fonction de certains fondamentaux macroéconomiques). C'est une contradiction : comme le modèle fait l'hypothèse qu'il n'y a jamais de défaut, la prime de risque cohérente avec le modèle est nulle !

Dans le chapitre 4, j'essaye de déterminer si le fossé entre les modèles SOE-RBC et les modèles de défaut endogène peut être comblé d'une façon simple. L'idée est d'introduire une possibilité de défaut dans un modèle SOE-RBC, sans pour autant briser la simplicité du paradigme RBC. L'idée est la suivante : à côté du modèle SOE-RBC original, j'introduis une fonction valeur qui correspond à ce que le pays obtiendrait s'il faisait défaut (en utilisant les outils de modélisation habituels du cadre Eaton and Gersovitz (1981)), et je compare cette fonction valeur du défaut à celle obtenue avec le modèle SOE-RBC (qui par construction ne prévoit pas la possibilité d'un défaut). De cette façon, il est possible de calculer une probabilité de défaut quasi-cohérente avec le modèle. Bien entendu cette probabilité n'est pas entièrement cohérente car le pays n'internalise pas le fait qu'il pourrait faire défaut dans le futur. Mais il n'est pas possible de faire mieux dans le cadre du paradigme RBC.

Une fois que cette extension a été ajoutée au modèle SOE-RBC, je regarde si le modèle résultant est capable de répliquer des probabilités de défaut et des niveaux de dette proches de ce qui est observé dans les données. Cet exercice permet également de répondre à une autre question, celle de savoir si les modèles SOE-RBC sont auto-cohérents lorsqu'ils font l'hypothèse qu'il n'y a jamais de défaut. Bien entendu, la réponse à ces deux questions ne peut pas être simultanément positive : soit le modèle a des probabilités de défaut réalistes et il n'est alors pas auto-cohérent, soit c'est l'inverse.

Les résultats auxquels je parviens montrent que, même si pour certaines valeurs des paramètres le modèle étendu est auto-cohérent en ce sens qu'il ne prédit aucun défaut, il lui est en tout cas difficile de reproduire des valeurs compatibles avec les données. En fonction des paramètres, la probabilité de défaut prédite par le modèle RBC étendu est en effet soit bien trop basse, soit bien trop élevée. Le modèle se comporte d'une façon très dichotomique, se situant soit dans un extrême soit dans l'autre.

Ainsi, en reprenant la calibration de Aguiar and Gopinath (2007), le pays se retrouve à accumuler une richesse extérieur nette moyenne égale à 65% de son PIB. Par voie de conséquence, la probabilité de défaut est quasi-nulle (elle n'est pas exactement nulle car le ratio dette sur PIB est très volatil et passe parfois en territoire positif). Ce résultat est la conséquence de deux effets : un besoin d'épargne de précaution, et une faible élasticité du taux d'intérêt au niveau d'endettement. Il est intéressant de noter que la plupart des auteurs ont généralement en tête l'idée que le modèle SOE-RBC standard prédit une position débitrice. Le problème semble venir de la méthode de résolution habituellement utilisée dans la littérature : la plupart des papiers linéarisent leur modèle, ce qui fait disparaître le motif d'épargne de précaution ; ici j'utilise une approximation du second ordre qui fait apparaître ce comportement prudent et engendre donc une accumulation de richesse.

À l'inverse, en modifiant les paramètres de la fonction de prime de risque pour la rendre plus rigide, il est possible d'obtenir une probabilité de défaut très élevée (jusqu'à 60% en termes trimestriels).

Il est ainsi clair que le modèle ne reproduit pas naturellement des probabilités de défaut réalistes, tout du moins au voisinage des calibrations de référence étudiées. Ce résultat soulève des interrogations sur la pertinence des modèles RBC pour l'étude des fluctuations du cycle des affaires dans les pays émergents où les *spreads* de taux d'intérêt jouent un rôle important. En particulier, si certains modèles SOE-RBC sont capables de produire un *spread* contracyclique, il est cependant difficile de soutenir que ce résultat a un sens économique lorsqu'il s'avère que le pays est un créditeur net la plupart du temps et qu'il devrait donc faire face à un taux d'intérêt constant.

À l'issue de cet exercice, il semble donc clair que les modèles SOE-RBC, bien que très utiles pour obtenir une meilleure compréhension du cycle des affaires dans les pays émergents, ne semblent pas amendables de façon simple afin de leur faire reproduire des faits stylisés relatifs à la dette souveraine. L'endogénéisation de la décision de défaut dans les modèles à la Eaton and Gersovitz (1981) n'est donc pas un accessoire ajouté pour la beauté du modèle ou son auto-cohérence : c'est un ingrédient qui modifie de façon déterminante le comportement du modèle. Une direction future de recherche dans le cadre des modèles SOE-RBC pourrait ainsi être d'améliorer la fonction de prime de risque (par rapport à celle utilisée dans ce chapitre et dans le reste de la littérature), car il est clair que c'est cette fonction qui conditionne les résultats relatifs à la dette.

Accélérer la résolution des modèles de dette souveraine avec une méthode de grille endogène

Le choix d'une méthode de résolution numérique est une décision importante pour l'étude d'un modèle de dette souveraine, comme pour l'étude de tout autre modèle quantitatif.

Comme l'ont montré Hatchondo et al. (2010), une méthode imprécise peut conduire à des erreurs numériques significatives lors de la résolution de modèles de dette souveraine, au point que certaines des principales conclusions d'articles innovants peuvent s'avérer fausses. Disposer d'une méthode suffisamment précise est donc fondamental.

Mais l'économiste est face à un dilemme : une méthode plus précise demande généralement un temps de calcul plus important, et est souvent (bien que pas toujours) plus difficile à implémenter. Dans le domaine des modèles de dette souveraine, le frontière vitesse/précision des méthodes de résolution est particulièrement défavorable en comparaison d'autres classes de modèles, tels que les RBC et DSGE. En effet, les modèles RBC et DSGE bénéficient de techniques avancées basées sur les conditions du premier ordre, tandis que les modèle de dette souveraine ont jusqu'à présent été limités à la méthode plus lente de l'itération sur fonction valeur (VFI). La raison principale de cet état de fait est que les modèles de dette souveraine ne peuvent pas être entièrement spécifiés en terme de conditions du premier ordre car la décision de défaut fait intervenir une comparaison entre deux fonctions valeur ; par conséquent les techniques standard pour les DSGE ne s'appliquent pas et d'autres solutions doivent être adoptées.

Dans le chapitre 5 je présente une nouvelle méthode pour résoudre les modèles de dette souveraine, qui améliore significativement la frontière vitesse/précision existante. Cette méthode est une adaptation aux modèles de dette souveraine de la méthode de grille endogène (EGM) introduite par Carroll (2006) et étendue par Barillas and Fernández-Villaverde (2007). La méthode EGM peut se résumer ainsi : au lieu d'utiliser une grille fixe pour la variable d'état (ici la dette héritée) comme dans la méthode VFI, l'idée d'EGM est d'utiliser une grille fixe pour la variable de contrôle (ici la dette émise aujourd'hui). Pour une valeur donnée de la dette émise, il est possible de déduire la dette héritée en utilisant la condition du premier ordre du problème. Ainsi la grille sur la dette héritée devient endogène, d'où le nom de la méthode. Le gain en performance vient du fait qu'il n'est plus nécessaire de faire une maximisation, mais simplement de résoudre une équation non-linéaire.

Cependant cette méthode ne peut pas s'appliquer directement aux modèles de dette souveraine. Le problème vient du fait que la règle de décision pour la dette émise en fonction de la dette héritée est très « plate » dans ces modèles. Ainsi, il est impossible de déterminer *ex ante* une grille fixe pour la dette émise. La solution consiste à rendre cette grille également endogène par un processus itératif. Pour cette raison, j'appelle « méthode de grille doublement endogène » (2EGM) la technique de résolution qui en découle.

Comme seconde contribution, j'explore la précision des méthodes de résolution (VFI et 2EGM) de manière plus systématique que ce qui a été fait auparavant dans la littérature sur la dette souveraine, en utilisant des tests basés sur les erreurs d'Euler. Le principal résultat auquel je parviens est que VFI comme 2EGM sont capables de calculer des solutions précises au modèle canonique de dette souveraine, mais que 2EGM est bien plus rapide (d'un facteur 5 à 10) que VFI pour un niveau de précision comparable. Par ailleurs, l'implémentation de 2EGM s'avère d'une complexité similaire à celle de VFI.

La disponibilité d'un algorithme rapide et précis comme 2EGM ouvre d'intéressantes possibilités. L'une d'elle est l'étude de modèles de dette souveraine de taille plus grande que ceux habituellement étudiés dans la littérature ; le modèle étudié au chapitre 2 entre justement dans cette catégorie. Les modèles qui seront développées dans un future proche, à la jonction avec la tradition RBC/DSGE comme celui de Mendoza and Yue (2012), auront certainement un espace d'état d'une dimension encore plus grande et pourront donc bénéficier utilement du gain de vitesse offert par 2EGM. Une autre possibilité à explorer est l'estimation de modèles de dette souveraine avec des techniques bayésiennes : comme de telles estimations nécessitent de résoudre le modèle un grand nombre de fois dans l'espace des paramètres, la méthode 2EGM pourrait s'avérer utile dans ce cas également.

Conclusion

Même si j'espère que les contributions de cette thèse aideront à parvenir à une meilleure compréhension des déterminants des crises de dette, beaucoup de questions restent en suspens.

Une première question non résolue consiste à comprendre pourquoi les instruments de dette *conditionnels* ne sont pas plus largement utilisés. Il est clair que le risque de défaut est diminué si on utilise des instruments dont les remboursements sont indexés sur une variable fondamentale affectant la capacité de remboursement du pays. Si, à l'extrême, les marchés étaient complets, alors le défaut serait inexistant. Les instruments conditionnels s'avèrent plus efficaces car ils aident à éviter les coûts sociaux associés aux situations de crise, et on comprend encore mal pourquoi ils sont si peu utilisés.

De nombreuses questions restent également ouvertes ou inexplorées du point de vue de la modélisation du risque souverain. Il serait notamment intéressant de mieux comprendre les conséquences du choix de la monnaie dans laquelle est libellée la dette. Cela pourrait notamment permettre de mieux comprendre les situations où le pays s'endette dans sa propre monnaie comme c'est de plus en plus souvent le cas, ou la situation d'un pays membre d'une union monétaire (comme la Grèce dans la zone Euro) et son incitation éventuelle à quitter l'union.

Une autre direction de recherche intéressante serait d'incorporer des règles fiscales plus élaborées dans les modèles de dette souveraine. Un tel travail est nécessaire afin d'étudier la soutenabilité à long terme de la dette tout en maintenant le caractère endogène du défaut. Bien que certains papiers (Martinez et al., 2012) ont commencé à étudier cette question, beaucoup de travail est encore nécessaire pour parvenir à une modélisation plus réaliste.

Enfin, comme cela a déjà été mentionné, la convergence des traditions RBC/DSGE avec celle des modèles de défaut endogène semble un axe très prometteur.

Sur un plan méthodologique, comme l'ajout de ces différents éléments de modélisation conduira à complexifier les modèles, il sera nécessaire d'améliorer les méthodes de résolution. L'obstacle principal réside dans le fait que toutes les méthodes existantes (y compris 2EGM) sont vulnérables au « fléau de la dimension », et sont donc incapables de résoudre des modèles avec un grand nombre de variables d'état. La solution réside probablement dans l'utilisation de méthodes de grilles clairsemées, de façon similaire à ce qui se fait pour les modèles DSGE de grande taille (Malin et al., 2011).

Acknowledgments

Many people have directly or indirectly contributed to this work.

First I would like to thank my fellow PhD students who were working in the same office than me when I started this work: Mathieu Valdenaire and Clément Carbonnier. Thanks also to all the students at Paris School of Economists with whom I was regularly having exciting discussions: Simon Gueguen, Abla Safir, Barbara Coello, Paula Español, Yannick Kalantzis, Thibault Fally, Diego Moccero, Cécile Valadier.

Later, when I worked at the Banque de France, I benefited from a friendly and stimulating environment thanks to my colleagues at SEMSI: Laure Frey, Alexandre Baclet, Enisse Kharroubi, Frédéric Lambert, Pavel Diev, Édouard Vidon, Sophie Guilloux-Nefussi, Nicolas Gobalraja.

The last part of this thesis was completed while I was working for the Dynare project. I owe a great debt to all the team members who helped me finding the right balance between my strong intellectual interest for macro-economics and my innate attraction towards computational techniques: Michel Juillard, Stéphane Adjemian, Ferhat Mihoubi, Houtan Bastani and Marco Ratto. Thanks also to the other people with whom I worked at Chevaleret: Pablo Winant, Christophe Cahn, Antoine Devulder, Tarik Ocaktan, Pierre Alary, Thomas Weitzenblum, Sumudu Kankanamge, Eleni Iliopoulos, Hanane Bahala.

I would also like to thank Romain Rancière, Xavier Ragot and Emmanuel Farhi, who gave me very useful feedback when I presented preliminary versions of the present work during seminars at Paris School of Economics.

And finally I thank my PhD advisor, Daniel Cohen, for dedicating me a lot of time and for giving me excellent scientific guidance, especially for the crafting of chapters 2 and 3.

Bibliography

Abramowitz, M. and I. A. Stegun (1964): *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, New York: Dover, ninth Dover printing, tenth GPO printing ed.

Adjemian, S., H. Bastani, M. Juillard, F. Mihoubi, G. Perendia, M. Ratto, and S. Villemot (2011): "Dynare: Reference Manual, Version 4," Dynare Working Papers 1, CEPREMAP.

Aguiar, M. and G. Gopinath (2004): "Defaultable Debt, Interest Rates and the Current Account," NBER Working Papers 10731, National Bureau of Economic Research, Inc.

——— (2006): "Defaultable debt, interest rates and the current account," *Journal of International Economics*, 69, 64–83.

——— (2007): "Emerging Market Business Cycles: The Cycle Is the Trend," *Journal of Political Economy*, 115, 69–102.

Alesina, A. and R. Perotti (1995): "The Political Economy of Budget Deficits," *IMF Staff Papers*, 42, 1–31.

Alessandro, M., G. Sandleris, and A. V. D. Ghote (2011): "Sovereign Defaults and The Political Economy Of Market Reaccess," Business School Working Papers 2011-08, Universidad Torcuato Di Tella.

Altug, S. G. and M. Bildirici (2010): "Business Cycles around the Globe: A Regime-switching Approach," CEPR Discussion Papers 7968, C.E.P.R. Discussion Papers.

Applebaum, D. (2004): *Lévy Processes and Stochastic Calculus*, Cambridge studies in advanced mathematics, Cambridge University Press, second ed.

Arellano, C. (2008): "Default Risk and Income Fluctuations in Emerging Economies," *The American Economic Review*, 98, 690–712.

Arteta, C. and G. Hale (2008): "Sovereign debt crises and credit to the private sector," *Journal of International Economics*, 74, 53–69.

Aruoba, S. B., J. Fernández-Villaverde, and J. F. Rubio-Ramírez (2006): "Comparing solution methods for dynamic equilibrium economies," *Journal of Economic Dynamics and Control*, 30, 2477–2508.

Barillas, F. and J. Fernández-Villaverde (2007): "A generalization of the endogenous grid method," *Journal of Economic Dynamics and Control*, 31, 2698–2712.

Benjamin, D. and M. L. J. Wright (2009): "Recovery Before Redemption: A Theory Of Delays In Sovereign Debt Renegotiations," CAMA Working Papers 2009-15, Australian National University, Centre for Applied Macroeconomic Analysis.

Bi, R. (2008): "Beneficial Delays in Debt Restructuring Negotiations," IMF Working Papers 08/38, International Monetary Fund.

Borensztein, E. and U. Panizza (2009): "The Costs of Sovereign Default," *IMF Staff Papers*, 56, 683–741.

Bulow, J. and K. Rogoff (1988): "Sovereign Debt Restructurings: Panacea or Pangloss?" NBER Working Papers 2637, National Bureau of Economic Research, Inc.

——— (1989a): "A Constant Recontracting Model of Sovereign Debt," *Journal of Political Economy*, 97, 155–78.

——— (1989b): "Sovereign Debt: Is to Forgive to Forget?" *American Economic Review*, 79, 43–50.

Byrd, R. H., P. Lu, J. Nocedal, C. Zhu, and C. Zhu (1994): "A Limited Memory Algorithm for Bound Constrained Optimization," *SIAM Journal on Scientific Computing*, 16, 1190–1208.

Calvo, G. A. (1988): "Servicing the Public Debt: The Role of Expectations," *American Economic Review*, 78, 647–61.

Campos, C. F., D. Jaimovich, and U. Panizza (2006): "The Unexplained Part of Public Debt," *Emerging Markets Review*, 7, 228–243.

Carré, S. (2011): "Modèles stochastiques à saut & dette souveraine de court terme," Mémoire de Master 2 – Analyse et Politique Économiques (APE), École Normale Supérieure et École d'Économie de Paris.

Carroll, C. D. (1997): "Buffer-Stock Saving and the Life Cycle/Permanent Income Hypothesis," *The Quarterly Journal of Economics*, 112, 1–55.

——— (2006): "The method of endogenous gridpoints for solving dynamic stochastic optimization problems," *Economics Letters*, 91, 312–320.

Catão, L. and S. Kapur (2004): "Missing Link: Volatility and the Debt Intolerance Paradox," IMF Working Papers 04/51, International Monetary Fund.

CHAMON, M. (2007): "Can debt crises be self-fulfilling?" *Journal of Development Economics*, 82, 234–244.

CHATTERJEE, S. AND B. EYIGUNGOR (2011): "Maturity, indebtedness, and default risk," Working Papers 11-33, Federal Reserve Bank of Philadelphia.

CHUAN, P. AND F. STURZENEGGER (2005): "Defaults Episodes in the 1980s and 1990s: What Have we Learned?" in *Managing Economic Volatility and Crises*, ed. by J. Aizenman and B. Pinto, Cambridge University Press.

COHEN, D. (1992): "The Debt Crisis: A Postmortem," in *NBER Macroeconomics Annual 1992, Volume 7*, National Bureau of Economic Research, Inc, NBER Chapters, 65–114.

——— (1993): "Low Investment and Large LDC Debt in the 1980's," *American Economic Review*, 83, 437–49.

COHEN, D. AND R. PORTES (2006): "Toward a Lender of First Resort," IMF Working Papers 06/66, International Monetary Fund.

COHEN, D. AND J. SACHS (1986): "Growth and external debt under risk of debt repudiation," *European Economic Review*, 30, 529–560.

COHEN, D. AND C. VALADIER (2011): "40 years of sovereign debt crises," CEPR Discussion Papers 8269, Centre for Economic Policy and Research.

COLE, H. L. AND T. J. KEHOE (1996): "A self-fulfilling model of Mexico's 1994–1995 debt crisis," *Journal of International Economics*, 41, 309–330.

——— (2000): "Self-Fulfilling Debt Crises," *Review of Economic Studies*, 67, 91–116.

CRUCES, J. J. AND C. TREBESCH (2011): "Sovereign Defaults: The Price of Haircuts," CESifo Working Paper Series 3604, CESifo Group Munich.

CUADRA, G. AND H. SAPRIZA (2008): "Sovereign default, interest rates and political uncertainty in emerging markets," *Journal of International Economics*, 76, 78–88.

DANIEL, B. C. AND C. SHIAMPTANIS (2008): "Fiscal policy in the European Monetary Union," International Finance Discussion Papers 961, Board of Governors of the Federal Reserve System (U.S.).

DELL'ARICCIA, G., I. SCHNABEL, AND J. ZETTELMEYER (2006): "How Do Official Bailouts Affect the Risk of Investing in Emerging Markets?" *Journal of Money, Credit and Banking*, 38, 1689–1714.

DEPETRIS-CHAUVIN, N. AND A. KRAAY (2005): "What Has 100 Billion Dollars Worth of Debt Relief Done for Low- Income Countries?" International Finance 0510001, EconWPA.

Detragiache, E. and A. Spilimbergo (2001): "Crises and Liquidity - Evidence and Interpretation," IMF Working Papers 01/2, International Monetary Fund.

Diamond, D. W. and P. H. Dybvig (1983): "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy*, 91, 401–19.

Eaton, J. and M. Gersovitz (1981): "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *Review of Economic Studies*, 48, 289–309.

Eaton, J. W., D. Bateman, and S. Hauberg (2008): *GNU Octave Manual Version 3*, Network Theory Limited.

Eslava, M. (2006): "The Political Economy of Fiscal Policy: Survey," RES Working Papers 4487, Inter-American Development Bank, Research Department.

Fink, F. and A. Scholl (2011): "A Quantitative Model of Sovereign Debt, Bailouts and Conditionality," Working Paper Series of the Department of Economics, University of Konstanz 2011-46, Department of Economics, University of Konstanz.

Galassi, M., J. Davies, J. Theiler, B. Gough, G. Jungman, M. Booth, and F. Rossi (2003): *GNU Scientific Library: Reference Manual*, Network Theory Ltd.

Gelos, R. G., R. Sahay, and G. Sandleris (2004): "Sovereign Borrowing by Developing Countries: What Determines Market Access?" IMF Working Papers 04/221, International Monetary Fund.

——— (2011): "Sovereign borrowing by developing countries: What determines market access?" *Journal of International Economics*, 83, 243–254.

Goodwin, T. H. (1993): "Business-Cycle Analysis with a Markov-Switching Model," *Journal of Business & Economic Statistics*, 11, 331–39.

Greenwood, J., Z. Hercowitz, and G. W. Huffman (1988): "Investment, Capacity Utilization, and the Real Business Cycle," *American Economic Review*, 78, 402–17.

Hamilton, J. D. (1989): "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle," *Econometrica*, 57, 357–84.

Hatchondo, J. C. and L. Martinez (2009): "Long-duration bonds and sovereign defaults," *Journal of International Economics*, 79, 117–125.

Hatchondo, J. C., L. Martinez, and H. Sapriza (2007a): "The economics of sovereign defaults," *Economic Quarterly*, 163–187.

——— (2007b): "Quantitative models of sovereign default and the threat of financial exclusion," *Economic Quarterly*, 251–286.

——— (2009): "Heterogeneous Borrowers In Quantitative Models Of Sovereign Default," *International Economic Review*, 50, 1129–1151.

——— (2010): "Quantitative properties of sovereign default models: solution methods," *Review of Economic Dynamics*, 13, 919–933.

Hausmann, R., F. Rodriguez, and R. Wagner (2006): "Growth Collapses," Working Paper Series rwp06-046, Harvard University, John F. Kennedy School of Government.

Heston, A., R. Summers, and B. Aten (2006): "Penn World Table Version 6.2," Tech. rep., Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania.

Imbs, J. and R. Rancière (2005): "The Overhang Hangover," CEPR Discussion Papers 5210, C.E.P.R. Discussion Papers.

Inter-American Development Bank (2007): *Living with Debt: How to Limit Risks of Sovereign Finance*, Economic and Social Progress Report (IPES), IADB.

International Monetary Fund (2006): *International Financial Statistics Yearbook, 2006*, IMF.

Jin, H.-H. and K. L. Judd (2002): "Perturbation methods for general dynamic stochastic models," Unpublished manuscript, Stanford University.

Judd, K. L. (1992): "Projection methods for solving aggregate growth models," *Journal of Economic Theory*, 58, 410–452.

——— (1998): *Numerical Methods in Economics*, MIT Press Books, The MIT Press.

Juillard, M. and S. Villemot (2011): "Multi-country real business cycle models: Accuracy tests and test bench," *Journal of Economic Dynamics and Control*, 35, 178–185.

Kletzer, K. M. (1994): "Sovereign Immunity and International Lending," in *The Handbook of International Macroeconomics*, ed. by F. van der Ploeg, Oxford: Blackwell, 439–479.

Kohlscheen, E. (2007): "Why Are There Serial Defaulters? Evidence from Constitutions," *Journal of Law and Economics*, 50, 713–730.

Kollmann, R., S. Maliar, B. A. Malin, and P. Pichler (2011): "Comparison of solutions to the multi-country Real Business Cycle model," *Journal of Economic Dynamics and Control*, 35, 186–202.

Kraay, A. and V. Nehru (2006): "When Is External Debt Sustainable?" *World Bank Economic Review*, 20, 341–365.

Krugman, P. (1988): "Financing vs. forgiving a debt overhang," *Journal of Development Economics*, 29, 253–268.

LEVY-YEYATI, E. AND U. PANIZZA (2011): "The elusive costs of sovereign defaults," *Journal of Development Economics*, 94, 95–105.

LUCAS, R. E. (1987): *Models of business cycles*, Basil Blackwell.

——— (1990): "Why Doesn't Capital Flow from Rich to Poor Countries?" *American Economic Review*, 80, 92–96.

——— (2003): "Macroeconomic Priorities," *American Economic Review*, 93, 1–14.

MALIN, B. A., D. KRUEGER, AND F. KUBLER (2011): "Solving the multi-country real business cycle model using a Smolyak-collocation method," *Journal of Economic Dynamics and Control*, 35, 229–239.

MANASSE, P. AND N. ROUBINI (2009): "'Rules of thumb' for sovereign debt crises," *Journal of International Economics*, 78, 192–205.

MANASSE, P., A. SCHIMMELPFENNIG, AND N. ROUBINI (2003): "Predicting Sovereign Debt Crises," IMF Working Papers 03/221, International Monetary Fund.

MARTINEZ, L., J. C. HATCHONDO, AND F. ROCH (2012): "Fiscal Rules and the Sovereign Default Premium," IMF Working Papers 12/30, International Monetary Fund.

MENDOZA, E. G. (1991): "Real Business Cycles in a Small Open Economy," *American Economic Review*, 81, 797–818.

MENDOZA, E. G. AND V. Z. YUE (2012): "A General Equilibrium Model of Sovereign Default and Business Cycles," *The Quarterly Journal of Economics*, 127, 889–946.

MITCHENER, K. J. AND M. D. WEIDENMIER (2005): "Supersanctions and Sovereign Debt Repayment," NBER Working Papers 11472, National Bureau of Economic Research, Inc.

MOODY'S (2008): "Sovereign Default and Recovery Rates, 1983–2007," Special comment, Moody's Investors Service.

MURPHY, K. M. AND R. H. TOPEL (1985): "Estimation and Inference in Two-Step Econometric Models," *Journal of Business & Economic Statistics*, 3, 370–79.

NEUMEYER, P. A. AND F. PERRI (2005): "Business cycles in emerging economies: the role of interest rates," *Journal of Monetary Economics*, 52, 345–380.

PATTILLO, C., H. POIRSON, AND L. A. RICCI (2011): "External Debt and Growth," *Review of Economics and Institutions*, 2.

PESCATORI, A. AND A. N. R. SY (2007): "Are Debt Crises Adequately Defined?" *IMF Staff Papers*, 54, 306–337.

PRESBITERO, A. F. (2008): "The Debt-Growth Nexus in Poor Countries: A Reassessment," *Economics - The Open-Access, Open-Assessment E-Journal*, 2, 1–28.

R Development Core Team (2011): *R: A Language and Environment for Statistical Computing*, R Foundation for Statistical Computing, Vienna, Austria, ISBN 3-900051-07-0.

Reinhart, C. M. (2002): "Default, Currency Crises, and Sovereign Credit Ratings," *World Bank Economic Review*, 16, 151–170.

Reinhart, C. M. and K. S. Rogoff (2004): "Serial Default and the 'Paradox' of Rich-to-Poor Capital Flows," *American Economic Review*, 94, 53–58.

——— (2009): *This Time is Different*, Princeton University Press.

——— (2011a): "The Forgotten History of Domestic Debt," *Economic Journal*, 121, 319–350.

——— (2011b): "From Financial Crash to Debt Crisis," *American Economic Review*, 101, 1676–1706.

Reinhart, C. M., K. S. Rogoff, and M. A. Savastano (2003): "Debt Intolerance," *Brookings Papers on Economic Activity*, 34, 1–74.

Rogoff, K. S. (2011): "Sovereign Debt in the Second Great Contraction: Is This Time Different?" *NBER Reporter*, 3, 1–5.

Rose, A. K. (2005): "One reason countries pay their debts: renegotiation and international trade," *Journal of Development Economics*, 77, 189–206.

Rose, A. K. and M. M. Spiegel (2004): "A Gravity Model of Sovereign Lending: Trade, Default, and Credit," *IMF Staff Papers*, 51, 50–63.

Ruiz-Arranz, M., T. Cordella, and L. A. Ricci (2005): "Debt Overhang or Debt Irrelevance? Revisiting the Debt Growth Link," IMF Working Papers 05/223, International Monetary Fund.

Sachs, J. (1989): "The Debt Overhang of Developing Countries," in *Debt, Growth and Stabilisation: Essays in Memory of Carlos Dias Alejandro*, ed. by J. de Macedo and R. Findlay, Oxford: Blackwell, 80–102.

Sachs, J. and D. Cohen (1982): "LDC Borrowing with Default Risk," NBER Working Papers 0925, National Bureau of Economic Research, Inc.

Schmitt-Grohé, S. and M. Uribe (2003): "Closing small open economy models," *Journal of International Economics*, 61, 163–185.

Stähler, N. (2011): "Recent developments in quantitative models of sovereign default," Discussion Paper Series 1: Economic Studies 2011,17, Deutsche Bundesbank, Research Centre.

Sturzenegger, F. and J. Zettelmeyer (2007): *Debt Defaults and Lessons from a Decade of Crises*, MIT Press Books, The MIT Press.

Tomz, M. and M. L. J. Wright (2007): "Do Countries Default in "Bad Times"?" *Journal of the European Economic Association*, 5, 352–360.

Uribe, M. and V. Z. Yue (2006): "Country spreads and emerging countries: Who drives whom?" *Journal of International Economics*, 69, 6–36.

World Bank (2004): *World Development Indicators 2004*, World Bank.

——— (2006a): *Global Development Finance 2006: The Development Potential of Surging Capital Flows*, World Bank.

——— (2006b): *World Development Indicators 2006*, World Bank.

——— (2010): *Global Development Finance 2011: External Debt of Developing Countries*, World Bank.

Yue, V. Z. (2010): "Sovereign default and debt renegotiation," *Journal of International Economics*, 80, 176–187.

Glossary

Roman letters

a_t transitory productivity shock. 101

$\mathcal{B}$ set of constraints over the parameters Θ for maximum likelihood estimation. 97

C_t consumption. 41, 67

C_t^d post-default consumption. 24, 41, 49, 66

C_t^r consumption in case of repayment. 24, 67

$\mathscr{D}$ default set. 67

D, D_t real external debt stock. 23, 40, 65

d, d_{it} debt-to-GDP ratio. 79

$\bar{d}$ steady-state debt-to-GDP ratio in RBC model. 102

$\tilde{D}'$ policy function for tomorrow's level of debt. 25, 42, 50, 69, 116

d^* debt-to-GDP ratio above which the country defaults. 44, 70, 105

$\mathbb{E}$ unconditional expectancy. 23

$\mathbb{E}_t$ expectancy conditional to the information available at date t. 25

$\mathcal{F}(g)$ cumulative density function of the growth rate (chapter 3) or of tomorrow's output given today's (chapter 5). 65, 116

$\mathcal{G}(\Lambda)$ cumulative density function of Λ. 66

$\hat{g}_{it}$ growth gap: expected growth conditionally on the absence of a crisis occurring minus the expected growth conditionally on the occurrence of a crisis. 84

g, g_t, g_{it} gross real growth rate. 23, 47, 79, 101

$g^-(h)$ gross growth rate in the bad state of nature. 45

$g^+(h)$ gross growth rate in the good state of nature. 45

h length of a time period (inverse of frequency). 39

h^* length of time period under which the Brownian process no longer generates defaults. 45

h_0 length of time period for which Brownian and Poisson are observationally equivalent. 45

$\mathcal{I}$ finite indexing set for grid over debt levels. 116

$\mathbb{J}(D', Q)$ expectancy of tomorrow's discounted value function. 119

$\mathcal{J}$ finite indexing set for grid over output levels. 116

J^d value function conditional to default. 24, 42, 68, 104, 116

J^r value function conditional to repayment. 24, 42, 68, 116

J^* value function (incorporating the optimal choice between repayment and default). 24, 42, 67, 116

K_t stock of capital. 101

$k(h)$ term correcting for discretization in the compound Poisson process. 40

$\mathcal{L}_\Theta$ likelihood of an observation. 95

$\mathscr{L}(Q)$ domain of definition of the policy function $\tilde{D}'$. 69

$\tilde{L}$ supply function of borrowing by the international investors, given the demand of the country for bonds due next period. 24, 42, 50, 116

L, L_t amount lent today by creditors in exchange of a promise of future repayments. 40, 66

ℓ_t supply of labor. 101

$\tilde{m}_t$ jump distribution in the compound Poisson process. 40

N "normal times" state. 48

n iteration counter in numerical algorithms. 116

o offset for updating the maximum debt value in 2EGM. 123

$\mathbb{P}$ probability of an event. 67

P_t amount captured by creditors after a default. 66

p in chapter 2, probability of a negative shock (moving to "trembling times") to the Poisson component of growth; in chapter 3, probability that the sunspot ζ is triggered. 47, 80

p_0 rate of the compound Poisson process. 40

Q, Q_t gross domestic product. 23, 40, 47, 65, 101

q probability of a positive shock (moving back to "normal times") to the Poisson component of growth. 47

Q_t^d post-default GDP. 23, 41, 49, 66

$\mathscr{R}$ repayment set. 67

r world riskless interest rate. 23, 26, 40, 65, 102, 105

$\mathcal{S}_\Theta$ *a posteriori* self-fulfilling probability. 96

s aggregate state of the model. 42, 50

T "trembling times" state. 48

TB trade balance. 27, 53

u utility function. 24, 42, 67

V recovery value after a default. 49, 67

$X_{i,t-1}$ regressors in panel regression. 79

x probability of a settlement after a default. 23, 26, 49, 104, 105

y_t Brownian component of growth. 23, 47, 101

z_t Poisson component of growth. 47

Greek letters

α share of labor in production function. 101, 105

β subjective discount factor. 24, 26, 51, 67, 105

Γ_t stochastic productivity trend. 25, 101

γ relative risk aversion. 24, 26, 42, 67, 105

Δ_t spread (over riskless interest rate). 24, 27, 53, 102

δ, δ_{it} credit history (1 if country is barred from financial markets, 0 otherwise). 50, 79

$\tilde{\delta}'$ default policy function (0 in case of repayment, 1 in case of default). 25, 42, 50, 116

ε_t^a innovation of the transitory productivity shock. 101

ε_{it}^d error term in the law of motion of the debt-to-GDP ratio. 75, 79

ε_{it}^δ error term in the equation determining if a debt crisis bursts or not. 79

ε_{it}^g error term in the law of motion of growth. 79

ε_t^y innovation of the Brownian component of growth. 23, 47, 101

ε_t^z innovation of the Poisson component of growth. 47

ζ_{it} sunspot variable (equal to 0 or 1). 80

η estimated coefficients in panel regression. 79

Θ vector of estimated parameters. 95

θ state of the economy, either N ("normal times") or T ("trembling times"). 50

κ depreciation rate of capital. 102, 105

Λ fraction of post-default output that creditors are able to capture. 66

λ loss of output in autarky (as a share of pre-default output). 23, 26, 41, 49, 66, 103, 105

μ percentage drift of geometric Brownian motion. 40

μ_g mean gross growth rate (ignoring Poisson shocks) (except in chapter 4 where it is the steady state growth rate of the stochastic trend). 23, 26, 101, 105

μ_y mean of the Brownian component of growth. 23, 47

ν parameter governing the Frisch elasticity of labor supply in GHH preferences. 101, 105

$\Xi_{t+1|t}$ Panglossian effect in linearized model. 74

ξ correction term to marginal price of debt when default is not smooth. 72

$\pi_{t+h|t}$ probability of default in $t+h$, from the perspective of date t. 41, 67

ρ_a auto-correlation of the transitory productivity shock. 101, 105

ρ_y auto-correlation of the Brownian component of growth. 23, 26, 47, 101, 105

ρ_z auto-correlation of the Poisson component of growth. 47

σ percentage volatility of geometric Brownian motion. 40

σ_a variance of the innovation of the transitory productivity shock. 101, 105

σ_d standard error of the error term in the law of motion of the debt-to-GDP ratio. 79

σ_g standard error of the error term in the law of motion of growth. 79

σ_y variance of the innovation of the Brownian component of growth. 23, 26, 47, 101, 105

τ parameter governing the disutility of labor in GHH preferences. 101, 105

Φ cumulative density function of the standard normal distribution. 95

φ probability density function of the standard normal distribution. 95

ϕ parameter governing the adjustment cost on capital. 102

ψ sensitivity of interest rate spread to debt level. 102, 105

Ω shadow price of foreign assets in terms of welfare units. 72

ω subjective discount rate. 42

List of Figures

List of Tables